ESTIMATION OF SPECTRAL CORRELATION USING FILTER BANKS

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ABSTRACT

This paper considers the use of filter banks for estimating the spectral correlation density function (SCD). Past techniques have used FFT techniques to lower the computational complexity of SCD estimation. We study an alternative approach of using FIR filter banks. The time resolution, bi-frequency resolution, and complexity of the two techniques will be compared. Examples from communication theory are considered.

1. INTRODUCTION

A process $x(t)$ is called cyclostationary if its statistics are invariant to a shift of the time origin by integral multiples of a constant T (the period). In this paper we consider only wide-sense cyclostationary (WSCS) processes, with mean $E\{x(t)\} = \mu(t) = \mu(t+mT)$ and autocorrelation $R(t+mT+\tau, t+MT) = R(t+\tau, t)$ [1]. As pointed out in [2], many man-made signals exhibit this property and it can be used to advantage in signal processing.

The cyclic autocorrelation function is a generalization of the autocorrelation function given by

$$R_x^\alpha(\tau) = E\left\{x(t+\tau/2)\, x^*(t-\tau/2)\, e^{-j2\pi\alpha t}\right\}. \tag{1}$$

Also, $R_x^\alpha(\tau)$ is a conventional cross-correlation function

$$R_{uv}(\tau) = E\left\{u(t+\tau/2)\, v^*(t+\tau/2)\right\}, \tag{2}$$

where

$$u(t) = x(t)\, e^{-j\pi\alpha t} \tag{3}$$

and

$$v(t) = x(t)\, e^{+j\pi\alpha t}. \tag{4}$$

When $\alpha = 0$ the conventional autocorrelation function is retrieved.

The power spectral density function (PSD) of a wide sense stationary (WSS) process $x(t)$ localizes in frequency the average power $E\left\{|x(t)|^2\right\} = R_x(0)$. Likewise, it can be very helpful to localize in frequency the correlation

$$E\{u(t)\, v^*(t)\} = E\left\{|x(t)|^2 e^{-j2\pi\alpha t}\right\} = R_x^\alpha(0). \tag{5}$$

Similar to the Wiener-Khinchine relation between the autocorrelation function and the PSD, the SCD is defined as

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau)\, e^{-j2\pi f\tau} d\tau, \tag{6}$$

which is also a cross-spectral density function between the two signals $u(t)$ and $v(t)$.

2. ESTIMATION OF THE SCD

Just as the PSD of a WSS process can be defined as

$$S_x(f) = \lim_{\Delta f\to 0} \frac{1}{\Delta f} \lim_{\Delta t\to\infty} \frac{1}{\Delta t} \int_{-\Delta t/2}^{+\Delta t/2} \left|h_{\Delta f}^f(t) \otimes x(t)\right|^2 dt, \tag{7}$$

so too can the SCD of a WSCS process be defined as

$$S_x^\alpha(f) = \lim_{\Delta f\to 0} \frac{1}{\Delta f} \lim_{\Delta t\to\infty} \frac{1}{\Delta t} \int_{-\Delta t/2}^{+\Delta t/2} \left(h_{\Delta f}^f(t) \otimes u(t)\right)\left(h_{\Delta f}^f(t) \otimes v(t)\right)^* dt. \tag{8}$$

The function $h_{\Delta f}^f(t)$ is a one-sided bandpass filter with center frequency f and bandwidth Δf, and $\otimes$ represents convolution.

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2.1. The Time Smoothed Cyclic Cross Periodogram

To estimate the SCD, assume $x(t)$ is bandlimited to $0 \le f \le \Omega$. We obtain the discrete time process $x(n) = x(t = nT_s)$, where $f_s = 1/T_s \ge 2\Omega$ is the sampling frequency. Equation (8) can be translated to a discrete version,

$$\hat{S}_x^\alpha(n,f) = \sum_{r=0}^{N-1} X\left(n-r, f+\frac{\alpha}{2}\right) \cdot X^*\left(n-r, f-\frac{\alpha}{2}\right) g(r). \quad (9)$$

This is referred to as the time smoothed cyclic cross periodogram [3][4]. The *complex demodulates* of $x(t)$ are computed as

$$X(n,f) = \sum_{r=0}^{K-1} a(r)\, x(n-r)\, e^{-j2\pi f(n-r)T_s}, \quad (10)$$

which is the complex envelope of the narrowband component of the signal centered at frequency f with bandwidth Δf. The data tapering window $a(r)$ is K samples long. Finally, $g(n)$ is an N point window for time smoothing the correlation $X(n, f+\alpha/2) X^*(n, f-\alpha/2)$.

It is shown in [3] that if the time windows are properly normalized, $\sum_n a^2(n) = \sum_n g(n) = 1$, then the time smoothed cyclic cross periodogram converges to the SCD in the limit, as $\Delta t \to \infty$, followed by $\Delta f \to 0$. For a reliable estimate it is necessary to have $\Delta t >> 1/\Delta f$, or equivalently, $\Delta t \Delta f >> 1$. Since $\Delta t \approx N T_s$ and $\Delta f \approx f_s/K$, then $\Delta t \Delta f \approx N/K$.

We are interested in efficient ways to compute the complex demodulates, which has been termed "channelizing" the data. This is done via filter bank decomposition, primarily through the windowed discrete Fourier transform approach. Ways of improving the efficiency include using FFT and fast subband filtering techniques, as well as decimation of the output signals. The demodulate signals have bandwidth on the order of $\Delta f \approx f_s/K$, and so they may be decimated by a factor $L \le K$. For higher decimation rates the aliasing distortion can become severe and reduce the quality of the estimate.

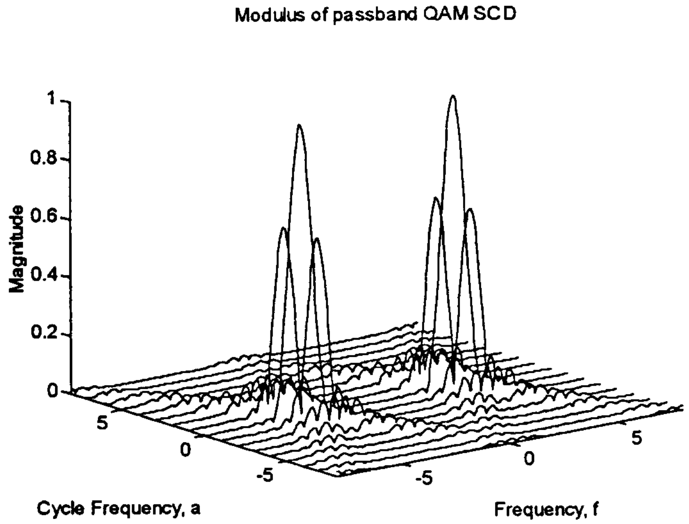

Figure 1. Theoretical spectral correlation density function of a passband QAM signal

2.2. Estimation based on the FFT

If we quantize the frequencies as $f_m = mf_s/M$, i.e., M frequency channels, then we can write

$$X(n,f_m) = \sum_{r=0}^{K} a(r)\, x(n-r)\, e^{-j2\pi m(n-r)/M}, \quad (11)$$

which can be directly computed using a DFT. This creates the diamond shaped tiling of the bifrequency plane, $\alpha_{i,j} = (i-j)f_s/M$, $f_{i,j} = (i+j)f_s/2M$, for $0 \le i,j < M$.

In this case the length of the data tapering window $a(n)$ is restricted to be $K \le M$. We consider only $K = M$. The system has computational complexity of $M + 4M + M\log M$ real additions and multiplications per M outputs. This is M for filtering, $4M$ for demodulation, and $M\log M$ for FFT operations. The frequency resolution is $1/M$. This system has time resolution of $1/N$.

2.3. Estimation based on filter banks

The frequency is again divided into M channels. We have considered using cosine modulated filter banks [5],

$$a_m(n) = a(n) \cdot \cos\left[\frac{\pi(n-n_a)}{M}\left(m+\frac{1}{2}\right)\right], \quad (12)$$

for $m = 0,1,\ldots,M-1$. Here $a(n)$ is a baseband prototype filter with nominal cutoff frequency $\pi/2M$ and n_a is the analysis filter bank delay. This creates the diamond shaped tiling of the bifrequency plane, $\alpha_{i,j} = (i-j)f_s/2M$, $f_{i,j} = (i+j+1)f_s/2M$, for $0 \le i,j < M$.

In general, the length of the baseband filter is not constrained, although there are certain lengths which give "easy" solutions, e.g., $K = 2M$ gives rise to the lapped orthogonal transform (LOT) [5]. The system has computational complexity of $K + 8M + 2M\log 2M$ real additions and multiplications per M outputs. This is K for filtering, $8M$ for modulation and demodulation, and $2M\log 2M$ for FFT operations. The frequency resolution is $1/K$. This system has time resolution of $1/N$.

EXAMPLE 1: Digital QAM

Figure 1 shows the theoretical SCD modulus for a digital quadrature amplitude modulation (QAM) signal [6]. The transmit filter was a 27 tap, 100% excess bandwidth, raised cosine approximation. Figure 2 shows the estimate using a length $M = K = 128$ FFT channelizer. Figures 3 and 4 show estimates for $M = 64$ band cosine modulated filter bank channelizers using $K = 128$ and $K = 256$ length baseband filters, respectively. The sampling frequency was $f_s = 16 f_{symbol}$, where f_{symbol} is the symbol rate. The carrier frequency is nominally $f_c = 3.5 f_{symbol}$, however it is varied slightly for worst case sibelode resolution (the filter bank and FFT approaches have different bin center frequencies). The record of data used was $N = 32768$ samples long. For brevity, these figures show only the lines of constant cycle frequency, $\alpha = n f_{symbol}$, for $n = 0,1,\ldots 7$ and $f \ge 0$. In all cases, the data tapering window, $a(n)$, was a Blackman window, and the smoothing window, $g(n)$, was a Hamming window [7].

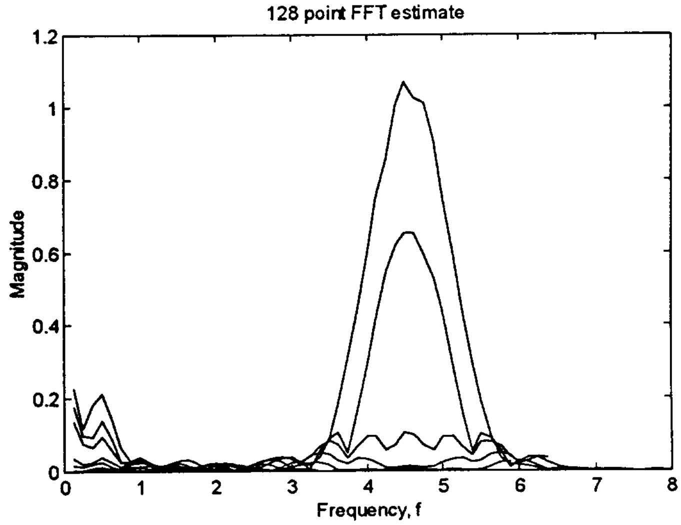

Figure 2. SCD estimate using 128 point FFT channelizer

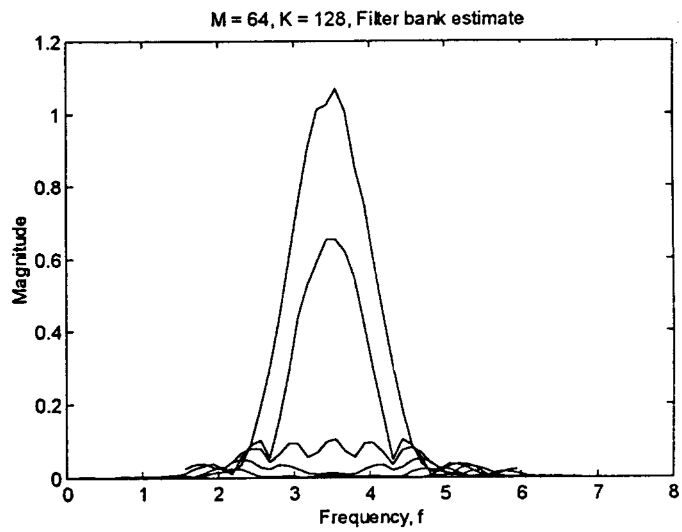

Figure 3. SCD estimate using $M = 64$, $K = 128$ filter bank

Note the roughly equivalent performance of the $M = 128$ FFT and $M = 64$, $K = 128$ filter bank. These two structures are closely related. The $M = 64$, $K = 256$ filter bank produces a less reliable estimate, a result of the N/K reduction by a factor of two.

EXAMPLE 2: LCAM

Consider the large carrier amplitude modulation (LCAM) test signal,

$$x(t) = \left(1 + \cos 2\pi f_{\mathrm{mod}} t\right)\cos 2\pi f_c t. \quad (13)$$

When measuring the SCD function at the point $(\alpha, f) = (0, 2f_c)$, the window function $a(n)$ must reduce the sideband spectral lines sufficiently as to not bias the estimate. This is the classic problem of spectral resolution and spectral leakage in harmonic analysis [7].

When $f_{mod} = 0.1 \cdot f_c$, the true value is $\hat{S}_x^{\alpha=2f_c}(f=0) = 0.25$. Using K = 128 FFT, the estimate is $\hat{S}_x^{\alpha=2f_c}(f=0) = 0.2662$. Using $M = 64$, $K = 256$ produces an estimate of $\hat{S}_x^{\alpha=2f_c}(f=0) = 0.2499$. The relative error is reduced by a factor of over 190. Again, this is using a Blackman filter.

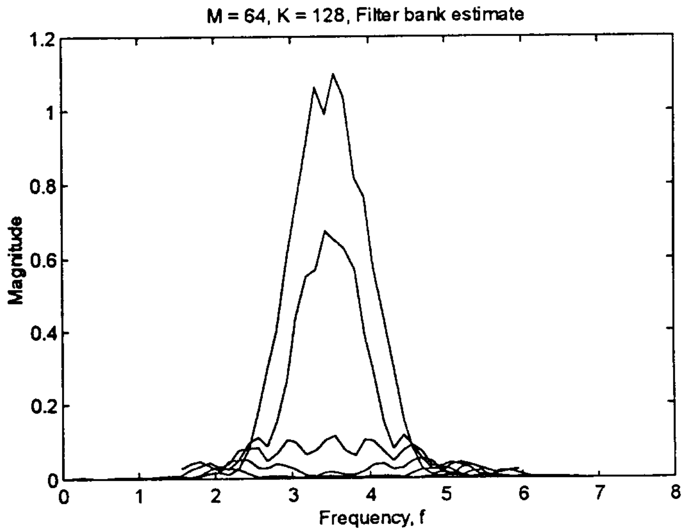

Figure 4. SCD estimate using $M = 64$, $K = 256$ filter bank

3. DISCUSSION

The time resolution, frequency resolution, time-bandwidth product, and computational complexity of the FFT and cosine modulated filter bank approach are shown in Table 1. Note the filter banks weaker dependence on K, which sets the spectral resolution and the bandwidth of the complex demodulates. Since the bandwidth is lower, the decimation rate of the channelizer output can be increased. This affects how often every block of M output samples needs to be computed. However, the time-bandwidth product is affected. For the filter bank case, using $K = 4M$ has produced good results. In this case, a filter with very low sidelobes can be used and still maintain spectral resolution on the order of the bin width. This also gives a factor of two higher decimation rate than in a FFT approach with the same bin width.

	FFT, $K = M$	Filter Bank	FFT, $M = 128$	FB, $M = 64$ $K = 128$	FB, $M = 64$ $K = 256$
Δf	f_s/M	f_s/K			
Δt	$N \cdot T_s$	$N \cdot T_s$			
$\Delta t \Delta f$	N/M	N/K			
Adds/ Mults	$M + 4M + M \log M$	$K + 8M + 2M \log 2M$	$12 \cdot 128$	$12 \cdot 128$	$13 \cdot 128$

Table 1. FFT and cosine modulated filter bank (FB) comparison

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