DETECTION OF SPREAD-SPECTRUM SIGNALS IN A MULTI-USER ENVIRONMENT

Urbashi Mitra & H. Vincent Poor***

* Department of Electrical Engineering, The Ohio State University

** Department of Electrical Engineering, Princeton University

ABSTRACT

Motivated by a previous study of adaptive multi-user demodulators for Direct-Sequence Spread-Spectrum Multiple Access, detectors for spread-spectrum signals are investigated. Due to the prohibitive complexity of the locally optimum detector for such a stochastic multi-variate signal in impulsive channel noise, moderate complexity, distribution-free detectors are pursued. In particular, the differential SNRs (processing gain) of correlator based structures are determined. This performance measure is apt given the relatively low signal strength of spread digital signals. The numerical results (in the context of the prior investigation of adaptive multi-user demodulators) impel the development of a hybrid detector which is composed of linear and non-linear structures. The asymptotic normality of the test statistics under study is also examined.

1. INTRODUCTION

Code-division multiple-access (CDMA) is emerging as a desirable protocol by which multiple users can simultaneously share a communication channel. CDMA enables the number of potential users to be increased in bursty or fading channels with cellular topologies, making it particularly attractive for applications such as mobile telephony and personal communications. In this work, we investigate the performance of several detectors for spread-spectrum signals modeled as stochastic multi-variate random variables embedded in channel noise.

As CDMA based digital communication networks proliferate, the need to determine the presence of a new user and integrate knowledge of this new user into the detection scheme becomes more important. It is expected that multiple network providers will share the communication space; as a result, security and privacy issues will gain greater prominence. It will be desirable not to broadcast the network determined communication parameters of the new users; thus schemes for integrating the new users into communication will be necessary. While the option for setting aside a separate channel for the transmission of such side information is possible, this results in a waste of bandwidth due to the bursty nature of much telecommunications traffic. In order to accommodate knowledge of new users into a receiver, one must first know that a new user has entered into communication. The techniques developed herein are tested with their application to signals resulting from adaptive multi-user detectors developed in [8] which require a signal detection algorithm to ascertain the presence of a digital signal prior to acquisition and demodulation.

Optimal detection of digitally modulated signals in the presence of Gaussian noise was examined by Krasner in [5]. Although the Gaussian noise model offers mathematical tractability in the determination of performance measures for various detectors it fails to capture all of the characteristics of noise found in many communication environments. Urban mobile-radio channels suffer from man-made electromagnetic noise which exhibits a noise density whose tails decay at a rate slower than that of the Gaussian noise density [7]. These *impulsive* noise sources produce large magnitude observations which must be contended with in the development of signal detectors. In particular, we shall consider non-linear detectors employing a hard-limiting operation. Such thresholding devices have proven to work well in impulsive environments studied previously (see *e.g.* [1]). The current exposition augments previous work [6] on locally optimal detectors. It also considers a multi-variate dependence structure that differs from prior research on detectors for strongly mixing data [3, 4, 10].

This paper is organized as follows. Section 2 describes the model of the random signal to be detected as well as the nature of the impulsive noise that will be considered. Next, in Section 3, we present the detection problem of interest. Moderate complexity, distribution-free detectors are developed in Section 4 and a performance measure (the differential SNR) is determined for these detectors. The performance curves of these detectors are presented in Section 5. Section 6 discusses the asymptotic normality of the detector statistics under investigation and final conclusions are drawn in Section 7.

2. PRELIMINARIES

The multiple access scheme considered is that of CDMA implemented via Direct-Sequence Spread-Spectrum Modulation (DS/SSM). In this form of CDMA, users modulate their transmitted data with a unique spreading sequence. We shall assume that the ambient channel noise is an additive impulsive noise process $(n(t))$. Denote the length N spreading sequence for each user as a_k, with $[a_k]_j \in \{-1, 1\}$. We consider coherent and syn-

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chronous communication. At the receiver, the signal is chip-matched filtered and sampled at the chip rate. We assume the existence of an orthogonalizing linear transformation V such that the signal to be used for detection, x, contains signal information about a particular user K and noise only (such a transformation is developed in [8]), that is,

$$x_i = Vr_i = Vn_i + VA_K b_i^{(K)} a_K = Vn_i + V\theta s_i, \quad (1)$$

where r_i, A_K, and $b_i^{(K)}$ are the received signal, the K'th user's received amplitude and information bit at time i, respectively. θ is a signal-to-noise ratio (SNR) parameter. We note that the matrix V is of size $L \times N$, where $L \leq N$. We assume that the original signal, s_i, is zero mean and stationary, with covariance matrix $\mathbf{E}\{s_i s_i^T\} = C_S$. Thus, after the linear transformation, the covariance matrix of the transformed signal is $R_S = VC_SV^T$.

Focusing on the noise process, we assume that the noise samples, n_i, are independent and identically distributed. We consider an ϵ-mixture model for the impulsive noise. Such densities can be described as follows:

$$f(n) = (1-\epsilon) f_\eta(n) + \epsilon f_I(n), \quad (2)$$

where $f_\eta(n)$ is deemed the *nominal* noise density and $f_I(n)$ is the *interfering* density. The degree of contamination is described by $\epsilon \in [0,1]$. Both the nominal and the interfering density considered in this work will be zero-mean Gaussian densities; however, the ratio of the variances of these two noise processes ($\gamma^2 = \frac{\sigma_I^2}{\sigma_\eta^2}$) will be on the order of 10−100 and will thus exhibit the impulsive aspect of noise seen in practical situations. The performance of various detectors is examined while the average noise variance is held constant and the shape parameters (ϵ and γ^2) are modified, *i.e.* $\sigma^2 = constant = (1-\epsilon)\sigma_\eta^2 + \epsilon\sigma_I^2$.

We shall assume that each component of the noise process prior to the application of V is distributed as $f(n)$ in (2). It can be shown for a given realization of the noise process, ω_i, that $n_i(\omega_i)$ has a Gaussian distribution with zero mean and covariance matrix $C(\omega_i)$,

$$C(\omega_i) = \sigma_\eta^2 \Lambda(\omega_i) + \sigma_I^2 (I - \Lambda(\omega_i)).$$

Clearly the probability of a particular realization ω_i is $p(\omega_i) = (1-\epsilon)^{(tr(\Lambda(\omega_i))}\epsilon^{(N - tr(\Lambda(\omega_i))}$. After the linear transformation V, the noise vector continues to be zero-mean Gaussian, but with covariance matrix $R(\omega_i) = VC(\omega_i)V^T$.

3. THE DETECTION PROBLEM

We shall study the detection of a stochastic signal using M samples of a received vector that is of size $L \times 1$. This is a hypothesis testing problem that can be cast as follows:

$$\begin{aligned} H_0 : x_i &= Vn_i \\ H_1 : x_i &= Vn_i + \theta V s_i \;\; for\, i \in [1, M], \end{aligned}$$

The alternative hypothesis introduces a random mean to the underlying noise process thereby altering the overall variance of the signal. We shall be examining detectors based on real-valued detection statistics, $T_M(x)$, compared to thresholds. The performance criterion that will be used to compare the various detectors under investigation is the *differential SNR*, The differential SNR for the stochastic mean signal case is defined as (see *e.g.* [4]),

$$\xi(T) \equiv \lim_{M\to\infty} \frac{1}{M'} \frac{\left[\lim_{\theta^m \to 0} \frac{\partial^2}{\partial\theta^2} \mathbf{E}_\theta\{T_M(x)\}\right]^2}{\mathrm{Var}_0\{T_M(x)\}}. \quad (3)$$

We note that in the definition above, $x = [x_1, x_2, \cdots, x_M]$. M' considers the number of sample operations being performed.

Embedded in the expression for the differential SNR is the locally optimal detector. It is easily shown the locally optimal detector maximizes the differential SNR. However one can show that the locally optimal detector for the impulsive noise case is impractical to implement due to the fact that one must consider all possible realizations of the noise process (on the order of 2^N terms). Thus we shall consider simpler distribution-independent structures.

4. DISTRIBUTION FREE DETECTORS

Thus impelled by the previous discussion, we consider several moderate complexity detectors whose implementations are independent of the distributions of the various random vectors. We shall study correlator based structures; we forego study of generalized energy detectors as we have observed them to have performance inferior to the correlator structures. The simple correlator is described as follows:

$$T_{SCO}(x) = \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} x_i^T x_j.$$

Due to the fact that the correlator detector is the sum of $O(\frac{M^2}{2})$ elements, $M'_{SCO} = \frac{M^2}{2}$. Note that this value of M' is consistent with the interpretation of our performance measure as an asymptotic average signal-to-noise ratio. For the remainder of this work, we shall assume that each sample of the noise process is independent and identically distributed; generalizing the subsequent results to the non-identically distributed case is straightforward. The differential SNR for the simple correlator detector is,

$$\xi(T_{SCO/mix}) = \frac{4\,(tr(R_S))^2}{\sigma^4 tr\,([VV^T]^2)}.$$

Note that the performance of the simple correlator is independent of the shape parameters, ϵ, γ^2.

We next examine the *polarity coincidence correlator*. Sign-based detectors have proven useful in combating impulsive noise [1]. The large magnitude observations that occur with significant probability are mitigated by the thresholding operation. The polarity coincidence correlator under study has the following form:

$$T_{PCC}(x) = \sum_{i=1}^{M}\sum_{j=i+1}^{M}\sum_{k=1}^{L} sgn(x_{ik})\, sgn(x_{jk}).$$

The differential SNR of the polarity coincidence correlator is,

$$\xi(T_{PCC/mix}) = \frac{4\left(tr\left(R_S\left[\mathbf{E}_\omega\{\Gamma(\omega)\}\right]^2\right)\right)^2}{\sum_{k=1}^{L}\sum_{l=1}^{L}\left[\mathbf{E}_\omega\left\{\arcsin\frac{\rho_{kl}(\omega)}{\sigma_k(\omega)\sigma_l(\omega)}\right\}\right]^2},$$

where $\sigma_k(\omega) = \sqrt{V_k^T\Lambda(\omega)V_k}$, $\rho_{kl}(\omega) = V_k^T\Lambda(\omega)V_l$, and V_k = k'th column of V. In addition, $\Gamma(\omega) = diag[\frac{1}{\sigma_1(\omega)}, \ldots, \frac{1}{\sigma_L(\omega)}]$.

The polarity coincidence correlator performs favorably if the background noise has an extremely impulsive shape; while the simple correlator performs well if the noise density is more Gaussian in nature. Thus, we consider the differential SNR of a composite structure, the *hybrid correlator*,

$$T_{HYB}(x) = \kappa\, T_{SCO}(x) + (1-\kappa)\, T_{PCC}(x),$$

where $\kappa \in [0,1]$. Although it is straightforward to optimize κ with respect to the differential SNR, it shall be observed that this detector with a fixed κ can provide consistent performance for a range of shapes of the impulsive noise density. We also note that Modestino [9] investigated an adaptive structure to determine κ for a similar detector for univariate data. The differential SNR is,

$$\xi(T_{HYB}) = \left\{2\kappa\, tr(R_S) + \frac{4}{\pi}(1-\kappa) tr\left(R_S\left[\mathbf{E}_\omega\{\Gamma(\omega)\}\right]^2\right)\right\}^2 \times \left\{\kappa^2\,(1-\epsilon)\sigma^4 tr(B^2) + \sum_{k=1}^{L}\sum_{l=1}^{L}\frac{4}{\pi^2}(1-\kappa)^2 \times \left[\mathbf{E}_\omega\left\{\arcsin\frac{\rho_{kl}(\omega)}{\sigma_k(\omega)\sigma_l(\omega)}\right\}\right]^2 + \frac{4}{\pi}\kappa(1-\kappa)\, tr\left(\left(\mathbf{E}_\omega\{\Gamma(\omega)R(\omega)\}\right)^2\right)\right\}^{-1}.$$

5. PERFORMANCE

The performance of the distribution independent detectors is studied in the context of the projection adaptive decorrelator introduced in [8] for CDMA communication implemented via DS/SSM. In order to accommodate a new user, the received multi-user signal is projected into the null-space of the existing users. This results in a residual signal of the form in (1) which can be employed to determine the presence of a new user.

The system under examination employed m-sequences of length $N = 15$ as spreading codes. We note that the length of the spreading sequence used in the investigation is limited by computational and memory capacity as the determination of the differential SNRs of the non-linear detectors require averaging over the 2^N realizations of the impulsive noise process. A topic for future study is to quantify the effect of truncating the averaging process to ignore low probability events.

The average per bit SNR was kept constant at 8dB. Figures 1 and 2 show performance of the cross-correlator structured distribution-free detectors for a two different impulsive noise shapes. As predicted, the hybrid correlator with optimized κ provides the optimal performance. However, it is worthy to note that the hybrid correlator with fixed parameter $\kappa = 0.5$ achieved consistently good performance for these two scenarios. Other shapes for the impulsive noise were studied and the trends observed above continued to hold.

6. ASYMPTOTIC NORMALITY OF TEST STATISTICS

The differential SNR defined in (3) is sometimes employed to calculate an asymptotic probability of detection for the test statistic in question. This calculation is greatly simplified by the reliance on the asymptotic normality (as $M \to \infty$) of the test statistic. In this section, we discuss the issue of asymptotic normality for the investigated test statistics; we shall assume that each sample vector is identically and independently distributed under both hypotheses.

We note that the correlator statistics cannot be written as a single sum of random variables indexed by a single counter (*e.g.* $T = \sum_{i=1}^{N} Z_i$). Prior work on deriving central limit theorems for dependent random variables hinged on the ability to write the desired statistic as such a sum. Thus we do not pursue the proof of asymptotic normality of those test statistics at this time. However, we are able to show asymptotic normality for a modified cross-correlative structure. In particular, we consider the following statistic,

$$T_{CCMOD}^{M}(x) = \sum_{i=1}^{M}\left(\sum_{j=1+(i-1)p}^{(i-1)p+r-1}\ \sum_{k=j+1}^{(i-1)p+r} g(x_j)^T g(x_k)\right),$$

where $g(\cdot)$ is the appropriate function for the detector of interest. Note that the statistic can be written in the following form: $T_{CCMOD}^{M}(x) = \sum_{i=1}^{M} Z_i'$. This detector determines the original statistic within a *sliding window* of length r. The statistic from each window is summed to form the final statistic. For practical purposes we assume that $r \geq p$. Let m be the smallest integer such that $mp > r$. Clearly the Z_i' defined above are m-dependent, also note that the sequence of Z_i' is stationary. It is a simple matter to show that the conditions for central limit theorems for strongly mixing, and thus m-dependent random variables (see *e.g* [2]) are met by the Z_i' above. Thus the statistic $T_{CCMOD}^{M}(x)$ is asymptotically normal as $M \to \infty$.

We next consider what would be lost with respect to the differential SNR by considering a test statistic of the form of $T^M_{CCMOD}(x)$ versus $T^M_{CC}(x)$. It can be shown that the ratio of the differential SNR of the original statistic with respect to the modified statistic is (this ratio is termed the *asymptotic relative efficiency*),

$$ARE_{CCMOD/CC} = \frac{1}{1+\frac{4(r-p)}{p^2}}. \quad (4)$$

Thus there is a loss in differential SNR, *unless* $r = p$. This is the scenario where the statistic is calculated over windows that do not overlap. There is no *asymptotic* loss as each Z'_{ij} is identically distributed. It is clear that the same results hold in the impulsive noise scenario as well.

7. CONCLUSIONS

In this paper we have developed techniques for the detection of a stochastic multi-variate signal embedded in dependent impulsive noise. We considered distribution-free correlator based detectors. After calculating the differential SNR for the simple correlator and the polarity coincidence correlator, a hybrid correlator structure composed of a weighted sum of these two detectors was proposed and investigated. This hybrid correlator proffers consistently favorable performance for a variety of impulsive noise density shapes. The asymptotic normality of the detectors under study was investigated. It was demonstrated that a modification of the correlator structure yielded asymptotically normal statistics with the same differential SNR. This result enables one to consider correlator structures that employ mutually independent pairs of independent samples of the received signal, thus simplifying calculation of the differential SNR.

As a final comment, we note that although the current investigation was motivated by the need to detect a spread-spectrum signal, the results are easily generalized for the detection of a large class of stochastic multivariate signals.

8. REFERENCES

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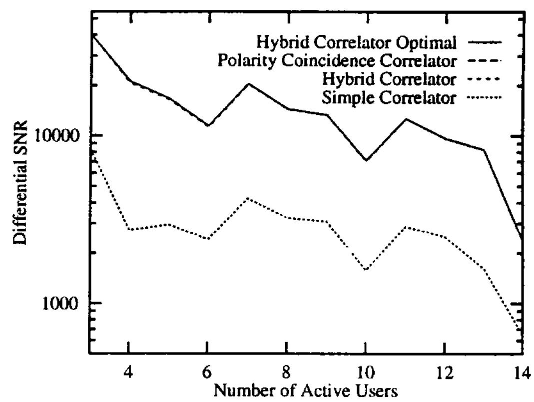

Figure 1: Differential SNRs of various detectors operating in very impulsive noise as a function of the number of active users; $\epsilon = 0.09$ and $\gamma^2 = 90$.

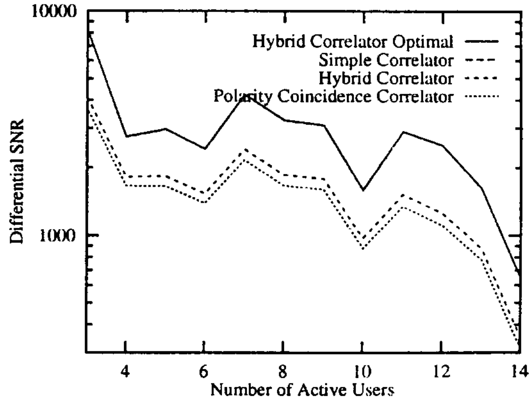

Figure 2: Differential SNRs of various detectors operating in mildly impulsive noise as a function of the number of active users; $\epsilon = 0.01$ and $\gamma^2 = 10$.