ACOUSTIC VECTOR-SENSOR BEAMFORMING AND CAPON DIRECTION ESTIMATION*

Malcolm Hawkes and Arye Nehorai

Department of Electrical Engineering
Yale University
15 Prospect Street
New Haven, CT 06520

ABSTRACT

We consider beamforming and Capon direction of arrival (DOA) estimation using arrays of acoustic vector sensors. We derive an expression for the Cramér-Rao bound (CRB) on the DOA parameters of a single source. Using this, we give conditions that minimize the lower bound on the asymptotic mean-square angular error, and conditions that ensure it is isotropic. The asymptotic performance of the Capon and beamforming estimators is analyzed and compared with a scalar-sensor array. The vector-sensor array is seen to have improved performance due to its elements' directional sensitivity. Large sample approximations for the mean-square error (MSE) matrices of the estimators are derived. Throughout, we compare vector-sensor arrays with their scalar-sensor counterparts.

1. INTRODUCTION

The use of acoustic vector-sensor arrays for DOA estimation has recently been proposed in [1]. Each vector sensor measures the acoustic pressure and the three components of acoustic particle velocity at a particular point in space, in contrast to traditional hydrophones (scalar-sensors) that only measure acoustic pressure. Vector sensors have already been constructed [2] and subject to sea trials [3]. In [3], a DOA estimation trial was conducted but no analysis was done on the performance of the estimators, nor any comparison made with a comparable scalar-sensor array.

We consider an array of vector sensors, illuminated by n narrowband, Gaussian signals in spatially and temporally uncorrelated Gaussian noise. In Section 2, we introduce the mathematical model. In Section 3, we use the results of [1] to derive expressions for the CRB and $\mathrm{MSAE}_{\mathrm{CR}}$ (a lower bound on the asymptotic mean-square angular error [4]), in the case of a single source in 3-D space. We give conditions on the array geometry such that the $\mathrm{MSAE}_{\mathrm{CR}}$ is minimized for a given array size, leading to better performance. We also give conditions such that the $\mathrm{MSAE}_{\mathrm{CR}}$ is independent of the actual source direction, i.e. the array's response is isotropic. The CRBs of scalar-sensor and vector-sensor linear arrays (LAs) are compared.

In Section 4.1, we derive expressions for the asymptotic beamforming and Capon spectra. It is seen that vector-sensor arrays have uniformly improved performance over scalar-sensor arrays, and suffer less from spatial aliasing when the acoustic field is undersampled. In Section 4.2, we derive approximations for the large sample MSE matrix of both estimators, when the DOA has two parameters. Section 5 concludes.

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2. THE MODEL

We consider n narrowband planewaves impinging on an array of m acoustic vector sensors. We wish to determine the DOA parameter vector $\boldsymbol{\theta} = [\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_n]^T$, where $\boldsymbol{\theta}_k = [\phi_k, \psi_k]^T$, and ϕ_k and ψ_k are the azimuth and elevation of the kth source. We assume that each vector sensor has its velocity sensors aligned with the x, y, and z axes. With this assumption, the array's steering vector is

$$\boldsymbol{a}(\boldsymbol{\theta}) = \boldsymbol{d}(\boldsymbol{\theta}) \otimes \boldsymbol{h}(\boldsymbol{\theta}), \tag{1}$$

where $\otimes$ is the Kronecker product,

$$\boldsymbol{d}(\boldsymbol{\theta}) = [e^{i\omega_c(\boldsymbol{u}(\boldsymbol{\theta})^T\boldsymbol{r}_1)/c}, \ldots, e^{i\omega_c(\boldsymbol{u}(\boldsymbol{\theta})^T\boldsymbol{r}_m)/c}]^T \tag{2}$$

and $\boldsymbol{h}(\boldsymbol{\theta}) = [1, \boldsymbol{u}(\boldsymbol{\theta})]^T$; here, $\boldsymbol{r}_k$ is the position vector of the kth sensor, $\boldsymbol{u}(\boldsymbol{\theta})$ is the unit vector in the direction from the array to the source, ω_c is the center frequency and c is the speed of sound (assumed constant). The vector $\boldsymbol{d}(\boldsymbol{\theta})$ is the steering vector of a pressure-sensor

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array of the same geometry, while $h(\theta)$ accounts for the directionality of each component: omni-directional for the pressure sensor and a cosine response for the velocity sensors. See [1] for further details.

With n sources, the output of the array at time t is

$$y(t) = A(\theta)x(t) + e(t), \quad (3)$$

where the transfer matrix is $A(\theta) = [a(\theta_1), \ldots, a(\theta_n)]$. The signal vector $x(t)$ contains the complex envelopes of the n sources. The vector $e(t)$ is a complex additive noise term. We assume that $x(t)$ and $e(s)$ both have the multivariate complex Gaussian distribution, and are independent for all s and t. The signal and noise covariance matrices are

$$\begin{aligned} \mathrm{E}\{x(t)x^H(s)\} &= P\delta_{t,s} & (4) \\ \mathrm{E}\{e(t)e^H(s)\} &= I_m \otimes \begin{bmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_v^2 I_3 \end{bmatrix} \delta_{t,s}, & (5) \end{aligned}$$

where the superscript ^{H} represents conjugate transposition, σ_p^2 and σ_v^2 are the pressure-sensor and velocity-sensor noise variances respectively, and I_m is the $m \times m$ identity matrix. The restriction on the form of the noise covariance is consistent with internal sensor noise.

3. CRAMÉR-RAO BOUNDS

When there is a single source, Capon and beamforming techniques are known to be asymptotically unbiased and asymptotically attain the CRB. A general expression for the CRB of an acoustic vector-sensor array is given in [1]. For a single source

$$\begin{aligned} \mathrm{CRB_S}(\theta) &= \frac{\rho_p}{2N}\left(1 + \frac{\rho_p}{m}\right) J^{-1} & (6) \\ \mathrm{CRB_V}(\theta) &= \frac{\rho_v}{2N}\left(1 + \frac{\rho_v}{(r^2+1)m}\right) K^{-1}, & (7) \end{aligned}$$

where $\rho_p = \sigma_p^2/\eta^2$, $\rho_v = \sigma_v^2/\eta^2$, $r = \sigma_v/\sigma_p$ and η^2 is the signal variance [(6) is the scalar-sensor and (7) the vector-sensor expression]. Taking the origin of the coordinate system to be the array centroid, i.e. $\sum_j r_j = 0$, the entries of the (symmetric) matrix J become

$$\begin{aligned} J_{\phi\phi} &= (\omega_c/c)^2 \cos^2\psi \sum_j (r_j^T v_1)^2 \\ J_{\phi\psi} &= (\omega_c/c)^2 \cos\psi \sum_j (r_j^T v_1)(r_j^T v_2) & (8) \\ J_{\psi\psi} &= (\omega_c/c)^2 \sum_j (r_j^T v_2)^2, \end{aligned}$$

where the sums are over the number of sensors, m. The entries of the (symmetric) matrix K are

$$\begin{aligned} K_{\phi\phi} &= (r^2+1)J_{\phi\phi} + m\cos^2\psi \\ K_{\phi\psi} &= (r^2+1)J_{\phi\psi} & (9) \\ K_{\psi\psi} &= (r^2+1)J_{\psi\psi} + m. \end{aligned}$$

The vectors v_1 and v_2 are defined by $v_1 = \frac{\partial u/\partial\phi}{\cos\psi}$ and $v_2 = \partial u/\partial\psi$. With these definitions, (u, v_1, v_2) is a right orthonormal triad.

Compare (6) with (7), and (8) with (9). The factor $r^2 + 1$ is a consequence of taking more measurements and is directly due to the fact that $|h(\theta)|^2 = r^2 + 1$. The extra additive terms in (9) also contribute to the improved vector-sensor array performance. Their presence shows that the use of velocity sensors provides more than a simple increase of signal-to-noise ratio (SNR).

For a single source, ϕ and ψ are uncoupled from η^2, σ_p^2 and σ_v^2 in the CRB. This was shown for a 2-D scalar-sensor array in [5], but is also true for 3-D scalar-sensor and vector-sensor arrays. Thus, knowledge of source or noise power does not affect the $\mathrm{MSAE_{CR}}$ in this case. Hence, (6) and (8) may be obtained from [6], which assumes *known* signal and noise powers.

Minimizing the bound on estimation accuracy by selection of array geometry will tend to improve array performance. A very natural criterion is the asymptotic mean-square angular error of a direction estimator. Introduced in [4], it is defined by

$$\mathrm{MSAE} \triangleq \lim_{N\to\infty} N\mathrm{E}\delta^2, \quad (10)$$

where δ is the angular error, and its lower bound is given by [4]

$$\mathrm{MSAE_{CR}} = N[\cos^2\psi \cdot \mathrm{CRB}(\phi) + \mathrm{CRB}(\psi)]. \quad (11)$$

The diagonal elements of J or K are good measures of the array's size. When they are fixed, the $\mathrm{MSAE_{CR}}$ is minimized by making the off-diagonal elements zero, i.e. decoupling ϕ and ψ in the bound. It was shown in [6], and may be seen from (8), that a set of sufficient conditions such that $J_{\phi\psi} = 0$ for all θ is

$$\sum_j r_{jx}^2 = \sum_j r_{jy}^2 \quad (12)$$

$$\sum_j r_{jx}r_{jy} = \sum_j r_{jx}r_{jz} = \sum_j r_{jy}r_{jz} = 0, \quad (13)$$

where r_{jx}, r_{jy} and r_{jz} are the x, y and z components of the jth sensor's position vector. It is clear from the form of (9) that these same set of conditions ensure $K_{\phi\psi} = 0$ for all θ. Arrays satisfying (12) and (13) have a certain symmetry in their x and y axes and include uniform circular and square arrays parallel to the x, y plane and uniform cylindrical, spherical, cubic and cuboidal arrays and vertical LAs. Under these conditions, the $\mathrm{MSAE_{CR}}$ becomes a function of elevation only. If we extend condition (12) to

$$\sum_j r_{jx}^2 = \sum_j r_{jy}^2 = \sum_j r_{jz}^2, \quad (14)$$

then when (13) and (14) hold, the $\mathrm{MSAE_{CR}}$ becomes independent of the source direction entirely; it is given by

$$\mathrm{MSAE_{CR,S}} = \rho_p \frac{1+\rho_p/m}{(\omega_c/c)^2 \sum_j r_{jx}^2} \quad (15)$$

$$\mathrm{MSAE_{CR,V}} = \rho_v \frac{1+\rho_v/(r^2+1)m}{(r^2+1)(\omega_c/c)^2 \sum_j r_{jx}^2 + m} \quad (16)$$

for the scalar-sensor and vector-sensor arrays respectively. This class of arrays, which has a symmetry in all three axes and includes cubic and spherical arrays, have isotropic performance, making them of great interest.

The LA is of particular interest because it is very often used in practice. For the common linear array (LA) $J_{\phi\phi} = J_{\phi\psi} = 0$ and

$$J_{\psi\psi} = (\omega_c/c)^2 \cos^2\psi \sum_j r_{jz}^2. \quad (17)$$

The entries of K are given by (9). Note that J is singular but K is not. Thus the vector-sensor LA has no ambiguity, whereas the scalar-sensor LA can resolve only up to a conic angle. Indeed just a single vector sensor is enough to resolve both azimuth and elevation. In addition $J_{\psi\psi} \to 0$ as $|\psi| \to \pi/2$, as discussed in [7], but $K_{\psi\psi}$ remains finite. Thus, beamforming and Capon methods (which asymptotically obtain the CRB with one source) will have much greater accuracy near endfire with the vector-sensor LA than the scalar-sensor LA.

4. DOA ESTIMATION

Beamforming and Capon's method [8] of DOA estimation involve estimating a spatial spectrum and maximizing over all possible directions. The beamforming (Bartlett) and Capon spectra are

$$f_{\mathrm{B}}(\boldsymbol{\theta}) = \boldsymbol{a}^H(\boldsymbol{\theta}) R \boldsymbol{a}(\boldsymbol{\theta}) \quad (18)$$

$$f_{\mathrm{C}}(\boldsymbol{\theta}) = \left[\boldsymbol{a}^H(\boldsymbol{\theta}) R^{-1} \boldsymbol{a}(\boldsymbol{\theta})\right]^{-1}, \quad (19)$$

respectively, where R is the data covariance matrix and $\boldsymbol{a}(\boldsymbol{\theta})$ is the steering vector. In general, these spectra will have many more local maxima than there are sources, thus the number of sources must be known *a-priori*. We shall denote the n largest maxima by $\{\bar{\boldsymbol{\theta}}_k\}_{k=1}^n$. In general, they are not the true DOAs, $\{\boldsymbol{\theta}_k\}_{k=1}^n$ if $n > 1$.

The estimates of the Bartlett and Capon spectra are obtained by replacing R by its maximum likelihood estimate $\hat{R}$ in (18) and (19). We denote these spectral estimates $\hat{f}_{\mathrm{B}}$ and $\hat{f}_{\mathrm{C}}$ respectively. Asymptotically, $\hat{R} \to R$, almost surely, hence $\hat{f}_{\mathrm{B}} \to f_{\mathrm{B}}$ and $\hat{f}_{\mathrm{C}} \to f_{\mathrm{C}}$ almost surely for all $\boldsymbol{\theta} \in \boldsymbol{\Theta}$. The distributions of $\hat{f}_{\mathrm{B}}$ and $\hat{f}_{\mathrm{C}}$ are given by [9]

$$\frac{N \hat{f}_{\mathrm{B}}(\boldsymbol{\theta})}{f_{\mathrm{B}}(\boldsymbol{\theta})} \sim \Gamma(N, 1) \quad (20)$$

$$\frac{N \hat{f}_{\mathrm{C}}(\boldsymbol{\theta})}{f_{\mathrm{C}}(\boldsymbol{\theta})} \sim \Gamma(N-m+1, 1), \quad (21)$$

for the scalar-sensor array, where $\Gamma(\alpha, \beta)$ is the gamma distribution with shape parameter α and scale parameter β. The arguments of [9] apply *mutatis mutandis* in the vector sensor case, giving that (20) and (21) hold if m is replaced by $4m$. The DOA estimates are the n values of $\boldsymbol{\theta}$ corresponding to the n largest maxima of $\hat{f}_{\mathrm{B}}$ or $\hat{f}_{\mathrm{C}}$ and are denoted $\{\hat{\boldsymbol{\theta}}_k\}_{k=1}^n$.

4.1. Asymptotic Spectra

When there is a single source from direction $\boldsymbol{\theta}_0$, the Bartlett spectra for scalar-sensor and vector-sensor arrays, normalized such that the maximum is unity, are

$$f_{\mathrm{B,S}}(\boldsymbol{\theta}) = \frac{|\boldsymbol{d}^H(\boldsymbol{\theta})\boldsymbol{d}(\boldsymbol{\theta}_0)|^2 + m\rho}{m^2 + m\rho} \quad (22)$$

$$f_{\mathrm{B,V}}(\boldsymbol{\theta}) = \frac{(1+\cos\gamma)^2 |\boldsymbol{d}^H(\boldsymbol{\theta})\boldsymbol{d}(\boldsymbol{\theta}_0)|^2 + 2m\rho}{4m^2 + 2m\rho}, \quad (23)$$

where γ is the angle between the direction to the source $\boldsymbol{u}(\boldsymbol{\theta}_0)$ and the direction of look $\boldsymbol{u}(\boldsymbol{\theta})$ (we have chosen $\sigma_p^2 = \sigma_v^2$ for convenience). Thus $f_{\mathrm{B,V}}(\boldsymbol{\theta}) \le f_{\mathrm{B,S}}(\boldsymbol{\theta})$ for all $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, with equality holding if and only if $\boldsymbol{\theta} = \boldsymbol{\theta}_0$. So for vector-sensor arrays, the Bartlett spectrum has a sharper peak and uniformly lower sidelobes, leading to better resolution and smaller estimation errors.

It is the term $(1+\cos\gamma)^2$ in (23) that provides the real impetus for using vector sensors. It arises from the inherent directional sensitivity of each sensor, and is what allows resolution of both azimuth and elevation by any array. The same term also means vector-sensor arrays suffer less from spatial aliasing. If the wavefield is undersampled, aliasing occurs in the spatial domain. With scalar-sensor arrays, grating lobes may appear — secondary peaks at the same height as the mainlobe that do not correspond to actual sources — hence, the DOA cannot be determined unambiguously, no matter how many snapshots are available. However, since $f_{\mathrm{B,V}}(\boldsymbol{\theta}) < f_{\mathrm{B,V}}(\boldsymbol{\theta}_0)$ if $\boldsymbol{\theta} \neq \boldsymbol{\theta}_0$ when a vector-sensor array is used, the grating lobes are no longer as high as the mainlobe, so we can determine the DOA given enough data. The term $(1+\cos\gamma)^2$ plays a similar role in the Capon spectrum, and the above comments apply. We also note that $f_{\mathrm{C}}(\boldsymbol{\theta}) \le f_{\mathrm{B}}(\boldsymbol{\theta})$ for both types of sensor with equality at $\boldsymbol{\theta} = \boldsymbol{\theta}_0$.

4.2. MSE Approximations

An MSE approximation for the Capon estimator has been presented in [10] for a 1-D DOA parameter space. Using a similar approach, we derive MSE approximations for both estimators for a 2-D parameter space, applicable to scalar-sensor and vector-sensor arrays.

For large N, $\mathrm{E}\{\hat{\boldsymbol{\theta}}\}$ is very nearly $\bar{\boldsymbol{\theta}}$, so the MSE is

$$\mathrm{MSE}(\hat{\boldsymbol{\theta}}) \approx (\bar{\boldsymbol{\theta}} - \boldsymbol{\theta})(\bar{\boldsymbol{\theta}} - \boldsymbol{\theta})^T + C(\bar{\boldsymbol{\theta}}), \qquad (24)$$

where

$$C(\bar{\boldsymbol{\theta}}) = \mathrm{E}(\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}})(\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}})^T. \qquad (25)$$

For the beamforming estimate

$$C(\bar{\boldsymbol{\theta}}) \approx [F(\bar{\boldsymbol{\theta}})]^{-1} G(\bar{\boldsymbol{\theta}}) [F(\bar{\boldsymbol{\theta}})]^{-T}, \qquad (26)$$

where

$$\begin{aligned} F(\bar{\boldsymbol{\theta}}) &= \frac{\partial^2 f(\bar{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^T} \\ &= 2\Re\left\{D^H(\bar{\boldsymbol{\theta}}) R D(\bar{\boldsymbol{\theta}}) + H(\bar{\boldsymbol{\theta}})\right\}, \qquad (27) \\ G(\bar{\boldsymbol{\theta}}) &= \mathrm{E}\left\{\left[\frac{\partial \hat{f}(\bar{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}}\right]\left[\frac{\partial \hat{f}(\bar{\boldsymbol{\theta}})}{\partial \boldsymbol{\theta}}\right]^T\right\} \\ &= \frac{1}{2N}\Re\Big\{\boldsymbol{a}^H(\bar{\boldsymbol{\theta}}) R \boldsymbol{a}(\bar{\boldsymbol{\theta}}) \left[D^H(\bar{\boldsymbol{\theta}}) R D(\bar{\boldsymbol{\theta}})\right] \\ &\quad - \left[D^H(\bar{\boldsymbol{\theta}}) R \boldsymbol{a}(\bar{\boldsymbol{\theta}})\right]\left[D^H(\bar{\boldsymbol{\theta}}) R \boldsymbol{a}(\bar{\boldsymbol{\theta}})\right]^H\Big\}, \qquad (28) \\ D(\bar{\boldsymbol{\theta}}) &= \left[\frac{\partial \boldsymbol{a}(\bar{\boldsymbol{\theta}})}{\partial \phi}, \frac{\partial \boldsymbol{a}(\bar{\boldsymbol{\theta}})}{\partial \psi}\right], \qquad (29) \\ H(\bar{\boldsymbol{\theta}}) &= \begin{bmatrix} \boldsymbol{a}^H(\bar{\boldsymbol{\theta}}) R \frac{\partial^2 \boldsymbol{a}(\bar{\boldsymbol{\theta}})}{\partial \phi^2} & \boldsymbol{a}^H(\bar{\boldsymbol{\theta}}) R \frac{\partial^2 \boldsymbol{a}(\bar{\boldsymbol{\theta}})}{\partial \phi \partial \psi} \\ \boldsymbol{a}^H(\bar{\boldsymbol{\theta}}) R \frac{\partial^2 \boldsymbol{a}(\bar{\boldsymbol{\theta}})}{\partial \phi \partial \psi} & \boldsymbol{a}^H(\bar{\boldsymbol{\theta}}) R \frac{\partial^2 \boldsymbol{a}(\bar{\boldsymbol{\theta}})}{\partial \psi^2} \end{bmatrix} \qquad (30) \end{aligned}$$

The above expressions hold for both scalar-sensor and vector-sensor arrays. For the Capon estimate R is everywhere replaced with R^{-1} above. Note that (28) then only holds approximately.

For any scenario, the MSE is estimated by numerically maximizing $f(\boldsymbol{\theta})$ to find $\bar{\boldsymbol{\theta}}$, then using (24) and (26)–(30). In the case of a single source, both the beamforming and Capon MSE approximations equal the CRB. This agrees with the fact that beamforming is known to coincide with maximum-likelihood estimation in the single-source case. This is true for both scalar-sensor and vector-sensor arrays and it follows that the MSAE attains its lower bound, the $\mathrm{MSAE}_{\mathrm{CR}}$.

5. CONCLUSION

The performance of acoustic vector-sensors for DOA estimation in 3-D space has been examined. The CRB for a single source was derived and conditions on the array geometry that minimized the $\mathrm{MSAE}_{\mathrm{CR}}$ and made performance isotropic were obtained. Expressions were given for beamforming and Capon spectra for the single source case. It was shown that the inherent directional sensitivity of vector-sensors gave them better performance and helped reduce spatial aliasing in the case that the wavefield is undersampled. An approximation was given for the large-sample MSE matrix of beamforming and Capon DOA estimates.

6. REFERENCES

[1] A. Nehorai and E. Paldi, "Acoustic vector-sensor array processing," *IEEE Trans. Sig. Proc.*, vol. 42, no. 9, pp. 2481–2491, Sept. 1994. A short version appeared in *Proc. 26th Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, Oct. 1992, pp. 192–198.

[2] J. Nickles et al, "A vertical array of directional acoustic sensors," *Proc. Mast. Oceans thru Tech. (Oceans 92)*, Newport, RI, Oct. 1992, pp. 340–345.

[3] G. D'Spain et al, "Initial analysis of the data from the vertical DIFAR array," *Proc. Mast. Oceans thru Tech. (Oceans 92)*, Newport, RI, Oct. 1992, pp. 346–351.

[4] A. Nehorai and E. Paldi, "Vector-sensor array processing for electromagnetic source localization," *IEEE Trans. Sig. Proc.*, vol. 42, no. 2, pp. 376–398, Feb. 1994. A short version appeared in *Proc. 25th Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, Nov. 1991, pp. 566–572.

[5] Y. Hua and T. Sarkar, "A note on the Cramér-Rao bound for 2-D direction finding based on 2-D array," *IEEE Trans. Sig. Proc.*, vol. 39, no. 5, pp. 1215–1218, May 1991.

[6] R. Nielson, "Azimuth and elevation angle estimation with a three-dimensional array," *IEEE Jour. of Oceanic Eng.*, vol. 19, no. 1, pp. 84–86, Jan. 1994.

[7] V. MacDonald and P. Schultheiss, "Optimum passive bearing estimation in a spatially incoherent noise environment," *Jour. Acous. Soc. Amer.*, vol. 46, no. 1, pt. 1, pp. 37–43, July 1969.

[8] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57, no. 8, pp. 1408–1418, Aug. 1969.

[9] J. Capon and N. Goodman, "Probability distributions for estimators of the frequency-wavenumber spectrum," *Proc. IEEE*, vol. 58, no. 10, pp. 1785–1786, Oct. 1970; corrections in vol. 59, no. 1, pp. 112, Jan. 1971.

[10] P. Stoica, P. Händel and T. Söderström, "Study of Capon method for array signal processing," *Circ. Syst. Sig. Proc.*, (to appear).