A Method for Reducing Computations in Cyclostationarity-Exploiting Beamforming *

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Abstract

The existing self coherence restoral (SCORE) beamforming techniques have been shown to be capable of blindly extracting a desired signal in the presence of unknown noise and interference by exploiting the cyclostationarity of the signal of interest. The versions of SCORE which offer the best convergence properties require computation of the observed data cyclic correlation matrix. This can be a large computational burden, particularly if the number of antennas in the array is large. This paper introduces a method which requires only a column-wise subset of the cyclic correlation matrix. It is shown that in many cases the new method performs as well as existing SCORE methods yet requires many fewer computations.

1 Introduction

The problem of cochannel interference is becoming increasingly important as spectral crowding increases and as the desire to increase capacity grows. Beamforming is an especially effective means for reducing cochannel interference. Applications where beamforming can be employed include signal reconnaissance, personal communications systems (PCS), and cellular communication systems. The class of self coherence restoral (SCORE) algorithms have been shown to be capable of extracting a desired signal without requiring the use of a known training signal, array calibration data, or knowledge of the spatial characteristics of the background noise and interference [1]. This algorithm seeks a beamformer weight vector that maximizes some measure of the cyclic feature strength of the beamformer output. If the environment does not contain correlated desired signals, this is essentially equivalent to maximizing the output SINR. Even in situations where coherent multipath is present, SCORE can be used to find an initial weight vector for other algorithms, such as CMA, that perform well in multipath but lack the signal selectivity of SCORE [2].

One drawback of existing SCORE techniques is the large number of computations required to compute the beamformer weight vector. A significant portion of these computations are used to form the observed data cyclic correlation matrix. This matrix is formed of the cyclic cross correlations between the data observed at all sensors in the array. As the number of sensors becomes large, forming the cyclic correlation matrix becomes computationally expensive. This paper introduces a new method that performs like cross-SCORE yet requires only a column-wise subset of the cyclic correlation matrix. The derivation of the new method is accomplished using the Programmable Canonical Correlation Analyzer (PCCA) framework [3].

2 Overview of SCORE

There are several different versions of SCORE, with each version behaving differently depending on the environment. The simplest version is Least Squares (LS) SCORE. The weight vector for this version of SCORE is typically computed as

$$\mathbf{w}_{ls} = \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{r}^{\alpha} \tag{1}$$

where $\mathbf{r}^{\alpha}$ is an $M \times 1$ vector given by

$$\mathbf{r}^{\alpha} = \left\langle \mathbf{x}(n)\, x_i(n-\tau) e^{-j2\pi\alpha n} \right\rangle, \tag{2}$$

$\mathbf{x}(n)$ is the $M \times 1$ vector of observed data, α is the cycle frequency being exploited, τ is the lag parameter, and $\langle\cdot\rangle$ denotes a time averaging operation. If only the desired signal exhibits cyclostationarity at cycle frequency α, then $\mathbf{r}^{\alpha}$ approaches the array response vector of the desired signal as the collect time approaches infinity. Thus $\mathbf{w}_{ls}$ weight vector converges to the optimal weight vector. However, in low SIR environments the convergence is very slow. In such environments, the cross-SCORE method will outperform LS-SCORE. Cross-SCORE can be motivated from the PCCA framework, as described below.

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The PCCA framework yields a set of weight vectors that maximize the correlation between a linear combination of the two sets of data $\mathbf{x}(n)$ and $\mathbf{y}(n)$. Typically $\mathbf{x}(n)$ is the observed data and $\mathbf{y}(n)$ is a training data set obtained through some transformation of the observed data. The PCCA weight vectors $\mathbf{W}_x$ for the observed data are given by the dominant eigenvectors of the matrix

$$\mathbf{T}_x = \mathbf{R}_{xx}\mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}. \quad (3)$$

The linear combiner weights $\mathbf{W}_y$ for the training data set $\mathbf{y}(n)$ are given by the dominant eigenvectors of the matrix

$$\mathbf{T}_y = \mathbf{R}_{yy}\mathbf{R}_{yx}\mathbf{R}_{xx}^{-1}\mathbf{R}_{xy}. \quad (4)$$

Note that the combiner weights for the training data set do not need to be computed. The vectors $\mathbf{W}_x$ are used as the beamformer weight vectors. The cross-SCORE algorithm can be derived using a frequency shifted (and possibly time delayed) version of the observed data for the training set. Thus for cross-SCORE $\mathbf{y}(n) = \mathbf{x}(n-\tau)e^{j2\pi\alpha n}$. Substituting this into (3) yields the matrix used in the cross-SCORE eigenequation

$$\mathbf{R}_{xx}^{-1}\mathbf{R}_{xx}^{\alpha}\mathbf{R}_{xx}^{-1}\left(\mathbf{R}_{xx}^{\alpha}\right)^{H} \quad (5)$$

where

$$\mathbf{R}_{xx}^{\alpha} = \left\langle \mathbf{x}(n)\,\mathbf{x}^{H}(n-\tau)e^{-j2\pi\alpha n}\right\rangle \quad (6)$$

is a finite-time estimate of the cyclic correlation matrix of $\mathbf{x}(n)$ for cycle frequency α and lag τ. Note that the vector $\mathbf{r}^{\alpha}$ used in LS-SCORE is the ith column of $\mathbf{R}_{xx}^{\alpha}$. A key point is that cross-SCORE requires computation of the complete $M \times M$ observed data cyclic correlation matrix, while LS-SCORE requires only one column of this matrix.

The PCCA framework can also be used, for example, to develop SCORE methods which exploit multiple cycle frequencies. In this case the training set $\mathbf{y}(n)$ has a larger dimension than the observed data set. The training set can be obtained by a number of other transformations, such as filtering, as well. This makes the PCCA useful as a general framework for developing blind adaptive algorithms.

It should be noted that neither cross-SCORE nor LS-SCORE is capable of separating multiple signals having the same cyclic feature. The phase-SCORE algorithm is capable of separating multiple signals exhibiting the same cyclic feature if those cyclic features have different phase. This would be the case, for example, with multiple bauded signals having the same baud rate but different baud timing. Unfortunately, phase-SCORE can not be motivated from the PCCA framework. The remainder of this discussion will assume that the desired signal is the only incident signal that exhbits the cyclic feature being exploited.

Results will be presented later for what will be referred to here as *principal components* versions of SCORE [4]. These versions are obtained by replacing the inverse correlation matrix of (1) and (5) with the pseudo-inverse. In a beamforming context, this is equivalent to constraining the weight vector to lie in the signal subspace. This modification can dramatically improve convergence in some cases.

3 Description of New Method

It is proposed here that a subset of $L < M$ sensors be used to form the training signal. This reduces the dimension of $\mathbf{R}_{xy}$ from $M \times M$ to $M \times L$. As an example, assume that sensors #1 and #2 are used to form the training signal. Examination of the general PCCA expression shows that for this particular choice of training signal, $\mathbf{R}_{xy}$ is equal to the first two columns of $\mathbf{R}_{xx}^{\alpha}$, and $\mathbf{R}_{yy}$ is a 2×2 sub-matrix of $\mathbf{R}_{xx}$. The technique can be described using a notation similar to MATLAB™ by the following. Denote by $\mathbf{k}$ the vector of sensor indices to be used. For the example above, $\mathbf{k} = [1\ 2]$. Then a matrix consisting of the first two columns of $\mathbf{R}_{xx}^{\alpha}$ is denoted by $\mathbf{R}_{xx}^{\alpha}(:,\mathbf{k})$. Using this notation, the dominant eigenvector of the following $M \times M$ matrix can be used in a manner analogous to cross-SCORE:

$$\mathbf{R}_{xx}^{-1}\mathbf{R}_{xx}^{\alpha}(:,\mathbf{k})\left\{\mathbf{R}_{xx}(\mathbf{k},\mathbf{k})\right\}^{-1}\left\{\mathbf{R}_{xx}^{\alpha}(:,\mathbf{k})\right\}^{H}. \quad (7)$$

Algorithms with the above form will be referred to here as subset-SCORE algorithms, because they use a subset of the cyclic correlation matrix. Note that cross-SCORE requires the inversion of $\mathbf{R}_{xx}$ (either explicitly or implicitly through the use of, e.g., QR decomposition and back substitution). The method described above requires inverting $\mathbf{R}_{xx}(\mathbf{k},\mathbf{k})$ as well. However, if $\mathbf{k}$ is small, this second required inversion may have a deterministic form (as for a 2×2 matrix) or may require a relatively small number of computations.

Obviously this approach reduces the number of computations required. The question that must be answered is how does this affect overall performance. In an environment with strong interferers, the training signal weight vector $\mathbf{W}_y$ rejects the interferers and extracts the portion of the training signal that is most strongly correlated with the output of $\mathbf{W}_x$. If $\mathbf{W}_y$ cannot reject the interference, performance will be degraded. In cross-SCORE, this is not an issue because $\mathbf{W}_y$ has the same aperture and same degrees of freedom as $\mathbf{W}_x$. When a subarray is used to

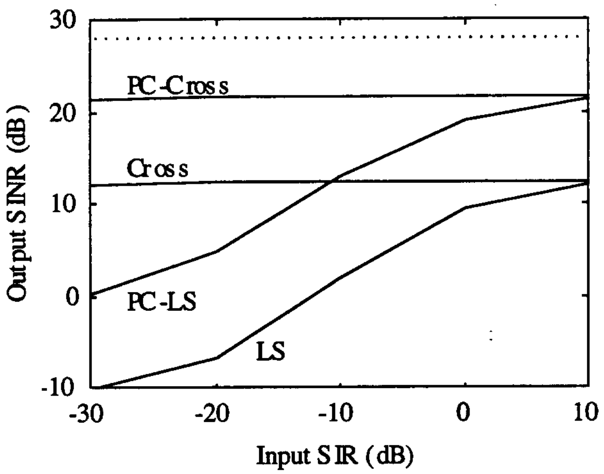

Figure 1: Comparison of several different versions of SCORE as a function of input SIR. Results shown for Principal Components Cross SCORE, Cross SCORE, Principal Components Least Squares SCORE, and Least Squares SCORE. The dotted line indicates the optimal output SINR.

form the training signal, $\mathbf{W}_y$ does not have the same aperture and degrees of freedom. Thus it would be expected that if the 'training array' cannot remove a strong interferer, the performance will be degraded. In such a case, the algorithm will not fail completely but will have performance similar to LS-SCORE. It can be seen that selection of the sensors to use in generating the training signal can have an impact on performance. It might be argued that simply reducing the number of sensors M used in the beamformer would have the same effect, i.e., reduce the number of computations and yield similar performance. However, reducing the number of sensors in the array can greatly limit the ability of the beamformer to separate signals.

4 Simulation Results

A general comparison of the SCORE methods described above will now be conducted through computer simulations. In all simulations an 8 element circular array configuration is used, with the diameter equal to one wavelength of the carrier frequency of the incident signals. The narrowband array model is used for the incident signals. A single 20 dB QPSK desired signal is incident from 0°. The baud rate is 0.25 relative to the sample rate of unity. The QPSK signal is generated using a 100% excess bandwidth Nyquist pulse shape. The cyclic correlation of the desired signal at the baud rate is exploited with the lag parameter τ set to zero. A varying number of gaussian interferers will also be incident on the array. Where results are presented for principal-components versions of SCORE, the number of incident signals is assumed to be known. The effect of varying the input SIR will be examined first.

It has been stated that the main advantage of cross-SCORE over LS-SCORE is the former's faster convergence, particularly in low SIR environments. This is clearly illustrated in Figure 1. This figure shows the mean output SINR (based on 100 independent trials) of several versions of SCORE as a function of input SIR. A single gaussian interferer is incident from 30°. The integration time is fixed at 1024 baud. Note that the performance of the PCCA-type methods (cross-SCORE and the principal components version of cross-SCORE) is essentially independent of input SIR. This is because the weight vector used to form the training signal is able to steer a null on the interferer. In contrast, LS-SCORE must completely rely on the frequency shift operation to decorrelate the interferer.

The performance of subset-SCORE and existing SCORE methods will now be compared. The environment is identical to that considered above with the power of the gaussian interferer set to 50 dB. This corresponds to the lowest input SIR considered earlier. Since the environment contains only one interferer, using two sensors in the training array should be sufficient. Figure 2 shows a comparison of cross-SCORE and subset-SCORE when two columns of the cyclic correlation matrix are computed. The subset-SCORE method uses two adjacent sensors to form the training signal. As can be seen, the performance is *nearly identical.* Thus there is no advantage to be gained from computing the entire cyclic correlation matrix. In this two signal environment, the two sensor subarray used to form the training signal could also be used to generate the actual beamformer output. However, if this were done the optimal output SINR would be only 14.4 dB, compared to the 28.1 dB optimal output SINR that can be achieved using the entire array. Figure 2 demonstrates that a two-sensor training signal can be used in principal-components versions of SCORE as well. Again the performance is nearly identical.

To further illustrate the behavior of the subset-SCORE approach, a more severe environment will be considered. This environment has four gaussian interferers in addition to the QPSK desired signal. The interferers are incident from 30°, −100°,−160°, and 120°, with SWNR of 50 dB, 50 dB, 20 dB, and 20 dB respectively. Note that two of the interferers are 30 dB stronger than the desired signal, while the other two are equally as strong as the desired signal.

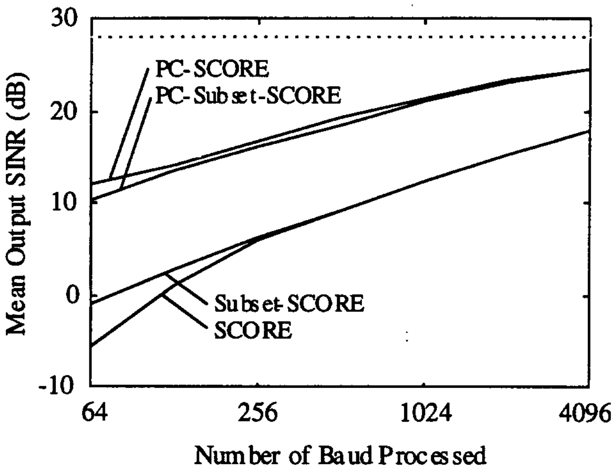

Figure 2: Comparison of: cross-SCORE, principal-components SCORE, and versions of these algorithms which use only two columns of the cyclic correlation matrix. The dotted line indicates the optimal output SINR.

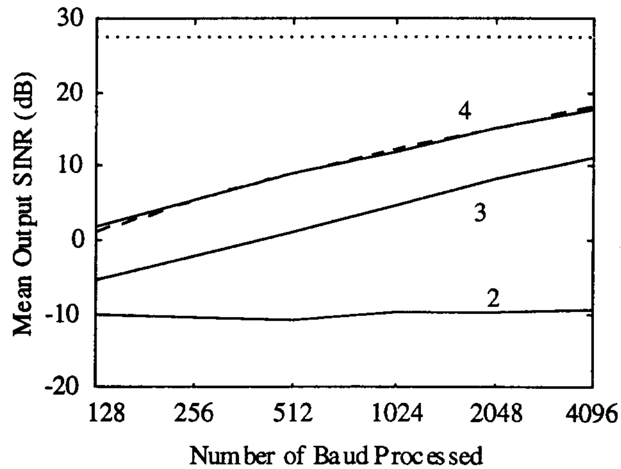

Figure 3: Comparison of cross-SCORE (dashed line) and subset-SCORE (solid lines) when four interferers are present. Parametric in the number of sensors used in the training array.

Results are shown in Figure 3. This plot is parametric in the number of sensors used in the training array, i.e., the number of columns of the cyclic correlation matrix that are computed. When 2 columns are used, the training array has only one degree of freedom, and the strong interferers can not be removed from the training signal. Thus the performance is very poor. When 3 columns are used, there are sufficient degrees of freedom to remove the strong interferers, but not the two weaker interferers. Thus the performance improves, but is still not as good as cross-SCORE. Interestingly, adding one more degree of freedom gives performance almost identical to that of cross-SCORE. Note that in this case, the training subarray is actually overloaded (four sensors, five signals). The general conclusion drawn from these simulation results is that the subset approach using only two columns will perform about as well as cross-SCORE except in very severe environments.

5 Conclusions

A new version of SCORE has been described that performs well in low SIR environments, like cross-SCORE, yet does not require computation of the complete observed data cyclic correlation matrix. This new version of SCORE is referred to as subset-SCORE, because it requires only a column-wise subset of the cyclic correlation matrix. In many cases as few as two columns is sufficient. Subset-SCORE is derived from the PCCA framework by reducing the number of sensors used to obtain the SCORE training signal set. In general, this procedure can used in any beamforming method that can be motivated from the PCCA framework. One drawback of this approach is that its usefullness is limited to environments where a single desired signal exhibits cyclostationarity at the cycle frequency being exploited.

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