A METHOD FOR REALIZATION OF AN ARMAX LATTICE FILTER

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ABSTRACT

This paper proposes a method for realization of an ARMAX lattice filter. ARMAX (Autoregressive Moving Average model with Exogenous Variable) model identification is significant because the ARMAX model is a standard tool in the control field, and it can be performed by the proposed algorithm. One of the recursive least-square methods for the ARMAX model identification is the ELS (Extended Least Squares). Applied to the ARMAX model identification, the ELS uses $o(N^2)$ multiplications, where $N \triangleq$ AR order + MA order + X order. When the proposed realization method of the ARMAX lattice filter is used, $o(M)$ multiplications are needed for the ARMAX model identification, where $M \triangleq \max\{\text{AR order, MA order, X order}\}$.

1 INTRODUCTION

When a reference model needs to be identified, a type of model and an identification method should be decided. Suppose the reference model is an ARMAX model (Autoregressive Moving Average model with Exogenous Variable)[1], its parameters can be computed by the least square method. Since the ARMAX model has become a standard tool in control, ARMAX model identification is significant. One of the recursive least-square methods for the ARMAX model identification is the ELS (Extended Least Squares)[1]. Applied to the ARMAX model identification, the ELS uses $o(N^2)$ multiplications, where $N \triangleq$ AR order + MA order + X order.

In this paper, a realization method of an ARMAX lattice filter is proposed. The proposed algorithm uses $o(M)$ multiplications for the ARMAX model identification, where $M \triangleq \max\{\text{AR order, MA order, X order}\}$.

First, the ARMAX model is introduced. Second the realization method of the ARMAX lattice filter is derived: It can be realized by the order-update procedures and the time-update procedures. Finally, experiments of the model identification are presented for verification of the proposed algorithm.

2 A LINEAR SYSTEM — AN ARMAX MODEL

The ARMAX model structure[1] is shown in Figure 1. In Figure 1, signals $y(k)$, $u(k)$, and $x(k)$ are an output signal, an input signal, and an exogenous variable, respectively. Figure 1 shows the following expression:

$$y(t) = \sum_{i=1}^{N} a_i y(t-i) + \sum_{l=0}^{M} c_l u(t-l) + \sum_{j=0}^{S} b_j x(t-j). \quad (1)$$

Using the proposed algorithm, suppose a reference model can be described with the ARMAX model defined in Eq. (1), then the estimated parameters $\hat{a}_i(k)$ $(i = 1, \ldots, n)$, $\hat{b}_j(k)$ $(j = 1, \ldots, m)$, $\hat{c}_l(k)$ $(l = 1, \ldots, s)$, that minimize the least-squares criterion, can be obtained where the estimated parameters are

$$\hat{y}_{n,m,s}(t|k) \triangleq \sum_{i=1}^{n} \hat{a}_i(k) y(t-i) + \sum_{l=0}^{m} \hat{c}_l(k) u(t-l) + \sum_{j=0}^{s} \hat{b}_j(k) x(t-j). \quad (2)$$

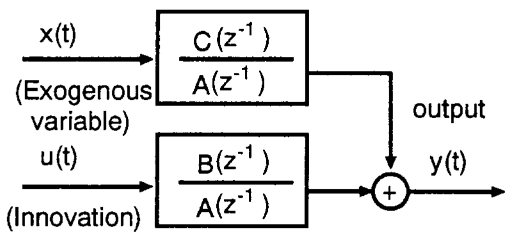

Figure 1: The ARMAX model

3 A REALIZATION METHOD

The ARMAX lattice filter is realized by cascading six kinds of elementary sections. The elementary sections can be designed by order-update recursive formulas. Further, the proposed algorithm can perform adaptive signal processing, by time-updating the coefficients of the ARMAX lattice filter. In this section, the elementary sections and the time-update recursive formulas of the lattice coefficients are derived.

3.1 *The Elementary Sections of the ARMAX Lattice Filter*

First, six kinds of prediction errors which are needed for the realization of the proposed ARMAX lattice filter are defined as follows, where it is assumed that the used input signal is not a white noise:

$\nu^f_{n,m,s}(i|k)$: a forward prediction error for the output signal $y(i)$

$\nu^b_{n,m,s}(i|k)$: a backward prediction error for the output signal $y(i-n)$

$\epsilon^f_{n,m,s}(i|k)$: a forward prediction error for the input signal $u(i)$

$\epsilon^b_{n,m,s}(i|k)$: a backward prediction error for the input signal $u(i-m)$

$\gamma^f_{n,m,s}(i|k)$: a forward prediction error for the exogenous signal $x(i)$

$\gamma^b_{n,m,s}(i|k)$: a backward prediction error for the exogenous signal $x(i-s)$

In the above definition, subscripts of the above prediction errors n, m, and s are AR, MA, and X orders respectively, and k is the number of the data samples which are used for computing the prediction errors.

Second, we derive order-update recursive formulas of the prediction errors in order to realize elementary sections of the ARMAX lattice filter. The order-update recursive formulas perform the following procedures:

AR order-update formulas:
: to increase the AR order of the prediction errors by one order when their MA and X orders are zero.

MA order-update formulas:
: to increase the MA order of the prediction errors by one order when their AR and X orders are zero.

X order-update formulas:
: to increase the X order of the prediction errors by one order when their AR and MA orders are zero.

ARMA order-update formulas:
: to increase both the AR order and the MA order of the prediction errors by one order when their X orders are zero.

ARX order-update formulas:
: to increase both the AR order and the X order of the prediction errors by one order when their MA orders are zero.

MAX order-update formulas:
: to increase both the MA order and the X order of the prediction errors by one order when their AR orders are zero.

ARMAX order-update formulas:
: to increase the AR order, the MA order, and X order by one simultaneously.

For example, the AR order-update recursive formulas are as follows:

$$\begin{aligned}
\nu^f_{n+1,0,0}(i|k) &= \nu^f_{n,0,0}(i|k) \\
&\quad + \mu^{\nu^b\nu^f}_{n,0,0}(k)\nu^b_{n,0,0}(i-1|k-1) \\
\nu^b_{n+1,0,0}(i|k) &= \nu^b_{n,0,0}(i|k) + \mu^{\nu^f\nu^b}_{n,0,0}(k)\nu^f_{n,0,0}(i|k) \\
\epsilon^f_{n+1,0,0}(i|k) &= \epsilon^f_{n,0,0}(i|k) \\
&\quad + \mu^{\nu^b\epsilon^f}_{n,0,0}(k)\nu^b_{n,0,0}(i-1|k-1) \\
\epsilon^b_{n+1,0,0}(i|k) &= \epsilon^f_{n,0,0}(i|k) + \mu^{\nu^f\epsilon^f}_{n,0,0}(k)\nu^f_{n,0,0}(i|k)
\end{aligned}$$

(The above equation is used only when an ARMA or ARMAX section is connected after this AR section.)

$$\begin{aligned}
\gamma^f_{n+1,0,0}(i|k) &= \gamma^f_{n,0,0}(i|k) \\
&\quad + \mu^{\nu^b\gamma^f}_{n,0,0}(k)\nu^b_{n,0,0}(i-1|k-1) \\
\gamma^b_{n+1,0,0}(i|k) &= \gamma^f_{n,0,0}(i|k) + \mu^{\nu^f\gamma^f}_{n,0,0}(k)\nu^f_{n,0,0}(i|k)
\end{aligned}$$

(The above equation is used only when an ARX or ARMAX section is connected after this AR section.)

(3)

where

$$\mu^{\nu^b\epsilon^f}_{n,0,0}(k) \triangleq -\frac{V^{\epsilon^f}_{n,0,0}(k)}{V^{\nu^b}_{n,0,0}(k-1)} \qquad \mu^{\nu^b\epsilon^b}_{n,0,0}(k) \triangleq -\frac{V^{\epsilon^f}_{n,0,0}(k)}{V^{\nu^b}_{n,0,0}(k)}$$

$$\mu^{\nu^b\gamma^f}_{n,0,0}(k) \triangleq -\frac{V^{\gamma^f}_{n,0,0}(k)}{V^{\nu^b}_{n,0,0}(k-1)} \qquad \mu^{\nu^b\gamma^b}_{n,0,0}(k) \triangleq -\frac{V^{\gamma^b}_{n,0,0}(k)}{V^{\nu^b}_{n,0,0}(k)}$$

$$\mu^{\nu^b\nu^f}_{n,0,0}(k) \triangleq -\frac{V^{\nu^f}_{n,0,0}(k)}{V^{\nu^b}_{n,0,0}(k-1)}$$

$$\mu^{\nu^f\nu^b}_{n,0,0}(k) \triangleq -\frac{V^{\nu^b}_{n,0,0}(k-1)}{V^{\nu^f}_{n,0,0}(k)} \tag{4}$$

$$V^{e^f_1 e^f_2}_{s,t}(k) \triangleq \sum_{i=1}^{k} e1^f_{n,m,s}(i|k) e2^f_{n,m,s}(i|k)$$

$$(\text{If } e^f_1 = e^f_2, \text{ then } V^{e^f_1 e^f_2}_{s,t}(k) = V^{e^f_1}_{s,t}(k).)$$

$$V^{e^b_1 e^b_2}_{s,t}(k) \triangleq \sum_{i=1}^{k} e1^b_{n,m,s}(i|k) e2^b_{n,m,s}(i|k)$$

$$(\text{If } e^b_1 = e^b_2, \text{ then } V^{e^b_1 e^b_2}_{s,t}(k) = V^{e^b_1}_{s,t}(k).)$$

$$V^{e^f_1 e^b_2}_{s,t}(k) \triangleq \sum_{i=1}^{k} e1^f_{n,m,s}(i|k) e2^b_{n,m,s}(i-1|k-1) \tag{5}$$

In the above equations, e_1 and e_1 are ν, ϵ, or γ, respectively.

Based on the order-update recursive formulas of the prediction errors, six kinds of elementary sections can be realized as shown in Figure 3. In Figure 3, all the lattice parameters are computed with the correlations of the prediction errors, Since the structure of the ARMAX section is complicated, the coefficients of the ARMAX elementary section are not described in Figure 3.

3.2 *Time-update recursive formulas for the coefficients of the ARMAX Lattice Filter*

In order to realize the ARMAX lattice filter, we need to time-update the values of the filter coefficients shown in Figure 3. As a consequence of Eqs. (3), (4), and, (5), the filter coefficients are calculated with some correlations of the prediction errors. Therefore, the filter coefficients can be time-updated by time-updating the correlations of the prediction errors. For example, the time-update recursive formula for $V^{\nu^f}_{n,m,s}(k)$ is shown as follows:

$$\begin{aligned}
V^{\nu^f}_{n,m,s}(k) &= \lambda\left\{V^{\nu^f}_{n,m,s}(k) + A_1(k-1)\nu^{f2}_{n,m,s}(k|k-1)\right\} \\
&\quad -A_2(k-w-1)\left\{\nu^f_{n,m,s}(k-w|k-1)\right. \\
&\quad \left. -\alpha^f_{n,m,s}(k-w-1|k-1)A_1(k-1)\right.
\end{aligned}$$

$$\cdot\nu^{f}_{n,m,s}(k-w|k-1)\Big\}^{2} \quad (6)$$

where λ is the forgetting factor[3], w is the sliding window length[4],

$$A_1(k-1) \triangleq \frac{A_2(k-w-1)}{(\lambda+\alpha_{n,m,s}(k-1))}$$

$$A_2(k-w-1) \triangleq \lambda^{w}\{1-\lambda^{w}\alpha_{n,m,s}(k-w-1)\}^{-1} \quad (7)$$

Further, in order to realize the ARMAX lattice filter, we compute $\alpha_{n,m,s}(k)$, $\alpha_{n,m,s}(k-w)$, and $\alpha_{n,m,s}(k-w|k)$ by order-update of them. For example, the AR order-update recursive formulas of α are shown as:

$$\alpha_{n+1,0,0}(k) = \alpha_{n,0,0}(k) + \nu^{b2}_{n,0,0}(k|k-1)$$

$$\alpha_{n+1,0,0}(k-w) = \alpha_{n,0,0}(k-w) + \nu^{b2}_{n,0,0}(k-w|k-1)$$

$$\alpha_{n+1,0,0}(k-w|k) = \alpha_{n,0,0}(k-w|k) + \nu^{b}_{n,0,0}(k-w|k-1)\nu^{b2}_{n,0,0}(k|k-1) \quad (8)$$

The above formulas are similar to Reference [4]. However, of course, the proposed formulas and Reference [4] differ in the prediction errors γ^f and γ^b.

3.3 *An input estimation method for the proposed algorithm*

One of the problems of ARMAX identification is estimating the input signal. In the proposed algorithm, the input signal, $\hat{u}(k)$ can be estimated by the following equation:

$$\hat{u}(k) \simeq \frac{\lambda}{\lambda+\alpha_{N,M,S}(k-1)}\nu^{f}_{N,M,S}(k|k-1) \quad (9)$$

where $\nu^{f}_{N,M,S}(k|k-1)$ is the prediction error which is output from the last section of the lattice filter.

4 EXPERIMENTAL RESULTS

The proposed method can perform the same ARMAX model identification problems as the ELS can perform. In order to verify the proposed algorithm, some of the experimental results are shown in this section.

In Figure 2, (a) and (c) show the frequency characteristics of the transfer functions $\frac{C(q^{-1})}{A(q^{-1})}$ and $\frac{B(q^{-1})}{A(q^{-1})}$ of a reference ARMAX model (c.f. Figure 1). In the experiments, since the output signal $(y(t))$ and the exogenous signal $(x(t))$ are given and the input signal $(u(t))$ should be estimated in the ARMAX model identification problems, an original input estimator is embedded in the proposed algorithm.

In the experiments, 700 samples of the data, which are sampled at 10 kHz are used, further, from Eq. (6), since the proposed realization algorithm implys the forgetting factor[3, 2] and the sliding window[3, 4], the forgetting factor is set at 0.98 and a window length is 200 samples. Figure 2 (b) and (d) show the frequency characteristics of the estimated models with an ARMA (8,4) order and an ARX (8,3) order, respectively. From Figure 2, it can been seen that the proposed algorithm can perform the ARMAX model identification.

5 CONCLUSIONS

In this paper, an ARMAX lattice filter realization method is proposed. By using the proposed algorithm, the ARMAX model identification can be achieved with fewer calculation

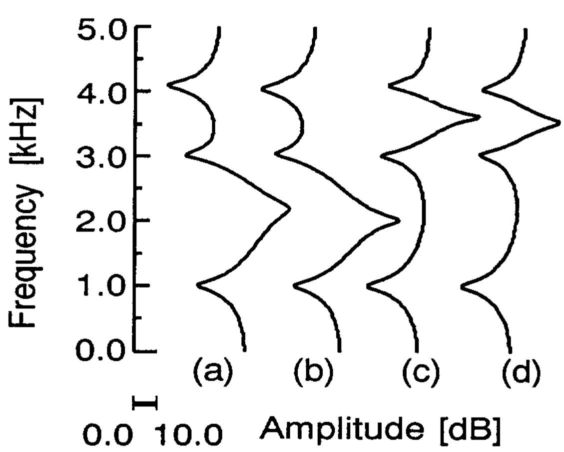

Figure 2: Experimental results

(a)	ARMA part of the reference model ($\frac{C(q^{-1})}{A(q^{-1})}$) (ARMA (6,2) order)	
(b)	An identified model	(ARMA (8,4) order)
(c)	ARX part of the reference model ($\frac{B(q^{-1})}{A(q^{-1})}$) (ARX (6,2) order)	
(d)	An identified model	(ARX (8,3) order)

costs than the ELS algorithm when the reference model has high ARMAX order.

Some considerations are still necessary for the comparison of the parameter-estimation accuracy of the proposed algorithm and the ELS. The structure of the proposed lattice filter also needs to be simplified. This can be done by one of the characteristics from the input signal that is an innovation.

ACKNOWLEDGMENT

The authors would like to thank Prof. Nagai of Hokkaido University for his advices. This research is supported in part by the Ministry of Education, Science and Culture of Japan under Grant-in-Aid for Scientific Research (No. 06855037).

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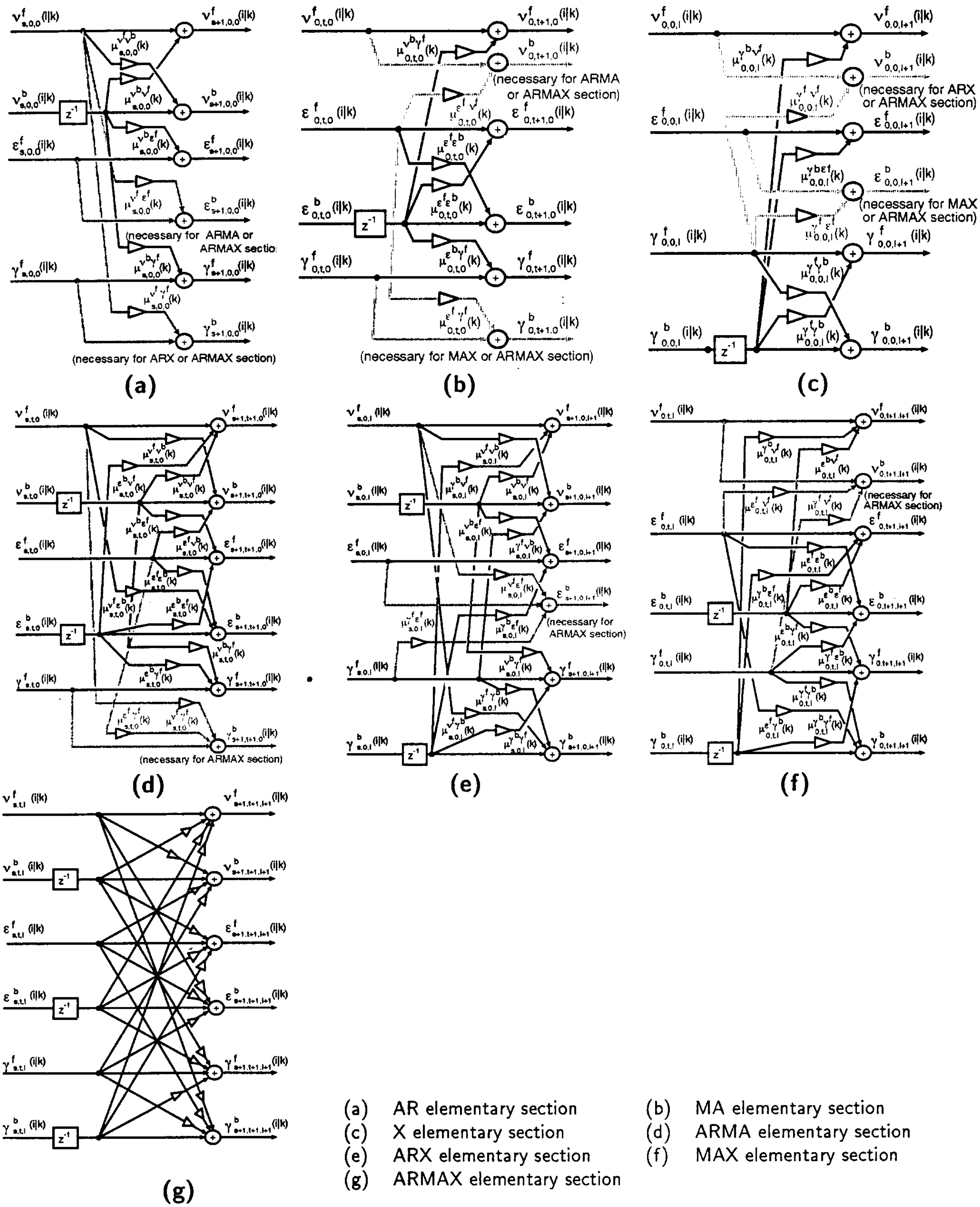

(a) AR elementary section
(b) MA elementary section
(c) X elementary section
(d) ARMA elementary section
(e) ARX elementary section
(f) MAX elementary section
(g) ARMAX elementary section

Figure 3: ARMAX lattice filter elementary sections