ESTIMATION OF CO-CHANNEL FM SIGNALS WITH MULTITARGET ADAPTIVE PHASE-LOCKED LOOPS AND ANTENNA ARRAYS

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ABSTRACT

A simple adaptive technique is proposed for separation and demodulation of multiple co-channel frequency modulated (FM) signals received at an antenna array. The proposed method, which for FM signals is embodied in an architecture referred to as a Multitarget Adaptive Phase-Lock Loop (MADPLL), exploits known signal structure through a complete demodulation and remodulation of the signals. The two properties of the signal that are exploited here are the known bandwidth of the information signal and the constant-modulus (CM) property of FM signals. It is shown that the proposed method can lead to significant improvements in performance over methods that exploit only the CM property.

1. INTRODUCTION

THE problem that we consider here is that of recovering multiple desired information signals from co-channel frequency-modulated (FM) signals received at an antenna array.

We propose a ***signal-structure*** solution to the co-channel FM signal problem *i.e.* a method that exploits only *a priori* information about the low-level structure of the signals of interest. Previously proposed signal-structure approaches to the co-channel signal problem include the so-called ***constant modulus*** (CM) methods (see [1] and references therein), which exploit the constant modulus property of signals derived from modulation schemes such as *e.g.* FM and QPSK, and ***finite alphabet*** (FA) methods [2], which exploit the finite alphabet property of digital information signals.

The method described in this paper essentially exploits two known properties of the signals: (i) the bandwidth of the desired information signals, and (ii) the constant modulus (CM) property FM signals. For FM signals, we refer to the proposed demodulator structure as a ***Multitarget Adaptive Phase-Lock Loop*** (MADPLL). The three main points intended to be conveyed by this paper are:

(i) Incorporating a full demodulation followed by remodulation can improve signal estimates by fully utilizing all of the known properties of the signals, *e.g.* with FM signals the known signal bandwidth can be used in addition to the constant-modulus property.

(ii) The incorporation of a forward channel (array response) model in addition to the traditional inverse channel (array response) model, facilitates separation of all signals in the multiple desired signal setting.

(iii) The simplicity of the the MADPLL makes it amenable to real-time implementation.

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2. PROBLEM FORMULATION

The model assumed for the signal measured at the output of the antenna array may be expressed as

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}, \tag{1}$$

where,

$\mathbf{S}$	=	$d \times n$ matrix of snapshots of the modulated information signals $\mathbf{Y}$.
$\mathbf{X}$	=	$m \times n$ matrix of measurements collected at the antenna array
$\mathbf{A}$	=	The array response matrix.
$\mathbf{N}$	=	Additive noise.

Here, d = number of signals, m = number of sensors (antennas), and n = number of measurements.

We assume that the signals are frequency modulated and that the information signal is band limited, *i.e.* the p^{th} row of $\mathbf{S}$ contains samples of the FM signal

$$s_p(t) = \exp\left(\omega_c + k\int_0^t y_p(t)dt\right),$$

where $y_p(t)$ (and therefore the phase of $s_p(t)$) is band limited. We will also assume that the signals are independent in the sense that the dimension of the signal subspace, given by the effective rank of the measurement matrix $\mathbf{X}$, is equal to the number of signals, *i.e.* ($\rho(\mathbf{X}) = d$).

The problem that is addressed in this paper is that of *finding estimates* $\hat{\mathbf{Y}}$ *of the desired (demodulated) signals contained in the rows of* $\mathbf{Y}$*, given the measurements* $\mathbf{X}$.

3. THE MADPLL

The proposed *Multitarget Adaptive Phase-Lock Loop* (MADPLL) demodulator structure for co-channel FM signals is depicted in Fig. 1. The MADPLL demodulator consists of two primary signal paths:

The Forward Path, which consists of a weight matrix $\overline{\mathbf{W}}$ (the *inverse array response model*), followed by a bank of independent PLL demodulators.

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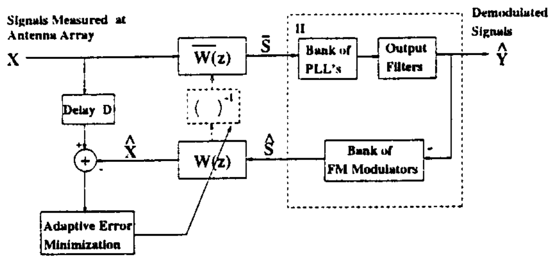

Figure 1: Block diagram of a MADPLL demodulator for estimation of co-channel FM signals.

The Feedback Path, which consists of a bank of FM modulators followed by the weight matrix $\mathbf{W}$ (the *array response model*).

The proposed MADPLL method may be described in the form of an adaptive algorithm as follows.

(i) Estimate the modulated signals $\mathbf{S}$ at time k using the current estimate $\overline{\mathbf{W}}$ for the inverse array response.
$$\tilde{\mathbf{S}}(k) = \overline{\mathbf{W}}\mathbf{X}(k),$$
where $\mathbf{X}(k)$ denotes the k^{th} column of $\mathbf{X}$.

(ii) Estimate the information signals $\hat{\mathbf{Y}}(k)$ by demodulating the $\tilde{\mathbf{S}}(k)$.

(iii) Remodulate $\hat{\mathbf{Y}}(k)$ to obtain new estimates $\hat{\mathbf{S}}(k)$ for the modulated signals.

(iv) Estimate the signals received at the antenna array based on the current estimate $\hat{\mathbf{S}}(k)$, of the modulated signals and the current estimate $\mathbf{W}$, of the array response, *i.e.*
$$\hat{\mathbf{X}}(k) = \mathbf{W}\hat{\mathbf{S}}(k).$$

(v) Update the array response model $\mathbf{W}$ to decrease the error between the estimate $\hat{\mathbf{X}}$ and the measured $\mathbf{X}$, *e.g.* by minimizing
$$\mathcal{E}(k) = \sum_{l=0}^{k} \lambda^k \|\mathbf{X}(l) - \hat{\mathbf{X}}(l)\|^2, \quad 0 < \lambda \leq 1,$$
using the *recursive least-squares* (RLS) algorithm. (Any delay in the demodulation/remodulation step can be accounted for here with a corresponding delay in the error definition (see Fig. 1)).

(vi) Update the inverse array response $\overline{\mathbf{W}}$ using the psuedoinverse of $\mathbf{W}$, *i.e.* $\overline{\mathbf{W}} = \mathbf{W}^{\dagger} = (\mathbf{W}^{*}\mathbf{W})^{-1}\mathbf{W}^{*}$.

(vii) $k \longleftarrow k+1$, Go to Step (i).

As mentioned earlier, the MADPLL exploits both the known bandwidth of the information signals $\mathbf{Y}$ and the CM property $\mathbf{S}$. The bandwidth information is introduced in the design of the PLL demodulators (and their associated output filters) to estimate $\mathbf{Y}$. The CM property is used by remodulating the estimated information signals to provide estimates of $\mathbf{S}$.

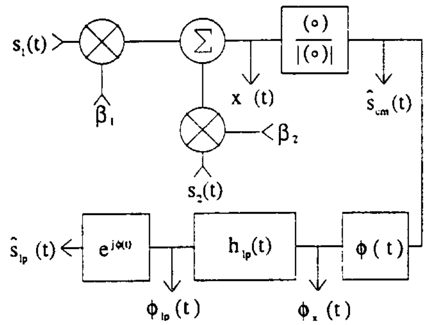

Figure 2: Phase Filtering Signal Estimation Block Diagram

A key difference between the MADPLL and combiner/receiver structures that are designed for the single signal case, is that the error used to update the combiner weights is computed at the the array output instead of at the output of the weight matrix. In the (noise free) case where two or more channels have converged to the same signal, the error between $\hat{\mathbf{S}}$ and $\tilde{\mathbf{S}}$ could be zero, while the error measured at the array output between $\hat{\mathbf{X}}$ and $\mathbf{X}$ is nonzero. Hence the incorporation of the array response model $\mathbf{W}$ in the feedback loop facilitates the separation of all signals present

4. ESTIMATION ERROR IMPROVEMENT

In this section, we analyze the effect of phase filtering on mean square signal estimation error given one desired constant modulus signal, and a second interfering constant modulus signal. Our purpose here is to motivate and provide insight into the phase filtering approach. We first analyze the improvement in mean square error if the received signal is simply forced to the constant modulus circle. We then show that phase filtering further improves the estimation error provided that the signal and interference CM waveforms possess certain properties. The process of phase filtering is presented in Fig. 2. We consider the received complex waveform

$$\begin{aligned} x(t) &= \beta_1 s_1(t) + \beta_2 s_2(t) \\ &= \beta_1[s_1(t) + \alpha s_2(t)] \qquad 0 \leq |\alpha| \leq 1 \qquad (2) \\ s_1(t) &= e^{j\phi_1(t)}, \qquad s_2(t) = e^{j\phi_2(t)} \qquad (3) \end{aligned}$$

We assume that the larger of the two received signal components ($\beta_1 s_1(t)$) is the desired signal. We also constrain $\phi_1(t)$ and $\phi_2(t)$ to be band limited information waveforms. In what follows, we normalize $x(t)$ by $\frac{1}{\beta_1}$ so that we evaluate mean square error normalized to the average desired signal energy. The mean square error between the received signal and the desired signal is

$$MSE_y = E\left\{|x(t) - s_1(t)|^2\right\} = \alpha^2 \qquad (4)$$

We can express $x(t)$ as an amplitude and phase modulation

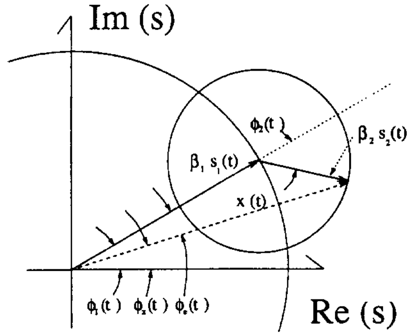

Figure 3: Received Waveform Phasor Diagram

$$x(t) = \gamma_y(t)e^{j\phi_x(t)} \tag{5}$$

Using the phasor diagram presented in Fig. 3, we can write the received signal phase term as

$$\begin{aligned}\phi_x(t) &= \phi_1(t) + \phi_e(t) \\ &= \phi_1(t) + \arctan\left[\frac{\alpha \sin(\phi_\Delta(t))}{1+\alpha\cos(\phi_\Delta(t))}\right]\end{aligned} \tag{6}$$

where $\phi_e(t)$ is the phase error term and $\phi_\Delta(t)$ is the difference in phase between the two received signal components. If we force the received composite signal to the unit, then the new CM signal is

$$\hat{s}_{cm}(t) = \frac{x(t)}{|x(t)|} = e^{j\phi_x(t)} \tag{7}$$

The mean square error for the CM signal is

$$\begin{aligned}MSE_{\hat{s}_{cm}} &= E\left\{|\hat{s}_{cm}(t) - s_1(t)|^2\right\} \\ &= 2(1 - E\{\cos(\phi_e(t))\})\end{aligned} \tag{8}$$

For a given CM distribution for $p(\phi_1(t))$ and $p(\phi_2(t))$, the distribution function $p(\phi_\Delta(t))$ can be determined and the MSE expression in Equation (8) may be evaluated.

We now investigate the improvement in MSE obtained from low pass filtering the phase waveform $\phi_x(t)$. We note that the expression for the phase error waveform in (6) is a nonlinear bandwidth expanding function. So we observe that if we low pass filter the phase error term, the phase error energy will be reduced. We define the low pass filtered phase waveform $\phi_{lp}(t) = \phi_x(t) * h_\phi(t)$ where $h_\phi(t)$ is the low pass phase filter function. We now form a second signal estimate $\hat{s}_{lp}(t) = e^{j\phi_{lp}(t)}$ We wish to show that $\hat{s}_{lp}(t)$ is an improved MSE signal estimate for certain signal waveforms. The MSE for the signal estimate $\hat{s}_{lp}(t)$ is found from the expression

$$MSE_{\hat{s}_{lp}} = 2(1 - E\{\cos(\phi_{lp}(t))\}) \tag{9}$$

While the expressions for MSE given in Equations (8) and (9) are usually quite difficult to reduce to analytical solutions, they can be evaluated numerically. For certain random process descriptions for $\phi_\Delta(t)$, the improvement in MSE after applying phase filtering can be significant. We illustrate this by considering two frequency modulated (FM) signals. We first express the signal MSE for the CM signal and the phase filtered signal assuming a fixed frequency difference between the received component signals. This characterizes the *instantaneous* phase filtered signal estimation improvement as a function of *instantaneous* frequency difference between the two component signals. We then evaluate the *average* signal MSE for the CM signal and the phase filtered signal for the specific case of band limited Gaussian frequency modulation for each signal.

For two FM signals $s_1(t)$ and $s_2(t)$, with a fixed frequency offset between the signals, the phase difference is given by

$$\phi_\Delta(t) = 2\pi(f_2 - f_1)t = 2\pi f_\Delta t \tag{10}$$

where $f_\Delta(t)$ is the frequency difference between the two signals. The distribution for $\phi_\Delta(t)$ is then uniform and the expression from (8) becomes

$$MSE_{\hat{s}_{cm}} = 2\left(1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos \left[\arctan\left(\frac{\alpha\sin(\phi_\Delta(t))}{1+\alpha\cos(\phi_\Delta(t))}\right)\right] d\phi_\Delta(t)\right) \tag{11}$$

Equation (11) is also the expression for the time averaged MSE. The MSE expression for the phase filtered signal, $\hat{s}_{lp}(t)$, is more difficult to evaluate because the time correlation for $\phi_\Delta(t)$ is essential; so this signal must be taken as a random process rather than a random variable. We can write the signal MSE conditioned on f_Δ in time average form as

$$MSE_{\hat{s}_{lp}}|f_\Delta = 2\left(1 - \lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} \cos \left[\arctan\left(\frac{\alpha\sin(2\pi f_\Delta t)}{1+\alpha\cos(2\phi_\Delta t)}\right) * h_\phi(t)\right] dt\right) \tag{12}$$

To illustrate the effect of phase filtering on signal MSE, using (11) and (12), we evaluate the improvement in MSE for the phase filtered signal waveform as compared to the MSE for the cm waveform. In Fig. 4, the ratio of phase filtered waveform error waveform error ($MSE_{\hat{s}_{lp}}|f_\Delta$) over CM waveform error ($MSE_{\hat{s}_{cm}}$) is plotted as a function of the frequency ratio $\frac{f_\Delta}{B_\phi}$ and α. B_ϕ is the cut off frequency of the low pass phase filter. Fig. 4 nicely illustrates the concept of phase filtering. By filtering the received phase waveform and re-modulating to obtain a signal estimate, we can significantly improve the signal estimation error provided that the frequency offset between the two signals is greater than the phase filter bandwidth. Based on these results, we can also see that the phase filtering operation will not result in a large signal estimation error improvement for all CM signal sets. Unless there is significant phase error energy beyond the bandwidth of the phase filter, little improvement is observed in Fig. 4.

To further illustrate the phase filtering concept, a second analysis has been performed for two FM signals which posses independent band limited Gaussian frequency waveforms. For this signal set, the MSE for the raw signal, the CM signal, and several phase filtered signals is presented in

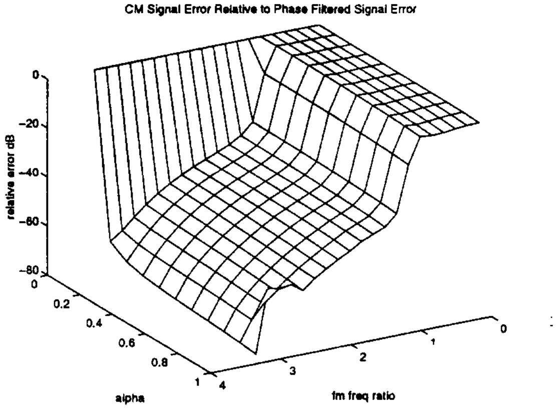

Figure 4: Estimation Error of Phase Filtered Signal Relative to CM Signal vs. Fixed Frequency Difference Between Component Signals

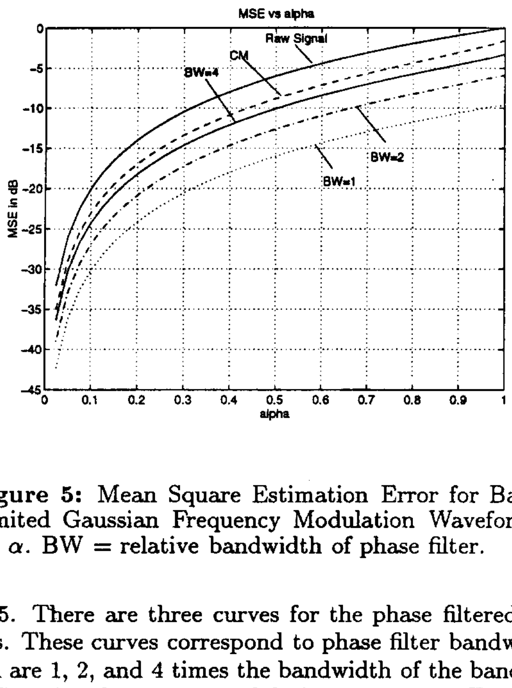

Figure 5: Mean Square Estimation Error for Band Limited Gaussian Frequency Modulation Waveforms vs. α. BW = relative bandwidth of phase filter.

Fig. 5. There are three curves for the phase filtered estimates. These curves correspond to phase filter bandwidths which are 1, 2, and 4 times the bandwidth of the band limited Gaussian frequency modulation processes. The MSE for the phase filtered waveform improves dramatically as the phase filter bandwidth is reduced.

5. SIMULATION RESULTS

The simulation results presented here are obtained for three sensors and three signals ($m = 3$, $d = 3$), with a sampling frequency f_s = 180KHz, carrier frequency f_c = 60 KHz, and signal bandwidth f_b = 10KHz. PLL FM demodulators were used to demodulate the signals, and the RLS algorithm was used to update the weight matrix. We compare the performance of the MADPLL described in Sec. 3, with a multitarget CM demodulator constructed by replacing the block labeled **II** in Fig. 1 by a projection onto the unit circle $(\cdot/|\cdot|)$.

The results of these simulations are shown in Fig. 6, where average output SNR's for the two cases are plotted as a function of average input CNR at the sensors. The output SNR's are computed using the output signal after a delay that accounts for the convergence time of the algorithm. It

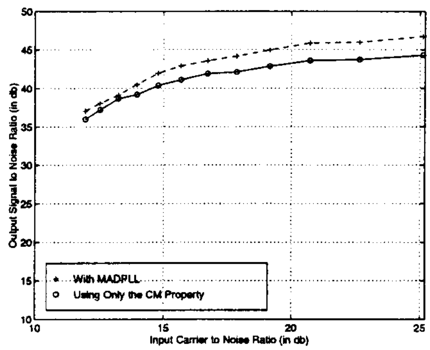

Figure 6: Average output SNR versus average input CNR for the constant gain channel. The averages are taken over 20 trials and over the three signals/sensors with random signals and array responses.

is improved performance of the MADPLL may be observed from these curves. As mentioned in the last section, the actual amount of improvement is dependent on both the signal and the array response.

6. SUMMARY AND DISCUSSION

In this paper we described a simple adaptive method that exploits known properties of the signal to estimate co-channel FM signals using measurements from an antenna array. The three distinguishing features of the method described are: (i) the use of a full demodulation followed by remodulation to improve signal estimates by utilizing the known bandwidth of the information signals in addition to the constant modulus (CM) property of FM signals, (ii) the use of a forward channel (array response) model in addition to the inverse channel (array response) model, to facilitate separation of all signals present, and (iii) the amenability of the MADPLL to real-time implementation. The improvement in performance over methods using only the CM property was shown through both analysis and simulations.

In the case of a multipath channels with significant delay-spreads, a variant of the MADPLL technique proposed here may be applied, which requires the use of MIMO filters $\mathbf{W}(z)$ and $\overline{\mathbf{W}}(z)$ in place of the weight matrices; this will be described elsewhere together with extensions to other modulation formats.

7. REFERENCES

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