ADAPTIVE MULTIUSER BOOTSTRAPPED DECORRELATING CDMA DETECTOR FOR ONE-SHOT ASYNCHRONOUS UNKNOWN CHANNELS[1]

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ABSTRACT

Vast research was recently performed on signal detection of multiuser Code Division Multiple Access (CDMA). Particularly for uplink (users to base station) the signals are asynchronous and the near-far problem is an important issue to deal with. All near-far resistant detectors, adaptive or non-adaptive, assume knowledge of the relative delays of the different users' signals. Among these are the one-shot detectors suggested by Verdu. In this paper we suggest an adaptive algorithm to decorrelate the outputs of the one shot matched filters that assumes no knowledge of the relative delay. The performance of this approach is shown to be better than the non-adaptive "zero-forcing" method previously used and which implements linear transformation via the inverse of matched filters' output cross-correlation. For simplicity a two-user case is presented. Extension to a higher number of users is relatively simple.

I. INTRODUCTION

CDMA is considered to be a promising multiplexing method for multiuser personal, mobile and indoor communications. One of the problems a designer of such a system is faced with is the so called "near-far problem," resulting from excessive Multiple Access Interference (MAI) energy from nearby users, compared with the desired user's signal energy. Power control, that is, adjustment of transmitter power, depending on its location and the signal energies of the other users, has been suggested as a solution to this problem. But it requires a significant reduction in the signal energies of the strong users in order for the weaker users to achieve reliable communication. This results in an overall reduction in communication ranges.

An optimum near-far resistant multiuser detector, without a need for power control has been proposed by Verdu[1]. Its complexity, however, is exponential in terms of the number of users, which makes it unsuitable for practical situations. Later, a sub-optimum detector whose complexity is linear was proposed[2]. However this detector, like Verdu's optimum detector, needs the knowledge of user signal energies.

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Adaptive near-far resistant CDMA detectors that do not need the knowledge of the received signal energies have been developed at NJIT in recent years[3–6]. All these detectors assume a non-fading multiuser environment.

All asynchronous detectors (except [6]), adaptive or non-adaptive, assume knowledge of the different users signals' relative delays.

In this paper we propose to use the decorrelation algorithm for separation of the user signals obtained at the outputs of Verdu's one-shot bank of matched filters. For this we do not require knowledge of the relative delays.

In the next section we present the system model and drive the relation between the matched filters' output and the users data. Since the delays are not known, the cross-coupling matrix is considered unknown. The decorrelating algorithm and the decorrelator's steady-state weights and outputs are given in Section III. In Section IV. we discuss performance evaluation and in Section V. we present an example and results.

II. SYSTEM MODEL

For the asynchronous multiuser, the equivalent low-pass signal at the input of the matched filter bank is given by,

$$r(t) = \sum_{k=1}^{K} \sum_{i} b_k(i)\sqrt{a_k(i)}s_k(t+T-\tau_k) + n(t), \quad (1)$$

where K is the number of users, $\alpha_k, \theta_k, b_k, s_k$ and τ_k are the signal amplitude, carrier phase shift, user bit, signature waveform and the relative delay of the kth user. $n(t)$ is the zero mean AWGN, with a two-sided power spectral density of $N_0/2$.

For the sake of simplicity we will restrict ourselves to two users only. Extension to a higher number is relatively simple.

Representing the signal of Eq. (1) in one-shot of the ith bit of user one, and without loss of generality, letting $i = 0$, we write,

$$\begin{aligned} r(t) &= \sqrt{a_1}s_1(t)b_1(0) + \sqrt{a_2\epsilon_2}\frac{1}{\sqrt{\epsilon_2}}s_2^L(t)b_2(-1) \\ &\quad + \sqrt{a_2(1-\epsilon_2)}\frac{1}{\sqrt{1-\epsilon_2}}s_2^R(t)b_2(0) + n(t), \quad (2) \end{aligned}$$

where $0 \leq \tau_2 \leq T$ is considered unknown or estimated with an error, and;

$$
\begin{aligned}
s_2^L(t) &= \begin{cases} s_2(t+T-\tau_2) & \text{if } 0 \leq t \leq \tau_2 \\ 0 & \text{if } \tau_2 < t \leq T \end{cases} \\
s_2^R(t) &= \begin{cases} 0 & \text{if } 0 \leq t \leq \tau_2 \\ s_2(t-\tau_2) & \text{if } \tau_2 < t \leq T \end{cases} \\
\epsilon_2 &= \int_0^{\tau_2} s_2^2(t+T-\tau_2)dt. \qquad (3)
\end{aligned}
$$

As shown in Fig. 1, we apply $r(t)$ to a bank of filters by using as a second input $s_1(t), s_2^{L'}(t), s_2^{R'}(t)$, respectively, the signature function of Eq. (3) extended to τ_2' instead of τ_2, normalized by ϵ_2' instead of ϵ_2.

The output of the first filter,

$$x_1'(0) = \sqrt{a_1}b_1(0) + \sqrt{a_2\epsilon_2}\rho_{12}b_2(0) + n_1(0), \qquad (4)$$

where $n_1(0) = \int_0^T n(t)s_1(t)dt$ is a zero-mean Gaussian variable with variance of $N_0/2$, and

$$
\begin{aligned}
\rho_{21} &= \frac{1}{\sqrt{\epsilon_2}}\int_0^T s_1(t)s_2(t+T-\tau_2)dt \\
\rho_{12} &= \frac{1}{\sqrt{1-\epsilon_2}}\int_0^T s_1(t)s_2(t-\tau_2)dt. \qquad (5)
\end{aligned}
$$

The subscript in $x'(0)$ indicates that we are using zero bit of user one in our one-shot.

The output of the second filter is given by,

$$
\begin{aligned}
x_2'(0) &= \frac{1}{\sqrt{\epsilon_2'}}\int_0^T r(t)s_2^{L'}(t)dt \\
&= \sqrt{a_1}\rho_{21}^{L'}b_1(0) + \sqrt{a_2\epsilon_2}\rho_{22}^{L'L}b_2(-1) \\
&\quad + \sqrt{a_2(1-\epsilon_2)}\rho_{22}^{L'R}b_2(0) + n_2(0), \qquad (6)
\end{aligned}
$$

where

$$
\begin{aligned}
\rho_{21}' &= \frac{1}{\sqrt{\epsilon_2'}}\int_0^T s_1(t)s_2(t+T-\tau_2')dt \\
\rho_{22}^{L'L} &= \frac{1}{\sqrt{\epsilon_2'\epsilon_2}}\int_0^T s_2(t+T-\tau_2)s_2(t+T-\tau_2')dt \\
\rho_{22}^{L'R} &= \frac{1}{\sqrt{\epsilon_2'(1-\epsilon_2)}}\int_0^T s_2(t-\tau_2)s_2(t+T-\tau_2')dt
\end{aligned}
$$

and

$$n_2(0) = \frac{1}{\sqrt{\epsilon_2'}}\int_0^T n(t)s_2(t+T-\tau_2')dt. \qquad (7)$$

The output of the third filter is given by,

$$
\begin{aligned}
x_3'(0) &= \frac{1}{\sqrt{1-\epsilon_2'}}\int_0^T r(t)s_2^{R'}(t)dt \\
&= \sqrt{a_1}\rho_{12}'b_1(0) + \sqrt{a_2\epsilon_2}\rho_{22}^{R'L}b_2(-1) \\
&\quad + \sqrt{a_2(1-\epsilon_2)}\rho_{22}^{R'R}b_2(0) + n_3(0), \qquad (8)
\end{aligned}
$$

where

$$
\begin{aligned}
\rho_{12}' &= \frac{1}{\sqrt{1-\epsilon_2'}}\int_0^T s_1(t)s_2(t-\tau_2')dt \\
\rho_{22}^{R'L} &= \frac{1}{\sqrt{(1-\epsilon_2')\epsilon_2}}\int_0^T s_2(t+T-\tau_2)s_2(t-\tau_2')dt \\
\rho_{22}^{R'R} &= \frac{1}{\sqrt{(1-\epsilon_2')(1-\epsilon_2)}}\int_0^T s_2(t-\tau_2)s_2(t-\tau_2')dt
\end{aligned}
$$

and

$$n_3(0) = \frac{1}{\sqrt{1-\epsilon_2'}}\int_0^T n(t)s_2(t-\tau_2')dt. \qquad (9)$$

Combining, Eqs. (4), (6) and (8) in matrix form,

$$\mathbf{x}'(0) = \mathbf{P}^T\mathbf{A}\mathbf{b}(0) + \mathbf{n}(0), \qquad (10)$$

where

$$\mathbf{P}^T = \begin{bmatrix} 1 & \rho_{21} & \rho_{12} \\ \rho_{21}' & \rho_{22}^{L'L} & \rho_{22}^{L'R} \\ \rho_{12}' & \rho_{22}^{R'L} & \rho_{22}^{R'R} \end{bmatrix}, \qquad (11)$$

$\mathbf{A} = \mathrm{diag}[\sqrt{a_1}, \sqrt{a_2\epsilon_2}, \sqrt{a_2(1-\epsilon_2)}]$ and $\mathbf{b}(0) = [b_1(0), b_2(-1), b_2(0)]^T$. $\mathbf{n}(t)$ is zero-mean Gaussian vector with covariance,

$$\mathbf{R}_n = \begin{bmatrix} 1 & \rho_{21}' & \rho_{12}' \\ \rho_{21}' & 1 & 0 \\ \rho_{12}' & 0 & 1 \end{bmatrix} \frac{N_0}{2}. \qquad (12)$$

2.1. Known Delay

If the relative delay τ_2 is known then we choose $\tau_2' = \tau_2$, and as a result $\rho_{21}' = \rho_{21}, \rho_{12}' = \rho_{12}, \rho_{22}^{L'L} = \rho_{22}^{R'R} = 1$, $\rho_{22}^{R'L} = \rho_{22}^{L'R} = 0$ and $\epsilon_2' = \epsilon_2 = 1$. The matrix $\mathbf{P}_D$ is known. If $|\rho_{21}| + |\rho_{12}| < 1$ then $\mathbf{P}_D$ is invertible. A simple linear transformation by $\mathbf{P}_D^{-1}$ results in,

$$\mathbf{z}(0) = \mathbf{A}\mathbf{b}(0) + \mathbf{P}_D^{-T}\mathbf{n}(0), \qquad (13)$$

where $\mathbf{P}_D$ is the modified cross-coupling matrix. Since the components of $\mathbf{b}(0)$ are assumed uncorrelated then the components of $\mathbf{z}(0)$ are also so.

III. THE DECORRELATING ALGORITHM

3.1. Unknown Delay

For this case the matrix $\mathbf{P}$ is unknown. We propose to use an adaptive algorithm to decorrelate the outputs of bank of matched filters (see Fig. 2). From this figure we have

$$\mathbf{z}'(0) = \mathbf{x}'(0) - \mathbf{W}^T\mathbf{x}'(0), \qquad (14)$$

where

$$\mathbf{W} = \begin{bmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{bmatrix}. \qquad (15)$$

The kth output of the decorrelator can be expressed as,

$$z_k = x_k - \mathbf{w}_k^T\mathbf{x}_k \qquad (16)$$

where $\mathbf{w}_k$ is the kth column of $\mathbf{W}$ with the element w_{kk} deleted and $\mathbf{x}_k$ is the vector obtained from $\mathbf{x}$ by deleting x_k.

The multidimensional decorrelator is the same as that frequently used in neural networks and other applications of signal separation[7–8].

For controlling the weights, we use the steepest descent algorithm which simultaneously reduces the absolute value of the correlation between the outputs of the decorrelator and the decision on all other outputs. That is, the weight, w_{lk} is controlled by the recursion,

$$w_{ml} \leftarrow w_{ml} - \mu(z_l \mathrm{sgn}(z_m)), \quad 1 \le l, m < 3, \ \ l \neq m. \quad (17)$$

Recursion in Eq. (17), reaches steady-state in the mean when $E\{z_l \mathrm{sgn}(z_m)\} = 0$. Note that w_{21}, for example, is used to cancel residue of $b_2(-1)$ at the output of z_1. It will settle down only when that residue (being correlated with $\hat{b}_2(-1)$) is zero. But any reduction of $b_2(-1)$ at z_1 will improve $b_1(0)$ (smaller error) and hence, will be more effective in reducing the residue of $b_1(0)$ at z_2 and z_3 through the control weights w_{12} and w_{13}. Therefore, the process of residue cancellation is enhanced successively which justify, using the name "bootstrap" in the past.

In the notation of Eq. (16), we write Eq. (17) in vector form,

$$\mathbf{w}_k \leftarrow \mathbf{w}_k - \mu(z_k \mathrm{sgn}(\mathbf{z}_k)), \quad (18)$$

where again $\mathbf{z}_k$ is obtained from the column of $\mathbf{z}$ by deleting z_k. The steady state is reached if $E\{z_k \mathrm{sgn}(\mathbf{z}_k) = \mathbf{0}$ for $k = 1, 2, 3$.

Now, $\mathrm{sgn}(\mathbf{z}_k) = \hat{\mathbf{b}}_k$, and if we assume the SNR's are large enough such that the main contribution to the decorrelator output error is the multiuser interference, then $E\{z_k \hat{\mathbf{b}}_k\}$ can be approximated by,

$$\begin{aligned} E\{z_k \hat{\mathbf{b}}_k\} &= E\{z_k \hat{\mathbf{b}}_k\}(\mathbf{I} - 2\Pr(\hat{\mathbf{b}}_k \text{ in error})) \\ &= E\{z_k \hat{\mathbf{b}}_k\}(\mathbf{I} - 2P_{ek}). \end{aligned} \quad (19)$$

Equating (19) to zero and using (16) together with (10) we get,

$$E\{z_k \hat{\mathbf{b}}_k\} = E\{\rho_{kk} a_{kk} b_k + \rho_k^T \mathbf{A}_k \mathbf{b}_k + n_k - \mathbf{w}_k^T \mathbf{x}_k\}\mathbf{b}_k = 0, \quad (20)$$

where ρ_{kk} is the kth term at the diagonal of $\mathbf{P}$, a_{kk} is the kth diagonal of $\mathbf{A}$, ρ_k is the kth column of $\mathbf{P}$ with its kth element deleted and so is $\mathbf{A}_k$.

3.2. Steady-State Weights and Decorrelator Output

Solving Eq. (20), we show in the Appendix that $E\{\mathbf{w}_k^T \mathbf{x}_k \mathbf{b}_k\} = \mathbf{A}_k \rho_k$ We also show that, $E\{\mathbf{w}_k^T \mathbf{x}_k \mathbf{b}_k\} = \mathbf{A}_k \mathbf{P}_k \mathbf{W}_k$. Therefore, the steady-state weight resulting from the decorrelating algorithm is given by,

$$\mathbf{w}_k = \mathbf{P}_k^{-1} \rho_k, \quad (21)$$

Substituting Eq. (21) in (16) with (A-2), we drive in the Appendix; Eq. (A-4), the decorrelator's steady-state outputs,

$$z_k = a_{kk}(\rho_{kk} - \rho_k^T \mathbf{P}_k^{-T} \mathbf{r}_k) b_k + n_k - \rho_k^T \mathbf{P}_k^{-T} \mathbf{n}_k. \quad (22)$$

where $\mathbf{P}_k$ is obtained from $\mathbf{P}$ by deleting the kth row and column, while $\mathbf{r}_k^T$ is the kth row without its kth element

IV. PERFORMANCE EVALUATION

From Eq. (22) for user one, after combining the noise terms, we have,

$$z_1 = \sqrt{a_1}(1 - \rho_1^T \mathbf{P}_1^{-T} \mathbf{r}_1) b_1 + \begin{bmatrix} 1 & \rho_1^T \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -\mathbf{P}_1^{-T} \end{bmatrix} \mathbf{n}. \quad (23)$$

Note that the decorrelating detector perfectly cancels the interfering signal energy. On the other hand, the desired user bit energy is modified by $(1 - \rho_1^T \mathbf{P}_1^{-T} \mathbf{r}_1)$, and the noise variance is given by

$$\sigma_N^2 = \rho_\epsilon^T \mathbf{R}^{-T} \mathbf{R}_n \mathbf{R}^{-1} \rho_\epsilon \frac{N_0}{2}, \quad (24)$$

where $\rho_\epsilon = \begin{bmatrix} 1 & \rho_1^T \end{bmatrix}^T$, $\mathbf{R}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -\mathbf{P}_1^{-T} \end{bmatrix}$ and $\mathbf{R}_n$ is the noise covariance matrix given in Eq. (12).

Let,

$$\gamma_1 = \frac{(1 - \rho_1^T \mathbf{P}_1^{-1} \mathbf{r}_1)^2 a_1}{\sigma_N^2}. \quad (25)$$

The BER for binary PSK as a function of SNR is given by $Q(2\gamma_1)$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$.

V. EXAMPLE AND RESULTS

In our numerical calculation and simulation, we assume $\epsilon_2 = 0.4, \rho_{21} = 0.2/\sqrt{0.4}, \rho_{12} = 0.6/\sqrt{0.6}$. If $\tau_2' < \tau_2$, then $\epsilon_2' < \epsilon_2$ (assumed 0.35) and hence the overlap of $s_2^{R'}$ with s_1 is larger than that of s_2^R. Therefore we take $\rho_{12}' = 0.25/\sqrt{0.35}$. With the same argument, $\rho_{21}' = 0.55/\sqrt{0.35}r$, $\rho_{22}^{L'L} = \sqrt{\epsilon_2'/\epsilon_2} = \sqrt{0.35/0.4}$ and $\rho_{22}^{R'R} = \sqrt{(1-\epsilon_2)/(1-\epsilon_2')} = \sqrt{0.6/0.65}$. Finally, $\rho_{22}^{R'L} = 0.05/\sqrt{(0.65)0.4}$. In summary,

$$\mathbf{P}^T = \begin{bmatrix} 1 & 0.3162 & 0.7746 \\ 0.3101 & 0.9354 & 0 \\ 0.9297 & 0.0981 & 0.9608 \end{bmatrix}. \quad (26)$$

Using matrix (26) the BER performance of both conventional and adaptive one-shot detectors is depicted in Figs. (3) and (4) for fixed interference, variable desired user energy and variable interference, fixed desired user energy respectively. In both figures, it can be observed that especially for high interfering energy the adaptive decorrelator, outperforms the conventional one.

Appendix

It is easy to show that, $E\{b_k \mathbf{b}_k\} = 0$, $E\{(\rho_k^T \mathbf{A}_k \mathbf{b}_k)\mathbf{b}_k\} = \mathbf{A}_k \rho_k$, $E\{n_k \mathbf{b}_k\} = 0$, so that

$$E\{(\mathbf{w}_k^T \mathbf{x}_k)\mathbf{b}_k\} = \mathbf{A}_k \rho_k. \quad (\mathrm{A}-1)$$

From (10)

$$\begin{aligned} \mathbf{x}_k &= \begin{bmatrix} \mathbf{P}^T & \mathbf{r}_k \end{bmatrix} \begin{bmatrix} \mathbf{A}_k & 0 \\ 0 & a_{kk} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ b_k \end{bmatrix} + \mathbf{n}_k \quad (\mathrm{A}-2) \\ &= \mathbf{P}_k^T \mathbf{A}_k \mathbf{b}_k + \mathbf{r}_k a_{kk} b_k + \mathbf{n}_k, \end{aligned}$$

Therefore,

$$E\{\mathbf{w}_k^T \mathbf{x}_k \mathbf{b}_k\} = \mathbf{A}_k \mathbf{P}_k \mathbf{w}_k. \quad (\mathrm{A}-3)$$

From Eq. (16) together with Eq. (21),

$$\begin{aligned} z_k &= \rho_{kk} a_{kk} b_k + \rho_k^T \mathbf{A}_k \mathbf{b}_k + n_k \\ &\quad - \rho_k^T \mathbf{P}_k^{-T} (\mathbf{P}_k^T \mathbf{A}_k \mathbf{b}_k + \mathbf{r}_k a_{kk} b_k + \mathbf{n}_k) \qquad (A-4) \\ &= a_{kk}(\rho_{kk} - \rho_k^T \mathbf{P}_k^{-T} \mathbf{r}_k) b_k + n_k - \rho_k^T \mathbf{P}_k^{-T} \mathbf{n}_k. \end{aligned}$$

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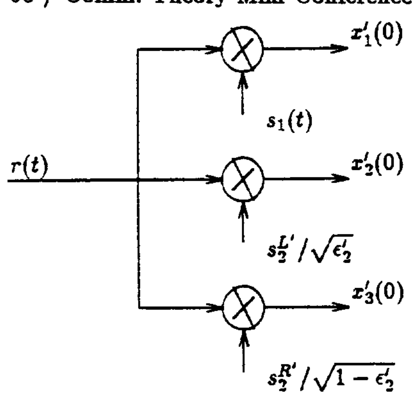

Fig. 1: Match Filter Bank.

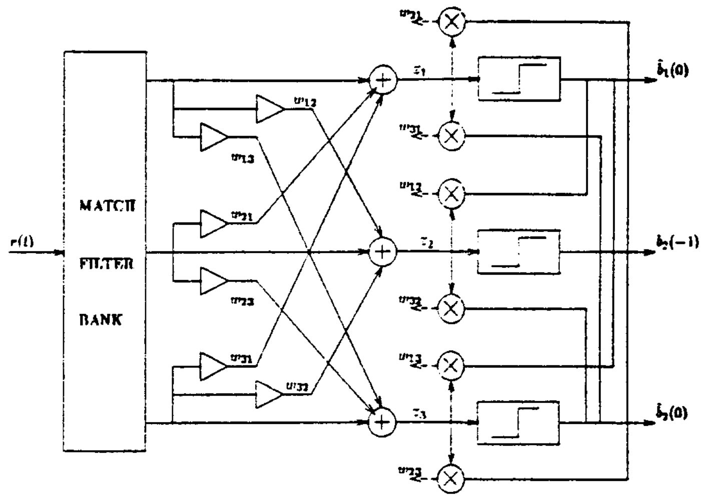

Fig. 2: Adaptive Decorrelator.

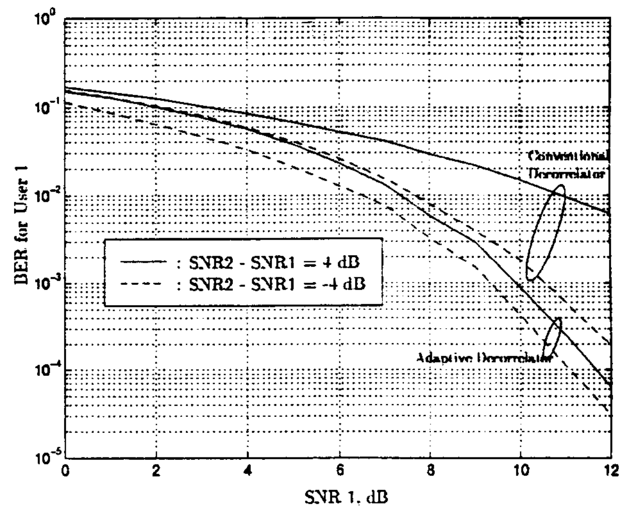

Fig. 3: Performance comparison of Adaptive and Conventional Decorrelator for fixed Interference and variable SNR1 values.

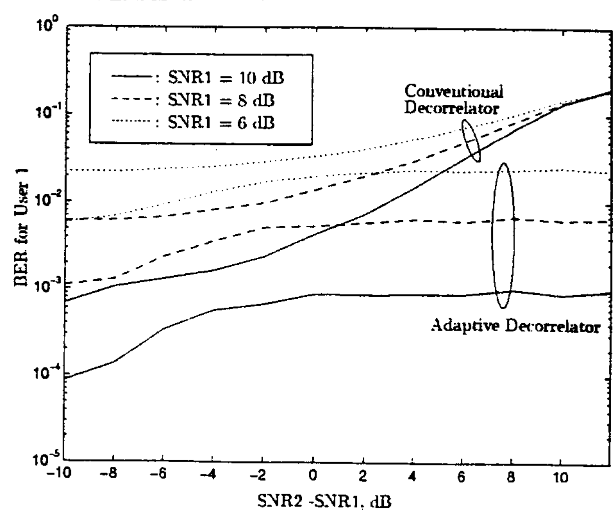

Fig. 4: Performance Comparison of Adaptive and Conventional Decorrelator for fixed SNR1 and variable Interference values.