A GENERALIZATION TO THE TEAGER-KAISER ENERGY FUNCTION & APPLICATION TO RESOLVING TWO CLOSELY-SPACED TONES

Wei Lin, Chris Hamilton, Prabhakar Chitrapu
Dialogic Corporation
1515 Route 10
Parsippany, NJ 07054, USA

ABSTRACT

Based on physical considerations, Kaiser recently proposed a new energy function for discrete time signals, known as Teager-Kaiser Energy Function (TKEF). An interesting property of the TKEF is that if the input signal consists of two closely-spaced tones (amplitude modulated signal), the TKEF produces the difference frequency tone (envelope signal). However, a drawback is that the TKEF is highly sensitive to additive noise. In this paper, we propose a Generalized TKEF (GTKEF) to reduce the noise sensitivity. Fortunately, it turns out that the generalization can be used to enhance the sum frequency tone as well. This enables us to apply the GTKEF to resolve of two closely-spaced tones. The result can be viewed as an interesting nonlinear preprocessing scheme to transform a signal consisting of two closeby frequencies at f_1 and f_2 into a signal consisting of frequencies at $(f_1 - f_2)$ and $(f_1 + f_2)$. Clearly, it is much easier to resolve the latter frequencies, whichever spectral analysis method is used.

1. MAIN RESULTS

The original Teager-Kaiser Energy Function [1] of a discrete time signal $x(n)$ is defined as

$$e_o(n) = x^2(n) - x(n+1)x(n-1) \tag{1}$$

Simple analysis shows that the TKEF computed for two noise-free closely-spaced tones yields the signal envelope, which is a sinusoid at the difference frequency. However, there are two shortcomings to this approach. One is that the TKEF is highly sensitive to noise. Secondly, it does not permit us to determine the sum frequency (which is required to estimate the individual tone frequencies). We solve both problems by generalizing the TKEF as follows:

$$e(n) = x^2(n) - x(n+M)x(n-M) \tag{2}$$

where M is an arbitrary integer, referred to henceforth as the *lag parameter*. We shall show that M can be chosen to enhance the difference frequency (say $M = M_d$) or the sum frequency (say $M = M_s$). Putting the two GTKEFs together, we can define the following nonlinear transformation of the signal $x(n)$ which translates a dual tone signal with frequencies at f_1 & f_2 into another dual tone signal with frequencies at $(f_1 + f_2)$ & $(f_1 - f_2)$.

$$y(n) = \{x^2(n) - x(n+M_d)x(n-M_d)\} + \{x^2(n) - x(n+M_s)x(n-M_s)\} \tag{3}$$

Spectral analysis of $y(n)$ can easily resolve the two tones. Note that a general nonlinear transformation of $x(n)$ would result in a quad frequency signal with tones at $2f_1$, $2f_2$, $(f_1 + f_2)$, $(f_1 - f_2)$. Since f_1 & f_2 are assumed to be close to each other, $(f_1 + f_2)$, $2f_1$, $2f_2$ are also close to each other and the problem of resolving the tones is not simplified.

2. SIMULATION STUDIES

Consider a noisy dual tone signal

$$x(n) = A_1 Cos(\Omega_1 n + \Phi_1) + A_2 Cos(\Omega_2 n + \Phi_2) + A_3 w(n) \tag{4}$$

where A_1 and A_2 are the amplitudes of the two sinusoids, Ω_1 and Ω_2 are the digital frequencies in radians/sample and are given by $\Omega_{1,2} = 2\pi f_{1,2}/f_s$, where $f_{1,2}$ are the analogy frequencies and f_s is the sampling frequency. Φ_1 and Φ_2 are the arbitrary initial phases in radians. $w(n)$ is a zero mean, unit variance white noise with A_3 determining the level.

Figs 1a&b show the original TKEF $e_o(n)$ and its magnitude spectrum for a signal $x(n)$ with $f_1 = 300$, $f_2 = 330$ Hz and $f_s = 8000$ Hz. Noise is assumed to be zero, i.e. $A_3 = 0$ and the sinusoid amplitudes are equal $A_1 = A_2 = 1$. Note that although the difference frequency is dominant, the sum frequency (630 Hz) can also be seen in the spectrum at an attenuated level. Moreover, when white noise $w(n)$ (zero mean, unit variance, Gaussian) is added with $A_3 = 0.177$ (signal to noise ratio SNR = 25 dB), the result becomes quite contaminated as illustrated in Figs 2a&b.

We first apply the generalized TKEF to the noiseless case $A_1 = A_2 = 1$ & $A_3 = 0$. With the lag parameter $M = 25$, the GTKEF $e(n)$ and its magnitude spectrum are computed and are shown in Figs 3a&b. Clearly the sum frequency component at 630 Hz is enhanced in both the time and frequency domains. This example demonstrates that the GTKEF can be used to determine the sum frequency of a dual frequency signal.

We now apply the GTKEF to the noisy case $A_1 = A_2 = 1$ & $A_3 = 0.177$ (SNR = 25 dB). Figs 4a&b depict the GTKEF and its magnitude spectrum with lag parameter $M_d = 6$. Clearly the difference frequency component at 30 Hz is enhanced considerably compared to the results produced by the original TKEF in Fig 2. Similarly, Figs 4c&d show the GTKEF with lag parameter $M_s = 25$, where we see that the sum frequency is enhanced as in Fig 3. This example demonstrates that the GTKEF can be used to suppress the effects of additive noise and also determine the sum frequency of a dual frequency signal.

0-7803-2431-5/95 $4.00 © 1995 IEEE

3. ANALYTICAL BASIS

We shall now produce analytical results to support the simulation studies.

3.1. Noiseless Case

Referring to (4), assume that there is no noise and that Φ_1 and Φ_2 are fixed, the GTKE function consists of three components: the DC component, and the two sinusoids with the sum and difference frequencies Ω_s and Ω_d respectively:

$$E(n) = D + A_s Cos(\Omega_s n + \Phi_s) + A_d Cos(\Omega_d n + \Phi_d) = D + e_s(n) + e_d(n) \quad (5)$$

where

$$D = A_1^2 Sin^2(M\Omega_1) + A_2^2 Sin^2(M\Omega_2) \quad \text{(the DC component)}$$

$$A_s = 2A_1A_2 Sin^2(M\Omega_d/2) \qquad A_d = 2A_1A_2 Sin^2(M\Omega_s/2)$$

$$\Omega_s = \Omega_1 + \Omega_2 \qquad \Omega_d = \Omega_1 - \Omega_2$$

$$\Phi_s = \Phi_1 + \Phi_2 \qquad \Phi_d = \Phi_1 - \Phi_2$$

We shall now show that M can be selected to make $A_d >> A_s$ or vice versa. Ignoring the DC component, define the *Signal Enhancement Ratio* (SER) to measure the relative enhancement quality of the sinusoids of Ω_d and Ω_s as follows:

$$SER_d \stackrel{\Delta}{=} \frac{A_d^2}{A_s^2} = \frac{Sin^4 \frac{M}{2}\Omega_s}{Sin^4 \frac{M}{2}\Omega_d} \quad (6a)$$

$$SER_s \stackrel{\Delta}{=} \frac{A_s^2}{A_d^2} = \frac{Sin^4 \frac{M}{2}\Omega_d}{Sin^4 \frac{M}{2}\Omega_s} = \frac{1}{SER_d} \quad (6b)$$

Fig 5 shows the dependence of SER_d (or $1/SER_s$) on M. Clearly, two M's, namely M_d and M_s, can be selected to maximize SER_d and SER_s. We can see $M = 1$ gives the maximum value of SER_d so that the original Teager-Kaiser energy function is optimal for the difference frequency in the noiseless case.

3.2. Noisy Case

Assume that $w(n)$ is a zero mean, unit variance, white Gaussian noise. We shall express the GTKE function $e(n)$ as follows:

$$e(n) = D + e_s(n) + e_d(n) + e_w(n) \quad (7a)$$

where the first three terms are defined in (5) and the last term contains the remaining noise-related quantities, with

$$\mathrm{E}[e_w(n)] = A_3^2 \quad (7b)$$

$$\mathrm{var}[E_w(n)] = 3(A_1^2 + A_2^2)A_3^2 + A_3^4 \mathrm{E}[w^4(n)] \quad (7b)$$

Furthermore, we shall measure the performance by the *Signal Enhancement Ratio* whose definition is extended to accommodate the stochastic case as the ratio of the variance of the signal term $e_s(n)$ to the variance of the noise term $e_n(n)$. Assuming that the desired term is the difference frequency tone, the Signal Enhancement Ratio (SER) turns out to be:

$$SER_d \stackrel{\Delta}{=} \frac{\mathrm{var}[e_d(n)]}{\mathrm{var}[e_s(n) + e_w(n)]} = \frac{\sin^4 \frac{M}{2}\Omega_s}{\sin^4 \frac{M}{2}\Omega_d + \frac{3}{2}\left(\frac{A_3^2}{A_1^2} + \frac{A_3^2}{A_2^2}\right)\sigma^2 + \frac{2A_3^4}{A_1^2 A_2^2}\sigma^4} \quad (8a)$$

Similarly, the SER for the sum frequency component is:

$$SER_s \stackrel{\Delta}{=} \frac{\mathrm{var}[e_s(n)]}{\mathrm{var}[e_d(n) + e_w(n)]} = \frac{\sin^4 \frac{M}{2}\Omega_d}{\sin^4 \frac{M}{2}\Omega_s + \frac{3}{2}\left(\frac{A_3^2}{A_1^2} + \frac{A_3^2}{A_2^2}\right)\sigma^2 + \frac{2A_3^4}{A_1^2 A_2^2}\sigma^4} \quad (8b)$$

The relationship between M and SER_d & SER_s is illustrated in Figs 6a&b. We can see $M = 1$ is no longer the optimal choice in the difference frequency case!

3.3. Nonlinear Preprocessor

This analysis suggests the general scheme shown in Fig-7 for resolving two closely spaced tones. The nonlinear processor is based on GTKEF and defined by (3). With M_d and M_s optimally chosen to enhance the sum and difference frequencies, this preprocessor can be seen as an interesting transformation taking a signal consisting of two closeby frequencies at f_1 and f_2 into a signal consisting of frequencies $(f_1 + f_2)$ and $(f_1 - f_2)$. The second block can be based on any spectral analysis method.

3.4. Approximate Formulas for Optimum Lag Parameters

Although the expressions for SER are rather complicated for determining the optimum lag parameters, we have produced the following approximate relations for the optimum lag parameters for the sum and difference frequencies. They are based on the assumption that the difference frequency is much smaller than the sum frequency and that the SNR is less than 50 dB, which is true in most of the practical situations. The formulas are:

$$M_d = \mathrm{int}\left[\frac{f_s/2}{f_1 + f_2} + 0.5\right] \quad (9a)$$

$$M_s = \mathrm{int}\left[\frac{f_s * i}{f_1 + f_2} + 0.5\right] \quad (9b)$$

where $i = 1, 2, 3...$ Larger values of the i produce greater signal enhancement but introduce longer delay (See Fig-6).

4. REFERENCE

[1] Kaiser, J.F., "On a simple algorithm to calculate the 'energy' of a signal", Proc. IEEE ICASSP-90, Albuquerque, NM, pp. 381-384, April 1990.

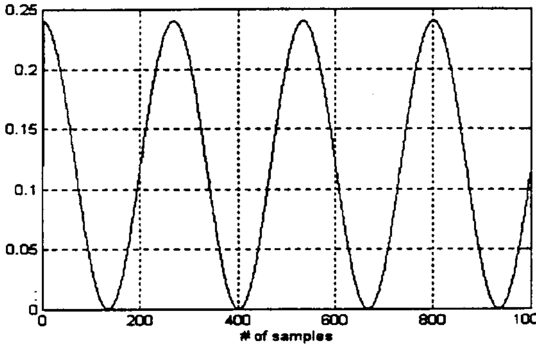

Fig. 1(a) TKEF $e_o(n)$: Noiseless & M = 1.

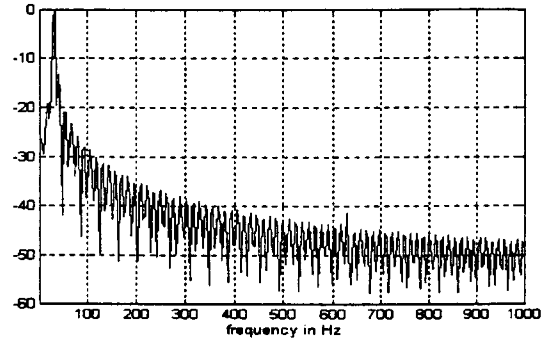

Fig. 1(b) Spectrum of $e_o(n)$.

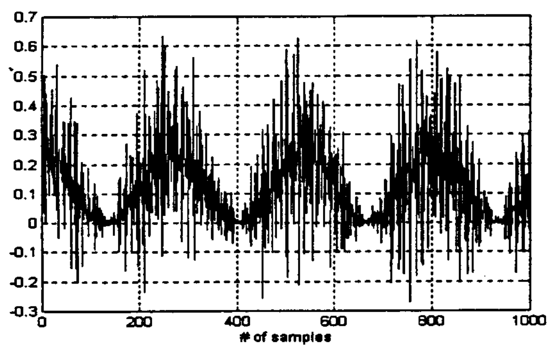

Fig. 2(a) TKEF $e_o(n)$: SNR_x = 25 dB & M = 1.

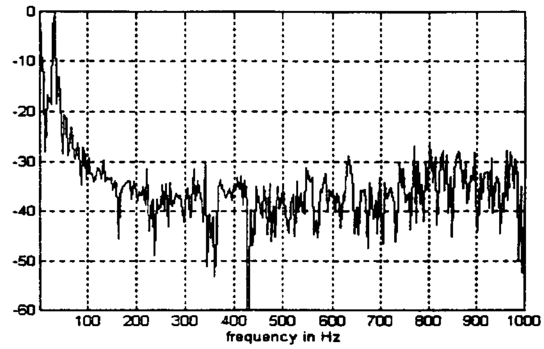

Fig. 2(b) Spectrum of $e_o(n)$.

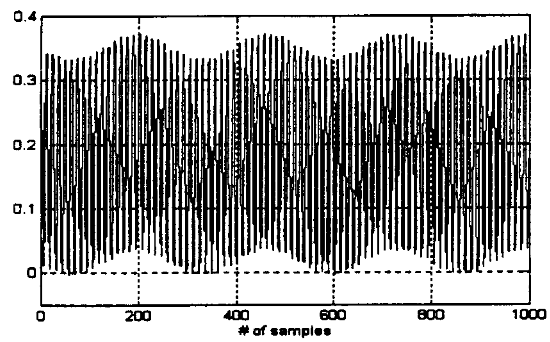

Fig. 3(a) GTKEF e(n): Noiseless & M = 25.

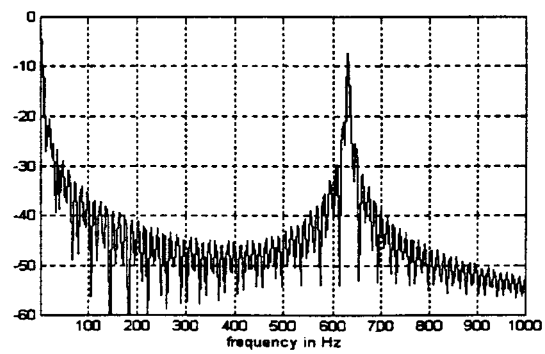

Fig. 3(b) Spectrum of e(n).

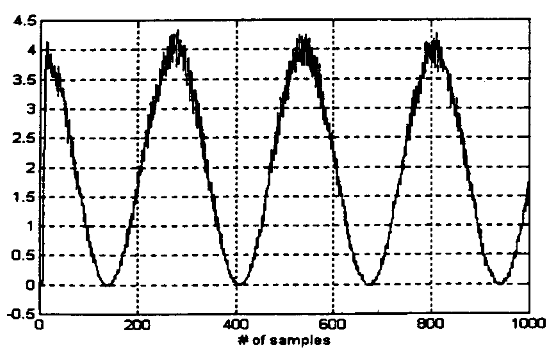

Fig. 4(a) GTKEF $e_d(n)$: SNR_x = 25 dB & M = 6.

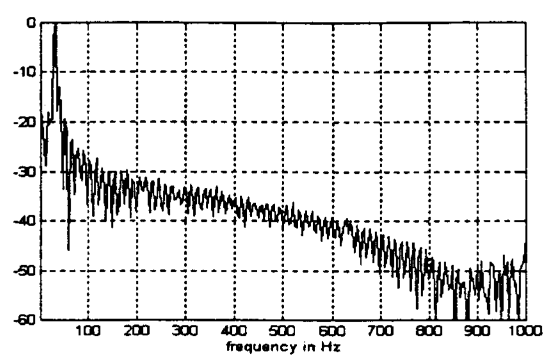

Fig. 4(b) Spectrum of $e_d(n)$.

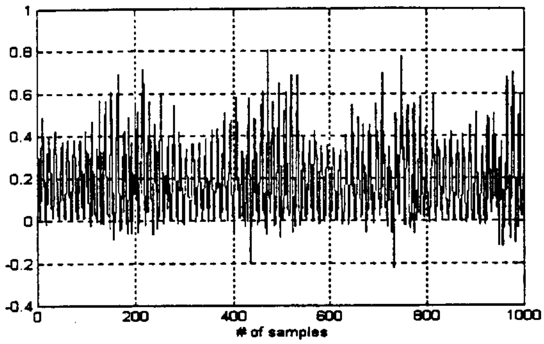

Fig. 4(c) GTKEF $e_s(n)$: $SNR_x = 25$ dB & M = 25.

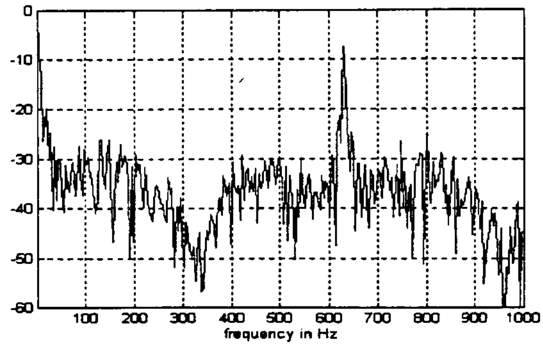

Fig. 4(d) Spectrum of $e_s(n)$.

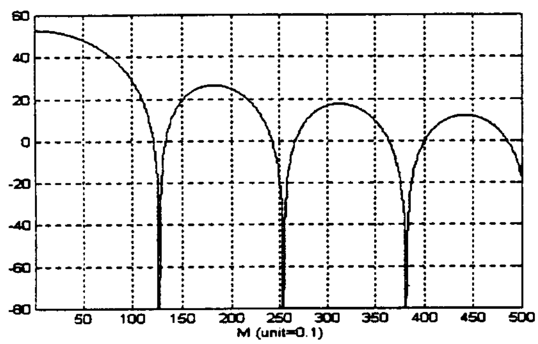

Fig. 5(a) $SER_d(M)$ in dB: Noiseless.

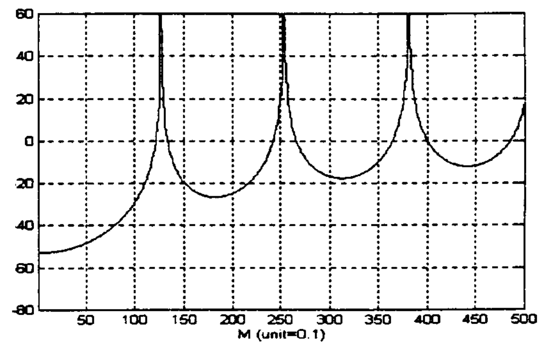

Fig. 5(b) $SER_s(M)$ in dB: Noiseless.

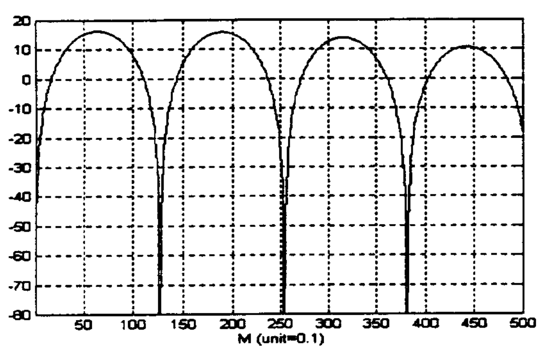

Fig. 6(a) $SER_d(M)$ in dB: $SNR_x = 25$ dB.

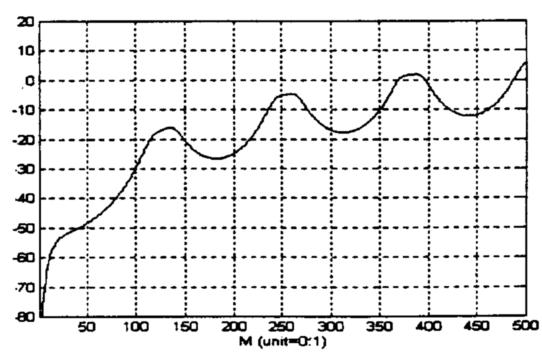

Fig. 6(b) $SER_s(M)$ in dB: $SNR_x = 25$ dB.

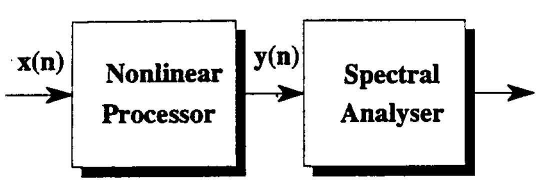

Fig. 7 General Scheme for Resolving Two Closely-spaced Tones.