AN UNIFIED PREFILTERING-BASED APPROACH TO HARMONIC RETRIEVAL IN NON-GAUSSIAN ARMA NOISE

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ABSTRACT

This paper addresses the harmonic retrieval problem in colored noise. As contrasted to the reported studies in which Gaussian noise was assumed, this paper focus on additive non-Gaussian ARMA noise. We propose an unified prefiltering-based approach to this problem. Our approach is hybrid in the sense that 3rd-order cumulants are first used to identify the AR part of the non-Gaussian noise process, and then correlation-based high resolution methods may be used for the filtered output process to estimate the parameters of harmonics. Simulation examples are presented to demonstrate the high resolution of this approach.

1. INTRODUCTION

The last decade has witnessed considerable research efforts aimed at harmonic retrieval in noise. Particularly, in recent years higher order cumulant-based harmonic retrieval has received increasing interest. The biggest advantage of cumulant-based methods over correlation-based methods is that the former handle additive colored Gaussian noise automatically, and thus boost signal-to-noise ratio in Gaussian noise cases, but the latter do not. The fourth-order cumulant based ESPRIT [1], MUSIC-like algorithm [2] and AR modeling approach [3] were recently developed.

Colored non-Gaussian noise environments are frequently related to sonar systems and signal detection. More recently, the problems of detecting sinusoids in colored non-Gaussian noise were studied in [4] and [5]. We remark that the work reported in [4] and [5] provides different non-parametric solutions to the hypothesis tests for detecting existence of sinusoids rather than harmonic retrieval for estimating the number of sinusoids and their frequencies. To our best knowledge, until now there are no reports on harmonic retrieval in additive non-Gaussian ARMA noise. The objective of this paper is just to deal with this problem.

We show that the AR coefficients of the non-Gaussian ARMA noise with asymmetrical distribution can be identified by using the 3rd-order cumulants of the output process, and that the parameters of harmonics can be estimated with the correlations of the output process filtered by the AR parameters of non-Gaussian ARMA noise. Therefore, an unified prefiltering-based high resolution approach is proposed for estimating the parameters of harmonics in non-Gaussian ARMA noise. Simulation results are given to illustrate the performance of the new approach.

2. PRELIMINARIES

Consider the following output time series with zero mean:

$$y(n)=s(n)+v(n) \tag{1}$$

where $s(n)$ is a noiseless real-valued harmonic signal specified by

$$s(n)=\sum_{i=1}^{p}\alpha_i \sin(\omega_i n+\phi_i). \tag{2}$$

where ω_i and α_i are the normalized frequency and amplitude of the ith harmonic, respectively. We assume that the harmonics are un-coupled, and the ϕ_i's are random variables uniformly distributed over $[-\pi,\pi]$. Phase randomization in the harmonics implies the availability of multiple records for consistent estimation of the cumulants (and correlation).

In comparison with most previous work, the additive noise $v(n)$ in (1) is assumed to be a non-Gaussian ARMA(p_1,q_1) process given by

$$v(n)=-\sum_{i=1}^{p_1}d(i)v(n-i)+\sum_{j=0}^{q_1}c(j)e(n-j) \tag{3a}$$

or

$$D(q)v(n)=C(q)e(n) \tag{3b}$$

where $D(q)=\sum_{i=0}^{p_1}d(i)q^{-i}$, $C(q)=\sum_{i=0}^{q_1}c(i)q^{-i}$ and $q^{-j}e(n)=e(n-j)$.

For the model (1), the following conditions are assumed to hold.

AS1) The transfer function $H(z)=C(z)/D(z)$ is free of pole-zero cancellations, and is exponentially stable. The noise is causal, but may be of nonminimum phase.

AS2) $\{e(n)\}$ is zero mean and i.i.d. with asymmetrically distribution. i.e., $\gamma_{3e}\equiv E[e^3(n)]\neq 0$ and finite moments.

AS3) $\{s(n)\}$ is independent of $\{e(n)\}$ and thus of $\{v(n)\}$.

Condition (AS2) is a key assumption in this work. However, this condition is satisfied for many distributions, such as exponential distribution.

Recall [3] that the third order cumulants of harmonic signal in (2) are identically zero. That is to say, for the model (1) we have $C_{3y}(m_1,m_2)=C_{3v}(m_1,m_2)$. Consequently, the AR order p_1 and AR coefficients $d(i), i=1,\ldots,p_1$ of the non-Gaussian ARMA noise model can be estimated by using $C_{3y}(m_1,m_2)$. By Giannakis and Mendel [6], the determination of the AR order and the estimation of AR parameters can be implemented by SVD-TLS method [7].

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In summary, the problem of interest is to estimate the number of harmonics, p, and their frequencies ω_i, provided that the AR order and AR parameters of noise process have been available.

3. THEORETICAL RESULTS

3.1 Prefiltering

Multiplying (1) by $D(q)$, we have

$$D(q)y(n)=D(q)s(n)+D(q)v(n). \quad (4)$$

Define

$$\tilde{y}(n)=D(q)y(n)=\sum_{i=0}^{p_1}d(i)y(n-i) \quad (5)$$

$$\tilde{s}(n)=D(q)s(n)=\sum_{i=0}^{p_1}d(i)s(n-i) \quad (6)$$

$$\tilde{v}(n)=D(q)v(n)=C(q)e(n)=\sum_{i=0}^{q_1}c(i)e(n-i) \quad (7)$$

then (4) can be rewritten as

$$\tilde{y}(n)=\tilde{s}(n)+\tilde{v}(n). \quad (8)$$

We refer to $\tilde{y}(n)$ as the filtered output process and $\tilde{s}(n)$ as the filtered harmonic signal. Interestingly, since $\tilde{v}(n)$ is a pure MA(q_1) process, we have

$$R_{\tilde{y}}(m)\equiv R_{\tilde{s}}(m), \quad m>q_1. \quad (9)$$

3.2 Autocorrelations of $\tilde{s}(n)$

It is well known that the autocorrelation of harmonic signal in (2) is given by

$$R_s(m)=\frac{1}{2}\sum_{i=1}^{p}\alpha_i^2 cos(\omega_i m). \quad (10)$$

For the filtered harmonic signal $\tilde{s}(n)$, we have the following result.

Proposition 1: The autocorrelation of the filtered harmonic signal $\tilde{s}(n)$ is

$$R_{\tilde{s}}(m)=\frac{1}{2}\sum_{i=1}^{p}\alpha_i^2\cdot\sum_{k=0}^{p_1}\sum_{l=0}^{p_1}d(k)d(l)cos[\omega_i(m+k-l)]. \quad (11)$$

Proof: From (6), we have

$$\begin{aligned}&E[\tilde{s}(n)\tilde{s}(n+m)]\\&=E[\sum_{k=0}^{p_1}d(k)s(n-k)\cdot\sum_{l=0}^{p_1}d(l)s(n+m-l)]\\&=\sum_{k=0}^{p_1}\sum_{l=0}^{p_1}d(k)d(l)E[x(n-k)x(n+m-l)]\end{aligned}$$

namely,

$$R_{\tilde{s}}(m)=\sum_{k=0}^{p_1}\sum_{l=0}^{p_1}d(k)d(l)R_s(m+k-l). \quad (12)$$

Combination of (12) and (10) yields (11).

Proposition 2: The autocorrelation of the filtered output process $\tilde{y}(n)$ satisfies

$$R_{\tilde{y}}(m)=\frac{1}{2}\sum_{i=1}^{p}\alpha_i^2\cdot\sum_{k=0}^{p_1}\sum_{l=0}^{p_1}d(k)d(l)cos[\omega_i(m+k-l)], \quad m>q_1. \quad (13)$$

Proof: Combination of (11) and (9) yields (13) directly.

From (13) it is seen that the autocorrelation of the filtered output process $\tilde{y}(n)$ contains the useful information of harmonic signal. i.e., the number of harmonics, p, the frequencies, ω_i's, and amplitudes, α_i's. Therefore, the existing high resolution methods for harmonic retrieval can be extended to new corresponding versions which are available for the filtered output process. In the next section, an unified prefiltering-based approach for harmonic retrieval in non-Gaussian ARMA noise is proposed.

4. PREFILTERING-BASED APPROACH

Harmonic signal in (2) satisfies the following Yule-Walker equation

$$\sum_{i=0}^{2p}a(i)R_s(m-i)=0, \quad any\ m. \quad (14)$$

The number of harmonics, p, can be determined via the rank of the autocorrelation matrix, and the frequencies ω_i's can be obtained by using the unit modulus roots z_i of $A(z)=\sum_{i=0}^{2p}a(i)z^{-i}=0$.

A logic question to ask is "What is the relationship between autocorrelations of the filtered output process and the AR parameters of harmonic signal, $a(i), i=1,...,2p$?". To answer this question, let us first consider the relationship between $R_{\tilde{s}}(m)$ and $a(i)$.

From (12), we have

$$\begin{aligned}&\sum_{i=0}^{2p}a(i)R_{\tilde{s}}(m-i)\\&=\sum_{k=0}^{p_1}\sum_{l=0}^{p_1}d(k)d(l)\times\sum_{i=0}^{2p}a(i)R_s(m+k-l-i)\\&\equiv 0\end{aligned} \quad (15)$$

where we have used (14) which yields directly $\sum_{i=0}^{2p}a(i)R_s(m+k-l-i)\equiv 0$. If $m>2p+q_1$ is taken in the above equation, then all lags used on $R_{\tilde{s}}(\cdot)$ are greater than q_1. Consequently, from (9) and (15) it is straightforward to see that

$$\sum_{i=0}^{2p}a(i)R_{\tilde{y}}(m-i)=0, \quad m>2p+q_1. \quad (16)$$

Several remarks about (16) are given.

Remark 1: For p complex harmonics in additive non-Gaussian ARMA(p_1,q_1) noise $\{v(n)\}$, the counterpart of (16) is given by

$$\sum_{i=0}^{p} a(i) R_{\tilde{y}}(m-i)=0, \quad m>p+q_1. \tag{17}$$

Remark 2: Equation (16) can be used to solve for the AR parameters $a(i)$'s of the harmonic signal model, based on the autocorrelations of the filtered output process $\tilde{y}(n)$. Although we typically do not know p and q_1 *a priori*, it is generally possible to make an educated guess of p and q_1 so as to ensure that $p_e>2p$ and $q_e>q_1$, then, the AR order $2p$ and AR parameters $a(i), i=1,...,2p$ can be estimated by using SVD-TLS method [7].

Equation (16) can also be considered as the modified Yule-Walker (MYW) equation for harmonic retrieval in non-Gaussian ARMA noise. Since the autocorrelations of $\{\tilde{y}(n)\}$ contain the useful information of the harmonic signal, it can be shown that by constructing a new matrix pencil, the ESPRIT can be extended to a new ESPRIT available for an additive MA noise with unknown correlation functions, and that by applying this new ESPRIT to the filtered output data $\{\tilde{y}(n)\}$, we can retrieve the harmonics in the non-Gaussian ARMA noise.

In summary, we can develop an unified approach to harmonic retrieval in non-Gaussian ARMA noise.

(1) Use the third order cumulant of the output data to determine the AR order p_1 and AR parameters, $d(i)$, $i=1,\cdots,p_1$ [6].

(2) Use the estimated AR polynomial and (5) to prefilter the output data.

(3) Apply any correlation-based method for harmonic retrieval in additive MA noise to the filtered output data in order to retrieve the harmonics.

Such an unified prefiltering-based approach is a hybrid method in the sense that a cumulant-based technique is first used to identify the AR part of the noise, and a correlation-based technique is then used to make harmonic retrieval in non-Gaussian ARMA noise. In the next section, we used the modified Yule-Walker equation method to simulate the harmonic retrieval problem.

5. SIMULATIONS

To demonstrate the effectiveness of the prefiltering-based approach for harmonic retrieval in non-Gaussian ARMA noise, we present a numerical example in this section.

Consider the following output process

$$y(n)=\sqrt{2}sin(2\pi 0.2n)+\sqrt{2}sin(2\pi 0.213n)+v(n)$$

where $v(n)$ was a non-Gaussian ARMA(2,2) noise process given by

$$v(n)-1.5v(n-1)+0.8v(n-2)=e(n)-0.75e(n-1)-2.5e(n-2).$$

It is noted that the noise spectrum has a strong pole at $f=0.1$. Independent exponentially distributed random deviates were generated for the input sequence of noise, $\{e(n)\}$, from the GGEXN subroutine in the International Mathematical and Statistical Library (IMSL). In the simulations, twenty independent realizations of noise data $y(n)$, each consisting of 512 samples, were generated at each fixed SNR, and three methods were used to compare their performances. The three methods used are the direct correlation method [7], the direct (4th-order) cumulant approach [3], and the prefiltering-based approach based on equation (16) of this paper.

We tested three SNR cases, i.e., $\sigma_v^2=1$(0dB), $\sigma_v^2=10$(−10dB) and $\sigma_v^2=100$(−20dB). By the SVD, $p_1=2$ and $p=2$ were determined in each run. Table I shows AR parameter estimates of the noise model. Table II shows the results of frequency estimation given by the three methods.

From the simulations, it is seen that the direct correlation based and direct cumulant based estimates are considerably biased and could not retrieve the given harmonics in each case, however, the prefiltering- based approach of this paper shows excellent resolution in the harmonic retrieval even at rather low SNR's. Although the AR parameter estimates of the noise model are biased and of high variance, the frequency estimates via the new approach show small bias and low variance.

6. CONCLUSIONS

This paper has analyzed the harmonic retrieval problem in additive non-Gaussian ARMA noise. An unified prefiltering-based approach has been proposed. The new approach consists of AR modeling of the noise process followed by filtering of the noisy time series followed by correlation-based harmonic retrieval. Simulations have shown the effectiveness and high resolution of this approach. Finally, we remark that although the whole ARMA modeling of the non-Gaussian noise is available, it is not a practical algorithm to whiten the non-Gaussian noise. This is because that the filter $D(z)/C(z)$ is non-causal and is not implementable if the noise is of non-minimum phase, or is unstable if the MA polynomial has zeros such that $C(z)=\sum_{i=0}^{q_1} c(i)z^{-i}=0$ for $|z|=1$.

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TABLE I

Statistics of AR Parameter Estimates Obtained via the Prefiltering-Based Approach (N=512 in Each Run, 20 Monte Carlo Runs)

Noise Model	$b(1)$	$b(2)$
True	-1.5	0.8
SNR = 0dB	-1.5876(0.2157)	0.9303(0.0831)
SNR = -10dB	-1.5723(0.1773)	0.9016(0.0799)
SNR = -20dB	-1.5102(0.1390)	0.8679(0.0772)

TABLE II

Statistics of Frequency Estimates Obtained via Three Methods (N=512 in Each Run, 20 Monte Carlo Runs)

Frequency	f_1	f_2	f_3
True	0.213	0.2	
SNR = 0dB			
Direct Correlation			
($2p$=4)	0.2075(0.0009)	0.0931(0.0058)	
($2p$=6)	0.2214(0.0143)	0.1825(0.0786)	0.0856(0.0368)
Direct Cumulant			
($2p$=4)	0.2004(0.0368)	0.0754(0.0207)	
($2p$=6)	0.2048(0.0342)	0.1143(0.0543)	0.0743(0.0632)
The New Approach	0.2129(0.0007)	0.2002(0.0006)	
SNR = -10dB			
Direct Correlation			
($2p$=4)	0.2079(0.0016)	0.0920(0.0057)	
($2p$=6)	0.2083(0.0086)	0.1065(0.0011)	0.0786(0.0161)
Direct Cumulant			
($2p$=4)	0.2042(0.0098)	0.1042(0.0084)	
($2p$=6)	0.2109(0.0502)	0.1058(0.0528)	0.0675(0.0530)
The New Approach	0.2128(0.0008)	0.2005(0.0015)	
SNR=-20dB			
Direct Correlation			
($2p$=4)	0.2061(0.0022)	0.0926(0.0067)	
($2p$=6)	0.2079(0.0008)	0.1095(0.0170)	0.0763(0.0153)
Direct Cumulant			
($2p$=4)	0.2008(0.0053)	0.0842(0.0076)	
($2p$=6)	0.2143(0.0068)	0.1145(0.0182)	0.0845(0.0253)
The New Approach	0.2148(0.0030)	0.1992(0.0031)	