JOINT PARAMETER ESTIMATION AND DEMODULATION OF SUPERIMPOSED CONVOLUTIONAL CODED SIGNALS

Gary D BRUSHE[1,2] and Langford B WHITE[1]

[1]Signal Analysis Discipline, Communications Division, Defence Science and Technology Organisation
PO Box 1500 Salisbury SA 5108, AUSTRALIA.
Tel 618 2596662, Fax 618 2596975, E-mail gdb@cd.dsto.gov.au
Cooperative Research Centre for Robust and Adaptive Systems
[2]Department of Systems Engineering, RSISE, Australian National University, Canberra

ABSTRACT

A method of jointly estimating the parameters of a number of superimposed convolutional coded communication signals incident on an antenna array and demodulating these signals is presented in this paper. The method would allow simpler arrays to be designed due to the threshold extension obtained by this method. It also has the potential to increase the throughput of current Multiple Access channel systems, for example, Satellite communications and digital Mobile Cellular Phones, by using an antenna array. The contribution of the paper is the use of sequence estimation combined jointly with parameter estimation in array processing problems. In the simulations it is shown that a significant improvement in the accuracy of the demodulated signals and in the estimation of the signals' angle of arrivals is obtained compared to a deterministic Maximum Likelihood estimation method.

1. INTRODUCTION

The estimation of superimposed signals incident on an array of sensors is not new. Traditional methods of demodulating superimposed signals involve the use of an array of sensors and beamforming techniques. These techniques first obtain an estimate of the parameters of the signals, i.e. amplitudes (ρ), phases (ψ) and angle of arrivals (AOAs), and then using these parameter estimates extracts each signal from the received observations, before demodulating each signal separately (see Figure 1). Expectation-Maximisation (EM) algorithms have been applied to this type of problem previously. Feder and Weinstein [1] introduced the use of the EM algorithm for direction finding. Miller and Fuhrmann [2] derived EM algorithms for the Maximum Likelihood (ML) estimation of the direction of arrivals of multiple narrow-band signals in noise, under both the deterministic and stochastic signal models. Ziskind and Hertz [3] derived an EM algorithm for Auto-Regressive (AR) processes. Malcolm and White [4] extend Ziskind and Hertz's EM algorithm for general linear Gaussian Markov processes by refining the E-step. Knowledge of the signals characteristics has also been used to improve the estimates [5], [6]. Jointly estimating the parameters and demodulating the signals using knowledge of the structure of the signals has only recently been applied [7], [8].

This paper investigates the sequence estimation problem of signals modelled as Markov sequences. These Markov sequences (convolutional coded signals [9]) have strongly constrained state sequences and therefore the estimation procedure should yield valid path constrained sequences. Knowledge of the signals' models is assumed (each signal is convolutional coded with known constraint length and generating polynomials) in order to estimate their maximum *a posteriori* probability (MAP) state sequences. These sequences are then used in estimating the signals' parameters.

The method used in this paper for jointly demodulating the signals and estimating their parameters is based on the iterative two step Segmental K-Means Algorithm (SKMA) [10]. The first step (segmentation step) uses Hidden Markov Model (HMM) methods [11] (leading to the well-known Viterbi algorithm [11]) to estimate the MAP state sequences of the convolutional coded signals. The second step (optimisation step) uses the estimated MAP state sequences to estimate the signals' parameters by maximising the state-optimised log likelihood function with respect to the parameters. A suboptimal method for decreasing the computational complexity of this problem and hence the processing time is also described.

The method described in this paper is shown to improve the demodulation of the signals as well as improve the accuracy in estimating the AOAs, when compared to a deterministic ML estimation method [12] for signals that are closely spaced. This improvement is shown to be significant for the examples discussed but are achieved through an increase in the computational complexity of the problem. Monte Carlo simulations are used to demonstrate the results and improvements obtained.

2. SIGNAL MODEL AND PROBLEM FORMULATION

Let the message sequence $b(t), t \geq 0$ denote a first order Markov process with transition probabilities a_{ij}, being the probability of transition to state j from state i (for $b(t)$ an independently and identically distributed (i.i.d.) equiprobable binary process, $a_{ij} = 0.5$). This sequence is convolutional coded with a constraint length N and rate Q/n, where Q message bits are produced for every n coded bits. This code is used to select one of $M = 2^Q$ possible phase signalling values. For a convolutional coded signal the transmitted phase is given by:

$$\phi(t) = \frac{2\pi}{M}\sum_{m=0}^{M-1} 2^m \sum_{n=0}^{N-1} b(t-n)G_m(n) \tag{1}$$

where $G_m(n)$ are known binary co-efficients of the convolutional code's generating polynomials.

$s(t) = [b(t), b(t-1), \ldots, b(t-N+1)]$ is a first order Markov process that has $F = 2^N$ states with transition probabilities:

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$$\Pr\{s(t)=[c(0),\ldots,c(N-1)] \mid s(t-1)=[d(0),\ldots,d(N-1)]\} = \begin{cases} a_{d(0)c(0)} & d(t-1-i)=c(t-i),\quad 0\le i\le N-1 \\ 0 & \text{else.}\end{cases} \quad (2)$$

We shall restrict attention to the uniform i.i.d. message case. Each $s(t)$ corresponds to a certain $\phi(t)$ hence the transmitted signal is modelled by

$$z(t)=x(t)+iy(t) \text{ with } x(t)=\rho\cos(\phi(t)+\psi)+w(t)$$
$$y(t)=\rho\sin(\phi(t)+\psi)+v(t) \quad (3)$$

where the amplitude (ρ) and phase (ψ) are slowly varying wrt time and are considered constant for some block length T, $w(t)$, $v(t)$ are i.i.d. white Gaussian noise (WGN) processes with zero mean and variance σ^2.

Consider L superimposed signals each generated as described above by Eqs. (1) to (3), except that the message sequences, $\{b^{(\ell)}(t)\}$ $1\le\ell\le L$, are generated independently, the generating polynomials $G_m^{(\ell)}(n)$ for each signal may be different, and that these signals are incident on a uniform linear array (ULA) of K sensors. The standard baseband model of the K-vector array outputs is:

$$\mathbf{U}(t)=\mathbf{A}(\Omega)\mathbf{Z}(t)+\mathbf{N}(t) \quad (4)$$

for $t=0,\ldots,T-1$, and where $\mathbf{A}(\Omega)$ is the so-called steering matrix that depends on the arrival angles $\Omega=\{\varphi^{(1)},\theta^{(1)},\varphi^{(2)},\theta^{(2)},\ldots,\varphi^{(L)},\theta^{(L)}\}$ of the signals $\mathbf{Z}(t)=[z^{(1)}(t),z^{(2)}(t),\ldots,z^{(L)}(t)]$ (where $\varphi^{(\ell)}$, $\theta^{(\ell)}$ refer to the elevation and azimuth of signal $z^{(\ell)}(t)$ respectively), together with the array geometry. $\mathbf{N}(t)$ is an i.i.d. WGN process with covariance matrix $R(t)=\sigma^2\mathbf{I}$. The problem of interest here is to estimate the amplitudes ($\rho^{(\ell)}$), phases ($\psi^{(\ell)}$), arrival angles ($\varphi^{(\ell)},\theta^{(\ell)}$) and the message sequences $\{b^{(\ell)}(t)\}$, $1\le\ell\le L$ from the observed data sequence $\{\mathbf{U}(t)\}$.

3. DEMODULATION OF THE SIGNALS AND ESTIMATION OF PARAMETERS

The observed signal model for the L superimposed signals is given by Eq. (4). For optimal demodulation, these L signals can be considered to be equivalent to a first order Markov process with F^L states given by:

$$S(t)=[s^{(1)}(t),s^{(2)}(t),\ldots,s^{(L)}(t)]=[b^{(1)}(t),b^{(1)}(t-1),\ldots,b^{(1)}(t-N+1),b^{(2)}(t),b^{(2)}(t-1),\ldots,b^{(2)}(t-N+1),\ldots,b^{(L)}(t),b^{(L)}(t-1),\ldots,b^{(L)}(t-N+1)] \quad (5)$$

with transition probabilities:

$$\Pr\{S(t)=[c^{(1)}(0),\ldots,c^{(1)}(N-1),c^{(2)}(0),\ldots,c^{(2)}(N-1),\ldots,c^{(L)}(0),\ldots,c^{(L)}(N-1)] \mid S(t-1)=[d^{(1)}(0),\ldots,d^{(1)}(N-1),d^{(2)}(0),\ldots,d^{(2)}(N-1),\ldots,d^{(L)}(0),\ldots,d^{(L)}(N-1)]\}$$
$$= \begin{cases} a_{d^{(\ell)}(0)c^{(\ell)}(0)} & d^{(\ell)}(k)=c^{(\ell)}(k+1);\ 0\le k\le N-1;\ 1\le\ell\le L \\ 0 & \text{else.}\end{cases} \quad (6)$$

where $a_{d^{(\ell)}(0)c^{(\ell)}(0)}=0.5^L$ when the L input message sequences $\{b^{(\ell)}(t)\}$ are binary & i.i.d.

The SKMA is used to demodulate the signals and estimate their parameters. The segmentation step estimates the i^{th} iterative MAP state sequence of the signals,

$$\{\hat{S}_i(t)\}=\hat{S}_i(0),\ldots,\hat{S}_i(T-1)=\arg\max_{S(0),\ldots,S(T-1)}\Pr\{S(0),\ldots,S(T-1)\mid U(0),\ldots,U(T-1);\hat{P}_i,\hat{\Psi}_i,\hat{\Omega}_i\} \quad (7)$$

for some block length T and i^{th} estimates $\hat{P}_i=\{\hat{\rho}_i^{(1)},\hat{\rho}_i^{(2)},\ldots,\hat{\rho}_i^{(L)}\}$, $\hat{\Psi}_i=\{\hat{\psi}_i^{(1)},\hat{\psi}_i^{(2)},\ldots,\hat{\psi}_i^{(L)}\}$ and $\hat{\Omega}_i=\{\hat{\varphi}_i^{(1)},\hat{\theta}_i^{(1)},\hat{\varphi}_i^{(2)},\hat{\theta}_i^{(2)},\ldots,\hat{\varphi}_i^{(L)},\hat{\theta}_i^{(L)}\}$.

The sequence $\{\hat{S}_i(t)\}$ is estimated as if it is one signal that has F^L states, using the Viterbi algorithm. From Eq. (5), the L estimated MAP state sequences $\{\hat{s}_i^{(\ell)}(t)\}$ and hence the L demodulated message sequences $\{\hat{b}_i^{(\ell)}(t)\}$, $1\le\ell\le L$ are then obtained.

To improve the demodulation of the signals updated estimates of the signals' parameters ($\hat{P}_{i+1},\hat{\Psi}_{i+1},\hat{\Omega}_{i+1}$) are determined to maximise the state-optimised log likelihood function:

$$(\hat{P}_{i+1},\hat{\Psi}_{i+1},\hat{\Omega}_{i+1})=\arg\max_{P,\Psi,\Omega}\{\log\Pr\{\{U(t)\}\mid\{\hat{S}_i(t)\};P,\Psi,\Omega\}\} \quad (8)$$

For convolutional coded plane wave signals, Eq. (8) is written as follows, with $\hat{\Omega}$ replaced by the signals' azimuth angles $\hat{\Theta}=\{\hat{\theta}^{(1)},\hat{\theta}^{(2)},\ldots,\hat{\theta}^{(L)}\}$ and $\mathbf{U}(t)$ written in quadrature baseband form, $u(t,k)=x(t,k)+iy(t,k)$ for each sensor, k.

$$(\hat{P}_{i+1},\hat{\Psi}_{i+1},\hat{\Theta}_{i+1})=\arg\max_{P,\Psi,\Theta}\Big\{-T\log(\sigma\sqrt{2\pi})-\frac{1}{2\sigma^2}\sum_{t=0}^{T-1}\sum_{k=0}^{K-1}\Big[\Big(x(t,k)-\sum_{\ell=1}^{L}\rho^{(\ell)}\cos(\hat{\phi}_i^{(\ell)}(t)+\psi^{(\ell)}+k\omega^{(\ell)})\Big)^2+\Big(y(t,k)-\sum_{\ell=1}^{L}\rho^{(\ell)}\sin(\hat{\phi}_i^{(\ell)}(t)+\psi^{(\ell)}+k\omega^{(\ell)})\Big)^2\Big]\Big\} \quad (9)$$

where $\hat{\phi}_i^{(\ell)}(t)$ is defined in Eq. (1), $\omega^{(\ell)}=\frac{2\pi d}{\lambda}\cos(\theta^{(\ell)})$, d is the spacing between sensors and λ is the signal wavelength.

There is no closed form solution to Eq. (9). Thus, numerical techniques [13] can be used to determine the values of $\hat{P}_{i+1},\hat{\Psi}_{i+1}$ and $\hat{\Theta}_{i+1}$ which maximise Eq. (9). Also a L-dimensional grid search could be applied. Both of these methods are computationally very intensive.

In order to reduce the computational load in estimating the signals' parameters, Eq. (9) is suboptimally solved using Feder and Weinstein's EM algorithm for deterministic signals [1]. The EM algorithm for this problem is summarised below.

For each signal ℓ, $1\le\ell\le L$

E-step: For each sensor k, $0\le k\le K-1$

$$\hat{u}_{i,b}^{(\ell)}(t,k)=\hat{\rho}_{i,b}^{(\ell)}e^{i\left(\hat{\phi}_i^{(\ell)}(t)+\psi_{i,b}^{(\ell)}+k\omega_{i,b}^{(\ell)}\right)}+\beta^{(\ell)}\left[U(t,k)-\sum_{r=1}^{L}\hat{\rho}_{i,b}^{(r)}e^{i\left(\hat{\phi}_i^{(r)}(t)+\hat{\psi}_{i,b}^{(r)}+k\hat{\omega}_{i,b}^{(r)}\right)}\right] \tag{10}$$

M-step:

$$\left(\hat{\rho}_{i,b+1}^{(\ell)},\hat{\psi}_{i,b+1}^{(\ell)},\hat{\theta}_{i,b+1}^{(\ell)}\right)=\arg\max_{\rho,\psi,\theta}\left\{\frac{-1}{2\sigma^2}\sum_{t=0}^{T-1}\sum_{k=0}^{K-1}\left[\left(\hat{x}_{i,b}^{(\ell)}(t,k)-\rho\cos\left(\hat{\phi}_i^{(\ell)}(t)+\psi+k\omega\right)\right)^2+\left(\hat{y}_{i,b}^{(\ell)}(t,k)-\rho\sin\left(\hat{\phi}_j^{(\ell)}(t)+\psi+k\omega\right)\right)^2\right]\right\}. \tag{11}$$

where $\sum_{\ell=1}^{L}\beta^{(\ell)}=1$, and $\hat{\phi}_i^{(\ell)}(t)$ & $\omega_{i,b}^{(\ell)}$ are similarly defined as in Eq. (9). Eq. (11) is now in an expanded form similar to Eq. (9) except that the $\sum_{\ell=1}^{L}$ terms are replaced by single terms. From Eq. (11) an expression for each of the parameters can be determined, however the $\hat{\Theta}_{i,b+1}$ values must still be approximated using numerical techniques and/or grid searches over L*H† values. ML estimates of $\hat{P}_{i,b+1}$ and $\hat{\Psi}_{i,b+1}$ can be evaluated in closed form once the values of $\hat{\Theta}_{i,b+1}$ have been computed. Further details can be found in [14].

The EM variant of the SKMA is a lot less computationally intensive than an L dimensional numerical technique or an L-dimensional search involving approximately H^L/L values. After a convergence criterion is satisfied the parameters $\left[\hat{P}_{i+1},\hat{\Psi}_{i+1},\hat{\Theta}_{i+1}\right]=\left[\hat{P}_{i,b+1},\hat{\Psi}_{i,b+1},\hat{\Theta}_{i,b+1}\right]$ are used in the next segmentation step.

The SKMA method and its suboptimal variant (denoted by SKMA-EM) are compared with the deterministic ML estimation method for the narrow-band problem as described by Hurt [12] and depicted in Figure 1.

4. SIMULATIONS AND RESULTS

Two signals are simulated to be incident on a ULA of 5 sensors spaced one-half wavelength. These signals are assumed to be convolutional coded quadriphase shift keyed (QPSK) with constraint length 7, rate ½. The generating polynomials are those found by Odenwalder as described in Sklar [9]. These two plane wave signals have a block length of 300 bits, $\rho=1.0$ and $\psi=0.0$ and are incident on the array at 23° and 28° from endfire. The signals have different in-dependent input message sequences. The results are calculated for 300 realisations per SNR‡/SENSOR value for each of the methods, a deterministic ML scheme, the SKMA and SKMA-EM schemes. The initial estimate of the AOA for each signal was randomly chosen in the range 13° to 38° and the ML search was also constrained to this region. The search grid was spaced 0.2° with H=126 values. The signals' amplitudes and phases are assumed known.

Figure 2 shows the RMS error of estimating the AOA for the two signal case using the three methods. The SKMA and the SKMA-EM methods are shown to be significantly more accurate at estimating the AOAs when compared to the deterministic ML method particularly at low SNR. It is also noted that there is little different between the accuracy of the SKMA and the SKMA-EM methods. However the SKMA-EM method is significantly less computationally intensive than the SKMA method.

The two signals are defined as being resolvable [15] if both signals' AOA estimates satisfy the condition:

$$\left|\theta^{(\ell)}-\hat{\theta}^{(\ell)}\right|<\frac{\theta^{(1)}+\theta^{(2)}}{2},\ \ell=1,2$$

where $\theta^{(\ell)}$ is the true AOA for signal ℓ,
$\hat{\theta}^{(\ell)}$ is the estimated AOA for signal ℓ.

Figure 3 shows the probability of resolving the signals. The signal resolution is far superior for the SKMA and SKMA-EM methods compared to the ML method at the SNR values shown, with no difference between the SKMA and SKMA-EM methods.

Figure 4 shows the average BER for the two signals as determined using the three methods. The BERs obtained using SKMA and SKMA-EM methods are almost identical. These two methods show a remarkable threshold extension (~20dB) over the deterministic ML method.

5. CONCLUSION

In this paper we have presented a method (using Juang and Rabiner's segmental k-means algorithm [10]) for spatial filtering of superimposed convolutional coded QPSK signals. This method was compared with a deterministic ML scheme [12]. For the examples discussed, we have shown that this method is significantly more accurate in its demodulation of the signals and in the estimation of the AOAs, particularly at low SNR. However, this improvement is achieved through an increase in the computational complexity of the problem and hence the processing time for obtaining solutions is also increased. The paper also described a means of reducing the computational complexity in the parameter estimation section of the SKMA method (using Feder and Weinstein's EM algorithm for deterministic signals [1]). This suboptimal method (SKMA-EM) significantly reduced the computational complexity involved in estimating the parameters without any significant loss of performance in the demodulation of the signals or in the estimation of the AOAs. The improved accuracy of the SKMA and SKMA-EM methods provide a remarkable threshold extension, ~20dB, compared to the deterministic ML scheme.

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† H is the number of values in a given search region for each signal.

‡ Signal to Noise Ratio = $10\log_{10}\left(\rho^2/\sigma^2\right)$

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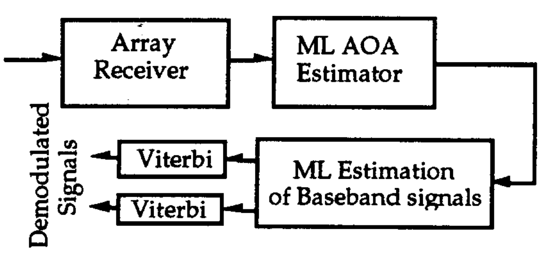

Figure 1: ML scheme for AOA estimation and demodulation for two signals.

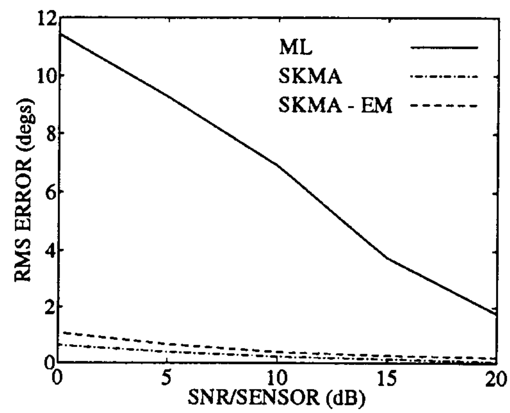

Figure 2: AOA estimation for two signals ML vs SKMA vs SKMA - EM methods.

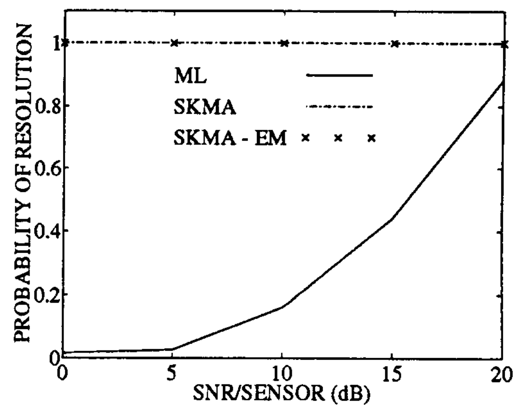

Figure 3: Probability of resolution for two signals: ML vs SKMA vs SKMA - EM methods.

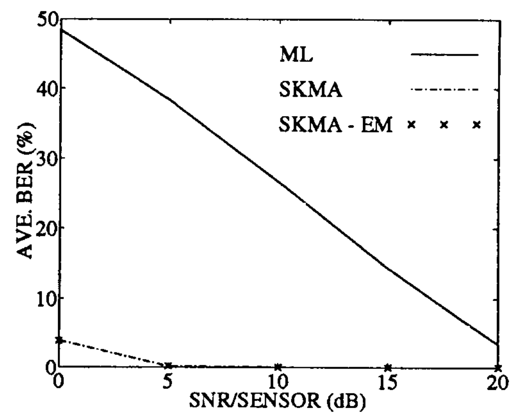

Figure 4: Average BER for two signals ML vs SKMA vs SKMA - EM methods.