# Blind Adaptive Beamforming for Narrowband Co-channel Digital Communications Signals in a Multipath Environment†

Michael D. Zoltowski
School of Electrical Engineering
Purdue University
West Lafayette, IN 47907 USA
mikedz@ecn.purdue.edu

Javier Ramos
E.T.S.I. Telecommunications
Polytechnic University of Madrid
28040 Madrid, Spain
jramos@gmr.ssr.upm.es

## ABSTRACT

The problem addressed is that of narrowband digital cellular communications in the presence of multipath and co-channel interferers having the same symbol rate and same overall signal characteristics as the desired source. For a given signal source, an algorithm is presented for blindly estimating the weight vector yielding the optimum SINR at the beamformer output. The instrumental quantity, denoted $\mathbf{S}_{xx}(f)$, is the Fourier Transform of the expected value of the zero-lag autocorrelation matrix for one symbol period. The proposed scheme employs the PRO-ESPRIT algorithm to exploit the relationship between the timing offset and optimum beamforming weight vector for each source and the principal generalized eigenvalues and eigenvectors of the matrix pencil $\{\mathbf{S}_{xx}(1/T_o), \mathbf{S}_{xx}(0)\}$, where $1/T_o$ is the symbol rate. Simulations are presented showing the rapid convergence of the algorithm and an improvement of several dB over the subspace-constrained Phase-SCORE algorithm.

## 1. NARROWBAND DIGITAL CELLULAR

As a consequence of multipath, the urban mobile communications environment is a time-varying, time-dispersive, and fading channel [1]. Consider for the moment that the multipath is specular. That is, view any diffuse multipath component as a sum of specular multipaths each having its own relative time delay, attenuation, phase, and angle of arrival. Based on experimental measurements, the maximum multipath time delay spread in an urban environment is on the order of 10 microseconds. With a symbol pulse waveform of duration commensurate with one of the 24 KHz channels in a TDMA commercial cellular system based on the IS-54 standard, multipath delays of this order are significant enough to necessitate the use of an adaptive equalizer.

In order to circumvent the need for an equalizer, Motorola has proposed and implemented in its Integrated Radio System (MIRS) a modulation scheme referred to as M-16QAM [1]. Note that MIRS is a TDMA system with six users per 24 KHz channel and has a cellular structure, but operates at around 800 MHz and at 1500 MHz (PCS applications). In M-16QAM, the TDMA bit stream with six users interleaved is split into M parallel lower-rate streams. The bits in each of the M parallel streams are encoded or mapped onto 16QAM symbols. Each composite symbol stream is then pulse shape filtered, modulated to its own subcarrier frequency, and then combined with the other subcarrier signals in a frequency division multiplex fashion. That is, the 24 HKz channel is split into M equi-spaced channels but each channel is not associated with a specific user as M can be greater than 6. The idea here is that since the bandwidth of each subchannel is less than 4 KHz, the symbol pulse waveform commensurate with each subchannel is very wide, i. e., its effective time duration is increased by a factor of M. Under these conditions, the maximum multipath time delay is negligible with respect to the 3 dB time duration of the symbol pulse waveform, which may be as high as 200 microseconds or more.

Thus, in this narrowband setting the time-dispersive effects of the channel are negligible. However, fading is still an issue as the multipaths associated with a particular source, though approximately time-aligned, have different relative phases due to the different path lengths traveled. The primary way to counter fading is through employing an array of antennas at either the base or the mobile.

It follows from the discussion above that the appropriate array data model is the standard narrowband array model with 100% correlated multipath. Since the multipath is generally diffuse with possibly some specular components, the array manifold for a given source is very complicated. Even if the multipath was specular, the presence of co-channel interferers each with its own set of specular multipaths, negates the economic feasibility of using angle-of arrival estimation techniques such as MUSIC or Maximum Likelihood. Yet, the beamforming weight vector yielding the optimum signal to interference noise ratio (SINR) for the desired source is obtained by pre-multiplying the array manifold for that source by the inverse of the autocorrelation matrix of array outputs. Thus, there is a need for blindly estimating the array manifold.

## 2. PREVIOUS BLIND ADAPTIVE BEAMFORMING SCHEMES

Previously proposed blind estimation schemes for interference cancellation using an array of antennas have been premised on either the signals having constant modulus, as in an MPSK signal constellation, or that the interferers have a different baud rate than the desired source. With respect to the constant modulus requirement, to achieve accept-

†This research was supported by NSF under contract no. MIP-9320890 and by AFOSR under contract no. F49620-92-J-0198 in conjunction with Wright Laboratories.



0-7803-2431-5/95 $4.00 © 1995 IEEE

able data rates in very narrowband channels, multi-level signal constellations must be used. Now, although it has been shown that the constant modulus algorithm (CMA) beamformer may be employed with nonconstant modulus signal constellations, the convergence rate is not nearly fast enough to track the multipath fluctuations.

In the case where the interferers are at different symbol rates, the principle of cyclostationarity dictates that if the autocorrelation of the data is beat against a complex sinusoid at the baud rate of the desired user over a long enough averaging interval, the desired source is the only source making a nonvanishing contribution to the resulting cyclic correlation matrix. However, due to the desired uniformity of receiver structure from cell to cell, the dominant interferers in a commercial narrowband cellular environment are at the *same* symbol rate as the desired user.

However, the recent cyclostationarity based Cross-SCORE and Phase-SCORE algorithms of Agee, Schell, and Gardner [2] are blind adaptive beamforming algorithms that can handle sources at the same baud rate. In fact, the subspace constrained version of Phase-SCORE proposed by Biedka [2], which converges much faster than Phase-SCORE, is closely related to the new algorithm developed in Section 3. Like the SCORE algorithms, the new method exploits the underlying periodic nature of the digital signal, but in a fundamentally different way. The instrumental quantity, denoted $\mathbf{S}_{xx}(f)$, is the Fourier Transform of the expected value of the zero-lag autocorrelation matrix for one symbol period. PRO-ESPRIT [3] is employed to exploit the relationship between the timing offset and optimum beamforming weight vector for each source and the principal generalized eigenvalues and eigenvectors of the matrix pencil $\{\mathbf{S}_{xx}\left(\frac{1}{T_o}\right), \mathbf{S}_{xx}(0)\}$, where $\frac{1}{T_o}$ is the symbol rate.

## 3. BLIND BEAMFORMING VIA PRO-ESPRIT

Consider $J$ co-channel digital communications signals in a multipath environment incident upon an array of $N$ antennas at either the base or mobile. As discussed previously, the appropriate array data model is the standard narrowband array model with 100% correlated multipath. The $N \times 1$ vector of baseband outputs from the $N$ antennas after complex demodulation is described by

$$\mathbf{x}(t) = \sum_{j=1}^{J} \sigma_j \mathbf{a}_j \sum_{m=0}^{M-1} b_j(m)\, p(t - mT_o - \tau_j) + \nu(t), \quad (1)$$

where $p(t)$ is the standard ISI pulse symbol waveform

$$p(t) = \frac{\sin\left(\pi \frac{t}{T_o}\right)}{\pi \frac{t}{T_o}} \frac{\cos\left(\beta\pi\frac{t}{T_o}\right)}{1 - \frac{4\beta^2 t^2}{T_o}}, \quad (2)$$

corresponding to a raised cosine spectrum with a bandwidth of $\frac{1+\beta}{2T_o}$, where $\beta$ is referred to as the excess bandwidth parameter and $\frac{1}{T_o}$ is the symbol rate (baud rate). To achieve as high a baud rate as possible given a certain bandwidth, $\beta$ is chosen to be between 0 and 1 with the current drive to get it below 0.5. Note that lowering the value of $\beta$ increases the number of tails of $p(t)$ outside the symbol period.

The various other quantities in (1) are described below·

- $\sigma_j$: complex amplitude factor dependent on distance between base and mobile
- $\mathbf{a}_j$: $N \times 1$ array manifold describing the relative amplitudes and phases across the array for the *j-th* signal source incorporating multipath effects
- $b_j(m)$: byte value for *j-th* source at *m-th* symbol period, member of finite alphabet (signal constellation)
- $\tau_j$: unknown relative time shifts, $-\frac{T_o}{2} \leq \tau_j \leq \frac{T_o}{2}$
- $M$: number of symbol periods for which it is assumed that $\mathbf{a}_j$ and $\tau_j$ are essentially constant
- $\nu(t)$: $N \times 1$ vector of additive receiver generated noise at each antenna element, $\mathcal{E}\{\nu(t)\nu^H(t)\} = \sigma_n^2 \mathbf{I}_N$

The zero-lag autocorrelation matrix for one symbol period is defined as

$$\mathbf{R}_{xx}(t) \stackrel{\text{def}}{=} \mathcal{E}\left\{\mathbf{x}(t)\mathbf{x}^H(t)\, rect\left(\frac{t}{T_o}\right)\right\}, \quad (3)$$

where $\mathcal{E}\{\bullet\}$ is the expectation operator and $rect\left(\frac{t}{T_o}\right)$ is unity over an interval of width $T_o$ centered at $t = 0$ and zero elsewhere. The instrumental quantity is $\mathbf{S}_{xx}(f) = \mathcal{F}\{\mathbf{R}_{xx}(t)\}$, where the Fourier Transform operates pointwise on the matrix. Given that the region of support of

$$S_p(f) = \mathcal{F}\{p^2(t)\} \quad (4)$$

is $-\frac{1+\beta}{T_o} < f < -\frac{1+\beta}{T_o}$ (due to squaring), define $\mathbf{S}_{xx}^{(\eta)}$ as

$$\mathbf{S}_{xx}^{(\eta)} \stackrel{\text{def}}{=} \mathbf{S}_{xx}\left(\frac{\eta}{T_o}\right) = \mathcal{F}\{\mathbf{R}_{xx}(t)\}\mid_{f=\eta/T_o} \quad (5)$$

$$\text{for:} \quad -(1+\beta) < \eta < 1+\beta$$

Assuming that $\mathcal{E}\{b_j(m)b_i^*(n)\} = \sigma_b^2 \delta_{ij}\delta_{mn}$, where $\delta_{ij}$ is the Kronecker delta, in the model for $\mathbf{x}(t)$ defined in (1), $\mathbf{S}_{xx}^{(\eta)}$ may be expressed as

$$\mathbf{S}_{xx}^{(\eta)} = \mathbf{A}\mathbf{Q}^{(\eta)}\mathbf{\Sigma}_P \mathbf{A}^H + \sigma_n^2 \,\text{sinc}(\eta)\, \mathbf{I}_N, \quad (6)$$

where $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_J]$, $\mathbf{\Sigma}_P = \sigma_b^2 \,\text{diag}\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_J^2\}$, $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, and $\mathbf{Q}^{(\eta)}$ is the $J \times J$ diagonal matrix

$$\mathbf{Q}^{(\eta)} = S_p\left(-\frac{1}{T_o}\right)\text{sinc}(\eta+1)\mathbf{\Phi}^* + S_p(0)\text{sinc}(\eta)\mathbf{I}_J + S_p\left(\frac{1}{T_o}\right)\text{sinc}(\eta-1)\mathbf{\Phi} \quad (7)$$

where $\mathbf{\Phi}$ is the $J \times J$ diagonal unitary matrix

$$\mathbf{\Phi} = \text{diag}\{e^{-j2\pi\frac{\tau_1}{T_o}}, e^{-j2\pi\frac{\tau_2}{T_o}}, \ldots, e^{-j2\pi\frac{\tau_J}{T_o}}\}. \quad (8)$$

Consider the $N \times N$ positive semi-definite matrix pencil $\{\mathbf{S}_{xx}^{(\eta_2)} - \sigma_n^2\text{sinc}(\eta_2)\mathbf{I}_N, \mathbf{S}_{xx}^{(\eta_1)} - \sigma_n^2\text{sinc}(\eta_1)\mathbf{I}_N\} = \{\mathbf{A}\mathbf{Q}^{(\eta_2)}\mathbf{A}^H, \mathbf{A}\mathbf{Q}^{(\eta_1)}\mathbf{A}^H\}$. With judicious selection of the two frequency values, $\frac{\eta_1}{T_o}$ and $\frac{\eta_2}{T_o}$, corresponding diagonal elements of $\mathbf{Q}^{(\eta_1)}$ and $\mathbf{Q}^{(\eta_2)}$ are distinct thereby facilitating the use of the PRO-ESPRIT [3] algorithm to compute the $J$ principal generalized eigenvalues and eigenvectors of

$\{\mathbf{AQ}^{(\eta_2)}\mathbf{A}^H, \mathbf{AQ}^{(\eta_1)}\mathbf{A}^H\}$ in terms of the eigenvalues and eigenvectors of a $J \times J$ matrix.

For example, consider $\eta_1 = 0$ and $\eta_2 = 1$ corresponding to evaluating $\mathbf{S}_{xx}(f)$ at $f = 0$ (DC) and $f = \frac{1}{T_o}$, the symbol rate, respectively. In this case, $\mathbf{S}_{xx}^{(1)} = S_p\left(\frac{1}{T_o}\right)\mathbf{A}\mathbf{\Sigma}_P\mathbf{\Phi}\mathbf{A}^H$ and $\mathbf{S}_{xx}^{(0)} - \lambda_{min}\mathbf{I}_N = S_p(0)\mathbf{A}\mathbf{\Sigma}_P\mathbf{A}^H$, where $\lambda_{min}$ is the smallest eigenvalue of $\mathbf{S}_{xx}^{(0)}$ equal to the noise power, $\sigma_n^2$. $\mathbf{S}_{xx}^{(0)}$ may be spectrally decomposed as

$$\mathbf{S}_{xx}^{(0)} = \mathbf{E}_S\mathbf{\Lambda}_S\mathbf{E}_S^H + \lambda_{min}\mathbf{I}_N, \tag{9}$$

where $\mathbf{\Lambda}_S$ is a $J \times J$ diagonal matrix containing the $J$ largest eigenvalues of $\mathbf{S}_{xx}^{(0)}$ with $\lambda_{min} = \sigma_n^2$ subtracted out, and $\mathbf{E}_S$ is $N \times J$ containing the corresponding signal eigenvectors. Without proof due to space limitations, we state the following results obtained from applying PRO-ESPRIT to this particular matrix pencil. First, the $J$ principal generalized eigenvalues and eigenvectors satisfying $\left\{\mathbf{S}_{xx}^{(1)} - \gamma_j\mathbf{C}_{xx}^{(0)}\right\}\mathbf{v}_j = 0$ may be computed via the eigenvalues and eigenvectors of the $J \times J$ *unitary* matrix

$$\mathbf{\Psi} = \frac{S_p(0)}{S_p(1/T_o)}\mathbf{\Lambda}_S^{-\frac{1}{2}}\mathbf{E}_S^H\mathbf{S}_{xx}^{(1)}\mathbf{E}_S\mathbf{\Lambda}_S^{-\frac{1}{2}} \tag{10}$$

referred to as the "core rotations" matrix. Specifically, if $\mu_j$ and $\mathbf{t}_j$, are the *j-th* eigenvalue and corresponding eigenvector of the unitary matrix $\mathbf{\Psi}$ above, then $\gamma_j = \mu_j$ and $\mathbf{v}_j = \mathbf{E}_S\mathbf{\Lambda}_S^{-\frac{1}{2}}\mathbf{t}_j$. PRO-ESPRIT also dictates that $\mathbf{a}_j \propto \mathbf{E}_S\mathbf{\Lambda}_S^{\frac{1}{2}}\mathbf{t}_j$. Since $\gamma_j = \mu_j = \mathbf{\Phi}_{jj} = e^{-j2\pi\frac{\tau_j}{T_o}}$, PRO-ESPRIT provides a means for estimating the unknown timing offsets, $\tau_j$, $j = 1, ..., J$, which are useful for synchronization purposes, and the unknown array manifolds, $\mathbf{a}_j$, $j = 1, ..., J$, which are useful for beamforming purposes.

Assuming ergodicity, the ensemble average is estimated via time averaging over an integer number of symbol periods. In the continuous time domain, for a duration of $MT_o$ seconds, we form for each interval of $T_o$ seconds $\hat{\mathbf{R}}_{xx}^{(n)}(t) = \mathbf{x}(t)\mathbf{x}^H(t)\ rect\left(\frac{t-nT_o}{T_o}\right)$, $n = 0, 1, ..., M-1$. $\mathbf{S}_{xx}(f)$ is then estimated as the Fourier Transform of $\hat{\mathbf{R}}_{xx}(t) = \frac{1}{M}\sum_{n=0}^{M-1}\hat{\mathbf{R}}_{xx}^{(n)}(t)$, denoted $\hat{\mathbf{S}}_{xx}(f)$. In practice, assuming $L$ samples per symbol period, $T_o$, where $L$ is an integer, $L$ time-averaged correlation matrices are formed as

$$\hat{\mathbf{R}}_{xx}\left(\ell\frac{T_o}{L}\right) = \frac{1}{M}\sum_{m=0}^{M-1}\mathbf{x}\left(\ell\frac{T_o}{L} + mT_o\right)\mathbf{x}^H\left(\ell\frac{T_o}{L} + mT_o\right)$$

$$\ell = 0, 1, ..., L-1, \tag{11}$$

and $\mathbf{S}_{xx}^{(0)}$ and $\mathbf{S}_{xx}^{(1)}$ are then estimated as

$$\hat{\mathbf{S}}_{xx}^{(0)} = \sum_{\ell=0}^{L-1}\hat{\mathbf{R}}_{xx}\left(\ell\frac{T_o}{L}\right)\ ;\ \hat{\mathbf{S}}_{xx}^{(1)} = \sum_{\ell=0}^{L-1}\hat{\mathbf{R}}_{xx}\left(\ell\frac{T_o}{L}\right)e^{-j2\pi\frac{1}{L}\ell}.$$

Note that due to the squaring inherent in forming the zero-lag autcorrelation matrix, the sampling rate must be at least four times the bandwidth of $p(t)$, i. .e, $L \geq 4$.

Note that given the definitions above, $\hat{\mathbf{S}}_{xx}^{(0)}$ is simply the standard sample autocorrelation matrix averaged over $LM$ snapshots ($L$ samples per each of $M$ symbol periods). It follows from classical adaptive beamforming theory that the beamforming weight vector yielding the optimum signal to interference plus noise ratio for the *j-th* source is $\mathbf{w}_j = \hat{\mathbf{S}}_{xx}^{-1}(0)\mathbf{a}_j$, where we have invoked the definition $\mathbf{S}_{xx}^{(0)} \stackrel{\text{def}}{=} \mathbf{S}_{xx}(0)$ to avoid cumbersome notation. Let $\hat{\mathbf{S}}_{xx}(0)$ be spectrally decomposed as

$$\hat{\mathbf{S}}_{xx}(0) = \hat{\mathbf{E}}_S\hat{\mathbf{\Lambda}}_{S+N}\hat{\mathbf{E}}_S^H + \hat{\mathbf{E}}_N\hat{\mathbf{\Lambda}}_N\hat{\mathbf{E}}_N^H, \tag{12}$$

where $\hat{\mathbf{\Lambda}}_{S+N}$ and $\hat{\mathbf{E}}_S$ contain the $J$ largest eigenvalues and corresponding eigenvectors, respectively, and $\hat{\mathbf{\Lambda}}_N$ and $\hat{\mathbf{E}}_N$ contain the $N$-$J$ smallest eigenvalues and corresponding eigenvectors. The core rotations matrix $\mathbf{\Psi}$ is estimated by substituting $\hat{\mathbf{E}}_S$ into (10) along with $\hat{\mathbf{\Lambda}}_S = \hat{\mathbf{\Lambda}}_{S+N} - \hat{\sigma}_n^2\mathbf{I}_J$, where $\hat{\sigma}_n^2$ is an estimate of the noise power, i. e., $\hat{\sigma}_n^2 = \text{trace}\{\hat{\mathbf{\Lambda}}_N\}/(N\text{-}J)$. Since $\hat{\mathbf{S}}_{xx}^{-1}(0) = \hat{\mathbf{E}}_S\hat{\mathbf{\Lambda}}_{S+N}^{-1}\hat{\mathbf{E}}_S^H + \hat{\mathbf{E}}_N\hat{\mathbf{\Lambda}}_N^{-1}\hat{\mathbf{E}}_N^H$ and $\mathbf{a}_j$ is estimated to within a multiplicative scalar as $\hat{\mathbf{E}}_S\hat{\mathbf{\Lambda}}_S^{\frac{1}{2}}\hat{\mathbf{t}}_j$, where $\hat{\mathbf{t}}_j$ is the *j-th* eigenvector of $\hat{\mathbf{\Psi}}$, it follows from the orthonormality of the eigenvectors in $\hat{\mathbf{E}}_S$ and $\hat{\mathbf{E}}_N$ that $\mathbf{w}_j$ may be estimated as

$$\hat{\mathbf{w}}_j = \hat{\mathbf{E}}_S\hat{\mathbf{\Lambda}}_{S+N}^{-1}\hat{\mathbf{\Lambda}}_S^{\frac{1}{2}}\hat{\mathbf{t}}_j. \tag{13}$$

Note that although asymptotically the noise contribution to $\hat{\mathbf{S}}_{xx}^{(1)}$ vanishes, relative to $\mathbf{S}_{xx}^{(0)}$ the signal part of $\mathbf{S}_{xx}^{(1)}$ is scaled by the ratio $\rho_s = S_p\left(\frac{1}{T_o}\right)/S_p(0)$. This ratio decreases with decreasing excess bandwidth parameter $\beta$. For $\beta = .3$, $\rho_s = 1/26$. Thus, in the practical case where $\mathbf{S}_{xx}^{(1)}$ is estimated from a limited number of symbol periods due to the high variability of the multipath environment, $\hat{\mathbf{S}}_{xx}^{(1)}$ can have a rather high variance. Observe that

$$\begin{aligned} S_p\left(\frac{1}{T_o}\right) &= \int_{-\infty}^{\infty} P\left(\frac{1}{T_o} - f\right)P(f)df \qquad (14) \\ &= \int_{\frac{1-\beta}{2T_o}}^{\frac{1+\beta}{2T_o}} \frac{1}{4}\left\{1 - \sin^2\left[\frac{\pi T_o}{\beta}\left(f - \frac{1}{2T_o}\right)\right]\right\}df \end{aligned}$$

where $P(f)$ is the standard raised cosine spectrum equal to the Fourier Transform of $p(t)$ in (1). Thus, we see that for both the signal as well as the noise, the only part of their respective spectra that contribute to $\hat{\mathbf{S}}_{xx}^{(1)}$ is that energy contained in the frequency interval $\frac{1-\beta}{2T_o} < |f| < \frac{1+\beta}{2T_o}$.

To enhance performance consider passing each receiver output through a pair of linear time-invariant filters with impulse responses $h_+(t)$ and $h_-(t)$, respectively. Collectively, the outputs may be expressed as

$$\mathbf{y}^+(t) = \mathbf{x}(t) * h^+(t)\ ,\ \mathbf{y}^-(t) = \mathbf{x}(t) * h^-(t) \tag{15}$$

where $*$ denotes linear convolution. The idea then is to replace $\mathbf{S}_{xx}^{(1)}$ in the algorithm developed thus far by

$$\mathbf{S}_{y^+y^-}^{(1)} \stackrel{\text{def}}{=} \mathcal{F}\left\{\mathcal{E}\left[\mathbf{y}^+(t)\,\mathbf{y}^{-H}(t)\ \text{rect}\left(\frac{t}{T_o}\right)\right]\right\}\Big|_{f=1/T_o}. \tag{16}$$

That is, the core rotations matrix is constructed according to (10) with $\mathbf{S}_{xx}^{(1)}$ replaced by $\mathbf{S}_{y^+y^-}^{(1)}$ and $S_p\left(\frac{1}{T_o}\right)$ replaced

$$\text{by}\quad S_{g^+g^-}\left(\frac{1}{T_o}\right) = \mathcal{F}\left\{g^+(t)g^{-*}(t)\right\}\big|_{f=1/T_o}, \tag{17}$$

where: $g^+(t) = p(t) * h^+(t)$ , $g^-(t) = p(t) * h^-(t)$

$\Lambda_S$ and $\mathbf{E}_S$ are determined from $\mathbf{S}_{xx}^{(0)}$ as before.

$h^+(t)$ and $h^-(t)$ should be selected as the solution to the following optimization problem:

$$\underset{h^+(t),\,h^-(t)}{\text{Maximum}} \quad \frac{\mathcal{F}\left\{g^+(t)g^{-*}(t)\right\}\big|_{f=1/T_o}}{\mathcal{E}\left\{\mathcal{F}\left\{\nu_\ell^+(t)\nu_\ell^{-*}(t)\right\}\big|_{f=1/T_o}\right\}} \tag{18}$$

where: $\nu_\ell^+(t) = \nu_\ell(t) * h_+(t)$ , $\nu_\ell^-(t) = \nu_\ell(t) * h_-(t)$

and $\nu_\ell(t)$ is the $\ell$-$th$ component of the $N \times 1$ noise vector $\nu(t)$. At the time of writing of this paper, this optimization problem is still being solved, but based on the form of the integrand in (14) we postulate that for the white noise distribution employed in the simulations the solution is

$$h^+(t) = \frac{-2\beta\frac{t}{T_o} + \sin\left(\beta\pi\frac{t}{T_o}\right)}{\beta\frac{t}{T_o} - 4\left(\beta\frac{t}{T_o}\right)^3} e^{-j\pi\frac{t}{T_o}} \tag{19}$$

with $h^-(t) = h^{+*}(t)$.

### 3.1. Summary of PRO-ESPRIT Based Blind Adaptive Beamforming Algorithm

(1) Perform principal component analysis of:

$$\hat{\mathbf{S}}_{xx}^{(0)} = \frac{1}{M}\sum_{\ell=0}^{L-1}\sum_{m=0}^{M-1} \mathbf{x}\left(\ell\frac{T_o}{L} + mT_o\right)\mathbf{x}^H\left(\ell\frac{T_o}{L} + mT_o\right)$$

(a) Compute $\hat{\lambda}_j$ and $\hat{\mathbf{e}}_j$, $j = 1, ..., N$, as eigenvalues and corresponding eigenvectors of $\hat{\mathbf{S}}_{xx}^{(0)}$ with $\hat{\lambda}_1 \geq ... \geq \hat{\lambda}_N$.

(b) Apply test (e.g., Akaike Information Criterion) to eigenvalues to estimate number of signals present, $\hat{J}$.

(c) Estimate noise power: $\hat{\sigma}_n^2 = \frac{1}{N-\hat{J}}\sum_{i=\hat{J}}^{N}\hat{\lambda}_i$.

(d) Form $\hat{\Lambda}_{S+N} = \text{diag}\left\{\hat{\lambda}_1, ..., \hat{\lambda}_{\hat{J}}\right\}$, $\hat{\Lambda}_S = \hat{\Lambda}_{S+N} - \hat{\sigma}_n^2\mathbf{I}_{\hat{J}}$, and $\hat{\mathbf{E}}_S = [\hat{\mathbf{e}}_1, ..., \hat{\mathbf{e}}_{\hat{J}}]$

(2) Perform filtering operations $\mathbf{y}^+(t) = \mathbf{x}(t) * h^+(t)$ and $\mathbf{y}^-(t) = \mathbf{x}(t) * h^-(t)$ in analog or discrete time domain, and then construct for $\ell = 0, 1, ..., L-1$:

$$\hat{\mathbf{R}}_{y^+y^-}\left(\ell\frac{T_o}{L}\right) = \frac{1}{M}\sum_{m=0}^{M-1}\mathbf{y}^+\left(\ell\frac{T_o}{L} + mT_o\right)\mathbf{y}^{-H}\left(\ell\frac{T_o}{L} + mT\right.$$

$$\text{and} \quad \hat{\mathbf{S}}_{y^+y^-}^{(1)} = \sum_{\ell=0}^{L-1}\hat{\mathbf{R}}_{y^+y^-}\left(\ell\frac{T_o}{L}\right)e^{-j2\pi\frac{1}{L}\ell}$$

(3) Apply PRO-ESPRIT algorithm to

$$\hat{\Psi} = \frac{S_p(0)}{S_{g^+g^-}(1/T_o)}\hat{\Lambda}_S^{-\frac{1}{2}}\hat{\mathbf{E}}_S^H\hat{\mathbf{S}}_{y^+y^-}^{(1)}\hat{\mathbf{E}}_S\hat{\Lambda}_S^{-\frac{1}{2}}$$

(a) compute SVD: $\hat{\Psi} = \mathbf{U}\Sigma\mathbf{V}^H$, and $\hat{\mu}_j$ & $\hat{\mathbf{t}}_j$, $j = 1, ..., \hat{J}$, as the eigenvalues and eigenvectors of $\hat{\Psi}_U = \mathbf{U}\mathbf{V}^H$

(b) estimate optimum beamforming weight vector for $j$-$th$ source: $\hat{\mathbf{w}}_j = \hat{\mathbf{E}}_S\hat{\Lambda}_{S+N}^{-1}\hat{\Lambda}_S^{\frac{1}{2}}\hat{\mathbf{t}}_j$, $j = 1, ..., \hat{J}$.

(c) estimate timing offset for $j$-$th$ source: $\hat{\tau}_j = -\frac{T_o}{2\pi}\arg\{\mu_j\}$

Note that relative to step (3a) if the SVD of a matrix is $\hat{\Psi} = \mathbf{U}\Sigma\mathbf{V}^H$, the closest unitary matrix to $\hat{\Psi}$ in a Frobinius norm sense is $\hat{\Psi}_U = \mathbf{U}\mathbf{V}^H$ [3].

## 4. SIMULATIONS

Simulations were conducted involving two BPSK signals with $\beta = 0.9$ incident upon an $N = 6$ element uniform linear array with half-wavelength spacing from the respective angles $\theta_1 = 0°$ and $\theta_2 = 30°$ relative to broadside, and having SNR's per element of $\text{SNR}_1 = 9$ dB and $\text{SNR}_2 = 3$ dB, respectively. To quantify the angular separation between the two sources, if a standard co-phasal beam was formed towards the first source at broadside, the second would be located at the first peak sidelobe. The relative timing offset between the two sources was $\tau_2 - \tau_1 = .3T_o$. The sampling rate was $F_s = \frac{10}{T_o}$, i. e., 10 samples per symbol period. The convergence speed of the PRO-ESPRIT based blind adaptive beamforming algorithm for source 1, averaged over 1024 independent runs, is plotted in Figure 1 and is observed to yield an improvement of several dB over the subspace-constrained Phase-SCORE algorithm [2]. Although these simulations are simplistic (no multipath), they illustrate the rapid convergence of the algorithm.

## REFERENCES


[1] M. A. Birchler and S. C. Jasper, "A 64 KBPS Digital Land Mobile Radio System Employing M-16QAM", *Motorola Land Mobile Products Sector*, 1994.

[2] T. E. Biedka, "Subspace-Constrained SCORE Algorithms", *Rec. of 27th Asilomar Conf. on Signals, Systems, and Computers*, pp. 716-720, Nov. 1993.

[3] M.D. Zoltowski and D. Stavrinides, "Sensor array signal processing via a Procrustes rotations based eigenanalysis of the ESPRIT data pencil", *IEEE Trans. Acoust., Speech, Signal Process.*, pp. 832-861, June 1989.


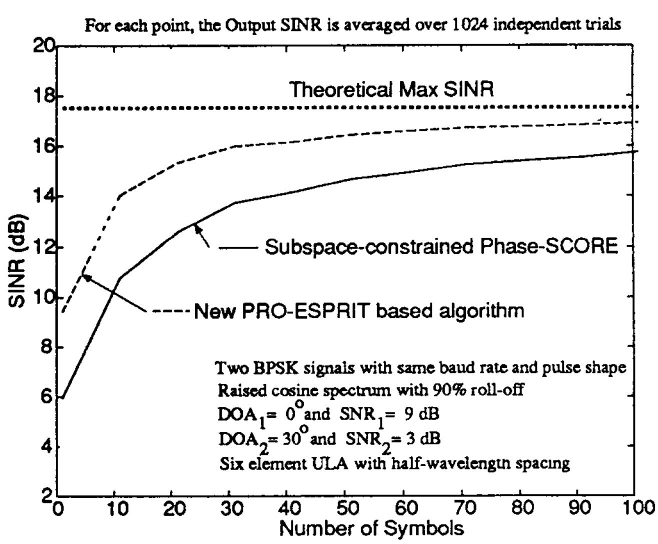


Figure 1: Convergence speed of new algorithm.