ADAPTIVE PERIOD ESTIMATION OF A CLASS OF PERIODIC RANDOM PROCESSES

Joanna M SPANJAARD[1] Langford B WHITE[1,2]

[1]Cooperative Research Centre for Robust and Adaptive Systems

[2]Signal Analysis Discipline, Comms Division, Defence Science and Technology Organisation
PO Box 1500 Salisbury SA 5108, AUSTRALIA.
E-mail jms@cd.dsto.gov.au

ABSTRACT

The problem of period uncertainty when evaluating spectrum estimates for wide sense cyclostationary processes is addressed in this paper. In particular, the extended Kalman filter (EKF) and a parallel bank of Kalman filters are investigated as different methods for adaptive estimation of a time-varying period. An example is given concerning an AR(1) process and a number of time-varying periods are adaptively tracked for different periodic functions. Convergence characteristics are also assessed. Finally, a combined detection-estimation approach is also investigated.

1. INTRODUCTION

This work is concerned with addressing the problems associated with period uncertainty when evaluating spectrum estimates for a class of periodic random processes known as wide sense cyclostationary (*wsc*) processes. Such processes can be found in a variety of disciplines ranging from rotating machinery to electrical communications. The influence of period uncertainty has previously been investigated in [1] and [2], providing the motivation for consideration of adaptive tracking algorithms for period estimation. By tracking a time-varying period, it may be possible to improve the estimate of the spectra density of a *wsc* process. Previous research [3] compares means of parameter estimation for non-stationary processes but only offers heuristic advice concerning the methods considered and the rate of convergence to the true parametric values. Period estimation is not considered. This paper investigates the use of the extended Kalman filter (EKF) [1],[4], for adaptive period estimation. An alternative method is also studied whereby a bank of parallel Kalman filters are employed (sometimes known as the Magill filter [5] or the multiple model approach [6]), each tuned to a particular period. Detection theory is then used to identify the true period. Convergence properties are available for this form of estimation [6] via parallel processing. Results are presented which compare the thresholding properties of both these methods to assess convergence performance. Time-varying periods are then adaptively tracked for a number of different periodic functions. A combined detection-estimation approach is also investigated. Detection is performed via the Magill filter determining the right local region of the period and utilising the filter's convergence properties. The EKF is then employed to fine tune the period estimate by adaptively tracking the time-varying period.

2. PROBLEM FORMULATION

A random process X_t is said to be a wide sense cyclostationary process if both its mean $E(X_t) \triangleq \mu_X(t)$ and autocorrelation $E(X_t, X_{t+\tau}) \triangleq R_X(t, t+\tau)$ are periodic functions of the variable t. A *wsc* system can then be described as follows,

$$\begin{aligned} X_{t+1} &= A(d_t)X_t + B(d_t)U_t \\ Y_t &= CX_t + W_t \end{aligned} \quad (1)$$

where X_t and U_t are n-dimensional, d_t and Y_t are one-dimensional, $A(\cdot)$ and $B(\cdot)$ are twice differentiable, d-periodic matrix valued functions, and U_t and W_t are white noise processes with known covariance and zero mean. For these investigations, a scalar AR(1) process with

$$A(d_t) = 0.9 + 0.1\cos(2\pi t / d_t)$$

$$B(d_t) = 0.2\sin(2\pi t / d_t)$$

is simulated and adaptive tracking of the period investigated via extended Kalman and Magill filtering.

0-7803-2431-5/95 $4.00 © 1995 IEEE

3. THE EXTENDED KALMAN FILTER

Adaptive estimation using extended Kalman filtering is performed by viewing the period as an additional state of the signal model (Eq. 1) and modelling it as a random walk (an AE(1) process)

$$d_{t+1} = d_t + \beta V_t \tag{2}$$

where V_t is a white noise process with known covariance and zero mean. Augmentation of the system in this way gives the well-known extended Kalman filter the ability to estimate slowly varying parameters. The period enters non-linearly in d_t so the model, Eq. (1), together with Eq. (2) is inherently non-linear. The EKF equations [4] are then applied to the augmented non-linear model.

4. THE MAGILL FILTER

The Magill filter [5] is a process which assumes the unknown period belongs to a discrete set $\{d_1, \ldots, d_N\}$, with a known or assumed initial probability for each d. The estimator consists of a bank of N standard Kalman filters each using the noisy signal measurements as input. Each Kalman filter is designed assuming a period from the discrete set thus making the technique linear. The estimate of the state and period is given by the weighted sum of the estimates from each Kalman filter. The weighting coefficient of the state of the ith Kalman filter is the a posterior probability that $d=d_i$ and this probability is updated recursively using the noisy signal measurements and the state of the ith Kalman filter. More precisely, the a posterior probability is denoted as $p(d_i \backslash Y_t)$ where Y_t is the sequence of measurements from $t = 0$ and is calculated as

$$p(d_i \backslash Y_t) = \frac{p(y_t \backslash Y_{t-1}, d_i)\, p(d_i \backslash Y_{t-1})}{\sum_{i=1}^{N} p(y_t \backslash Y_{t-1}, d_i)\, p(d_i \backslash Y_{t-1})}. \tag{3}$$

The likelihood functions, $p(y_t \backslash Y_{t-1}, d_i)$, are readily available using the innovations and error covariances from the Kalman filter equations. Thus, the state estimate is the sum of the contributions from each of the filters and is expressed as

$$\hat{x}_{t/t-1} = \sum_{i=1}^{N} \hat{x}_{t/t-1,d_i}\, p(d_i \backslash Y_t). \tag{4}$$

An advantage of this form of adaptive estimation is that convergence results exist for the stationary case. It is shown in [7] that if the true parameter values lies within the discrete set, the corresponding a posteriori probability converges exponentially to 1.

For the situation where the unknown parameter is, in fact, time varying, various modifications to this parallel processing scheme are available. One approach requires reinitialisation which effectively throws away old data. Any $p(d_i \backslash Y_t)$ which are zero are set to a nonzero value. The states in the bank of Kalman filters are also reset to the current state estimate. The frequency of reset should be related to the temporal structure of the unknown parameters.

5. SIMULATIONS AND RESULTS

Before adaptive tracking of the period of the *wsc* process described in Section 2 is considered, convergence properties are first investigated. A Magill filter and an extended Kalman filter were applied to the system. Suppose $d_t = d = 50$ and $\sigma_U^2 = \sigma_V^2 = 1$. The extended Kalman filter of Section 3 has $\beta = 1\text{x}10^{-5}$ (Eq. 2) and an initial error covariance of $\Sigma_{0/-1} = \begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}$. The Magill filter consists of a bank of 5 standard Kalman filters each modelled as in Eq. 1 but with a constant d_i taken from the discrete set {42, 46, 50, 54, 58}.

The period estimation threshold behaviour for the Magill and extended Kalman filters, over a range of σ_W^2, is shown in Figure 1. The results are averaged over 100 realisations and are for t = 0,..,9999. Initialisation in this instance is at the correct value. However, it is noted that convergence of the EKF can be dependent on initial assumptions of period and error covariance and convergence properties of the EKF are not well understood. Estimation of the period utilising the adaptive EKF is the preferred method. However, performance of the Magill filter is highly dependent on the discretisation of the parameter space and the Kullback information function [7] can be used to specify a sensible discretisation of the parametric space. One realisation for both the EKF and the Magill filter is shown in Figure 2 for $\sigma_W^2 = 0.5$. Clearly, the Magill filter has a faster convergence rate than the EKF but a higher initial error due to the choice of the parametric space.

The EKF is then used to adaptively track a time-varying period. Three periodic functions and their associated period tracks are shown in Figures 3, 4 and 5 It is apparent that the algorithm can adaptively track slowly time-varying changes in periods as well as abrupt steps but over a small range. As the period is modelled as an AR(1) process in this instance, the performance of the filter is directly related to the type of autoregressive model which fits the time-varying period. The Magill filter is also used to adaptively track a strongly time-varying period (Figure 6). Periodic reinitialisation assuming prior knowledge of the temporal structure is employed with this particular filter to allow for time-varying parameters. Performance of the Magill filter was poor when adaptively tracking a rapidly time varying period such as a sine or ramp.

Analysis of the Magill filter and the EKF leads naturally to a combined detection-estimation approach. A tandem arrangement was formulated where the Magill filter employs the detector algorithms to determine which region of the parameter space the true parameter lies. Once it has been established that the right local region has been located, initial estimates of the period and error covariance are passed to the EKF and smaller variations in the period are adaptively tracked. This approach is shown to be effective in alleviating convergence problems with the EKF due to poor initialisation and reduces the mean square error as the system settles. The asymptotic steady state errors remain unchanged. Improvement in threshold performance for estimation of period for this tandem arrangement is illustrated in Figure 7.

6. CONCLUSION

The goal of this work was to investigate the potential of the extended Kalman filter and the Magill filter for adaptive estimation of period for cyclostationary processes. Convergence of both filters has been investigated for the AR(1) process described earlier. The extended Kalman filter exhibits good thresholding behaviour even at a high level of noise in the measurement signal provided care is taken when choosing initial conditions. The Magill filter is shown to be more robust but performance is lost as a result of the discretisation of the parametric space. For the process considered, the extended Kalman filter is an effective means of tracking a time-varying period. In comparison, the Magill filter can also yield good estimates of period but the method does require some prior knowledge with regards to the temporal structure of the period. Finally, it is possible to obtain significant performance benefits by combining a Magill filter with an extended Kalman filter to accommodate for poor initialisation. Thus, it is fair to suggest that the adaptive methods discussed here for period estimation could be incorporated into the estimation of time-varying spectra for wide sense cyclostationary processes.

ACKNOWLEDGMENT

The authors wish to acknowledge the funding of the activities of the Cooperative Research Centre for Robust and Adaptive Systems by the Australian Commonwealth Government under the Cooperative Research Centre Program.

REFERENCES

[1] P.J. Sherman, L.B. White and R.R. Bitmead, *On AR Representations for Cyclostationary Processes*, *Proc. ICASSP '93*, Minneapolis 1993.

[2] L.B. White and P.J. Sherman, *Periodic Uncertainty in Periodic Spectral Analysis of Processes associated with Periodic Phenomena*, *Proc. ICASSP '94*, Adelaide 1994.

[3] R.J. Bennett, *Non-stationary Parameter Estimation for Small Sample Situations; A Comparison of Methods*. *Int. J. Systems Sci.* v. 7, no. 3, 1976. pp. 257-275.

[4] B.D.O. Anderson and J.B. Moore, Optimal Filtering, Prentice Hall, 1979.

[5] R. Grove Brown, *A New Look at the Magill Adaptive Filter as a Practical Means of Multiple Hypothesis Testing*. *IEEE Transactions on Circuits and Systems*, v. 30, no. 10, 1983, pp. 765-768.

[6] Y. Bar-Shalom and Xiao-Rong Li, Estimation and Tracking: Principles, Techniques and Software. Artech House, 1993.

[7] J. Moore and R.M. Hawkes, *Decision Methods in Dynamic System Identification*. *14th IEEE Symposium on Adaptive Processes*, 1975, pp. 645-650.

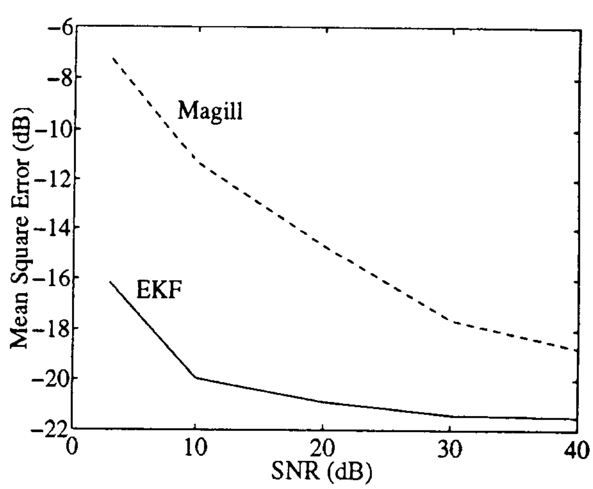

Figure 1 Thresholding Behaviour for the Extended Kalman Filter and the Magill Filter

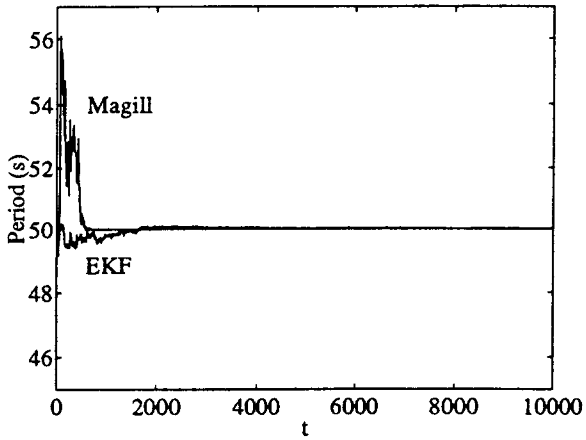

Figure 2 Comparison of the Extended Kalman Filter and the Magill Filter $\sigma_W^2 = 0.5$

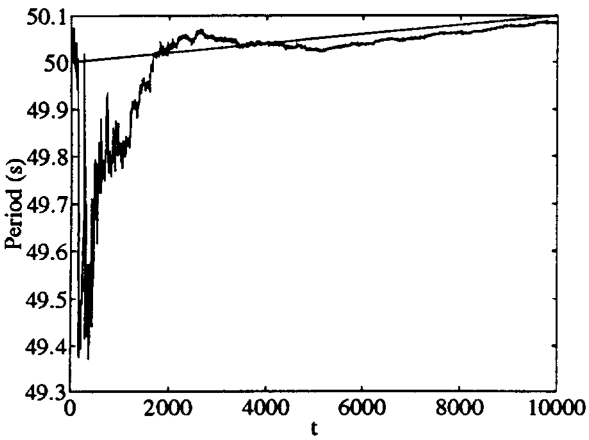

Figure 3 Adaptive Estimation - EKF Ramp, $\sigma_W^2 = 0.01$

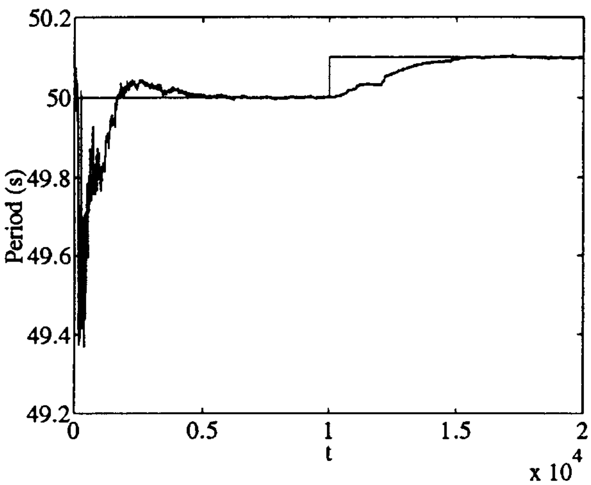

Figure 4 Adaptive Estimation - EKF Step, $\sigma_W^2 = 0.01$

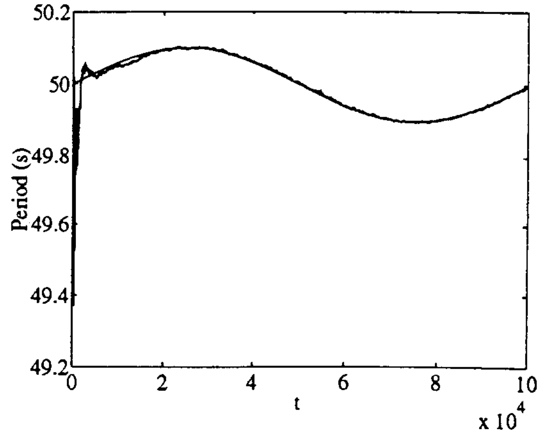

Figure 5 Adaptive Estimation - EKF Sine, $\sigma_W^2 = 0.01$

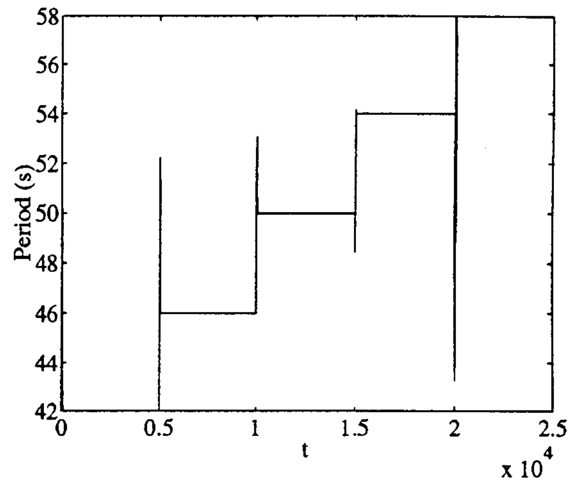

Figure 6 Adaptive Estimation - Magill Step, $\sigma_W^2 = 0.01$

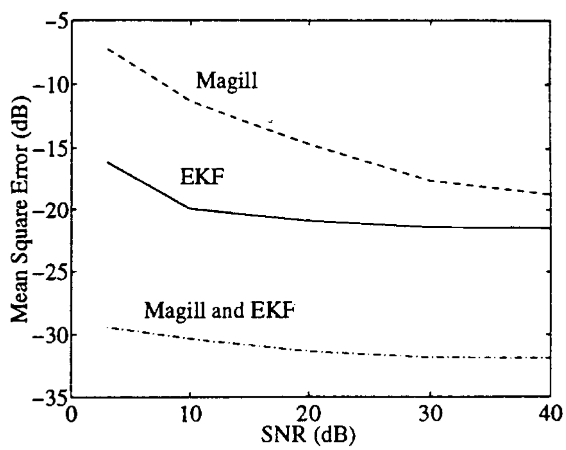

Figure 7 Thresholding Behaviour for Combined Magill and Extended Kalman Filters