BLIND IDENTIFICATION OF NONLINEAR MODELS USING HIGHER ORDER SPECTRAL ANALYSIS

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ABSTRACT

A simple method is proposed for blind identification of discrete-time nonlinear models consisting of two Linear Time Invariant (LTI) subsystems separated by a polynomial-type Zero Memory Nonlinearity (ZMNL) of order N (the LTI-ZMNL-LTI model). When the input to the model is a circularly symmetric Gaussian sequence, the linear subsystem of the model can be identified efficiently using slices of the $N+1^{th}$ order polyspectrum of the output signal, even when the second linear subsystem is of *Non-Minimum Phase* (NMP). The ZMNL coefficients need not be known. The order N of the nonlinearity can, in principle, be estimated from the received signal. The methods possess noise suppression characteristics. Computer simulations support the theory.

1. INTRODUCTION

Linear channel models, because of their simplicity, have traditionally been used in a large number of practical applications and a variety of methods have been suggested for identification of such models [1]. However, most practical circuits and channels are better approximated by nonlinear models. Examples include hard-clippers, rectifiers, envelope detectors, data generation and other communication circuits, the magnetic recording channel, telephone channels operating at high data rates, digital microwave and satellite communication links with High Power Amplifier (HPA) nonlinearities [2], among others. In several of these applications, the input signal (or a training sequence) is unavailable, and it is desirable that identification of the nonlinear channel be based only on the received sequence and known statistics of the input (Blind Identification). Traditional methods based on cross-correlation of the input and the output are clearly inapplicable here.

Though the ability of polyspectra to extract nonlinearly phase coupled components has been exploited for *detection* of nonlinearities [1], blind identification of nonlinear channels has remained an intractable problem, except for a restricted range of inputs, primarily Gaussian [3]. Alshebeili and Venetsanopoulos [4] showed that a quadratic Volterra model with Gaussian input can be identified by cumulant matching [4].

The LTI-ZMNL-LTI model finds wide application in characterization of distortion in complex envelopes of signals in communication channels and circuits with HPA nonlinearities. It is this model that is of primary interest in this paper. It was shown by Rozario and Papoulis [3] that such models with *minimum phase* linear subsystems can in principle be identified with Gaussian input when the ZMNL is a *monotonic* function. The method is based on the observation that since the input to the ZMNL is Gaussian, its output possesses a Polyspectrum that is real. Identification of the first linear subsystem involves comparing the Probability Density Functions (PDF) of the ZMNL input and output. Besides being tedious, the method places strong restrictions on the model.

In this paper, we discuss a method for blind identification of linear subsystems of a discrete-time LTI-ZMNL-LTI model. The linear subsystems are allowed to be of NMP, and the N^{th} order polynomial ZMNL is not necessarily monotonic. However, the first linear subsystem of the model can be completely identified only if it is of minimum phase. Only slices of the $N+1^{th}$ order moment-based polyspectrum of the output are required for identification. The ZMNL coefficients are not estimated and need not be known. In theory, the methods suppress the effect of noise.

These methods find application in analysis of distortion in QAM type signals due to a HPA nonlinearity. The transmitted data, however, is assumed to possess a Gaussian PDF. It should be noted in this connection that of all the nonuniform symbol PDFs, the Gaussian PDF is the optimal (in the maximum entropy sense) [5] for several signal constellations. For this reason, shaping data to have a Gaussian PDF is an increasing trend in digital communications.

Problem assumptions are presented in section 2. Some blind identification methods are proposed in section 3. In section 4, the results of computer simulations are presented to verify the theory.

2. PROBLEM FORMULATION

Consider the discrete-time LTI-ZMNL-LTI model in figure 1. The input sequence $\{x(k)\}$ is a zero-mean, independent, identically distributed (*iid*) complex process whose real and imaginary parts are Gaussian with equal variance. The data

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is then *circularly symmetric*:

$$E\{x(k_1)\dots x(k_m)x^*(k_{m+1})\dots x^*(k_{m+r})\} \neq 0 \quad \text{only when } m = r \qquad (1)$$

where the symbol $*$ in the superscript denotes conjugation and E is the expectation operator.

We start for the sake of generality with the moving average Volterra model of order N. The output signal then is:

$$r(k) = \sum_{i=1}^{N} \sum_{m_1,\dots,m_i=-\infty}^{\infty} \underline{h}_i(m_1,\dots,m_i)\prod_{j=1}^{i} x(k-m_j) + n(k) \qquad (2)$$

where $\underline{h}_i(,.,)$ is the i^{th} dimensional Volterra kernel sequence and $n(k)$ is the additive noise. The LTI-ZMNL-LTI model of figure 1 is a special case of the above with factorizable kernels. If the input $u(k)$ and output $v(k)$ of the ZMNL depicted in the figure are related by $v(k) = \sum_{i=1}^{N} C_i u^i(k)$ where $\{C_i, i = 1,2,\dots,N\}$ are the complex ZMNL coefficients (C_1 is assumed to be unity), then the kernels of the LTI-ZMNL-LTI model can be factored in the frequency domain as follows [6]:

$$\underline{H}_i(z_1,z_2,\dots,z_i) = C_i\prod_{j=1}^{i} H_1(z_j)H_2\Big(\prod_{j=1}^{i} z_j\Big) \qquad (3)$$

where $\underline{H}_i(z_1,z_2,\dots,z_i)$ is the i^{th} Volterra transfer function and $H_1(z)$ and $H_2(z)$ are the transfer functions of the two linear subsystems. Identification of such LTI-ZMNL-LTI models is of primary interest in this paper,though some restricted Volterra models and LTI-ZMNL models are also considered. The objective then is to estimate the impulse responses $h_1(k)$ and $h_1(k)$ using only the statistics of the output.

3. IDENTIFICATION METHOD

We rewrite equation 2 as:

$$r(k) = \sum_{i=1}^{N} \mathcal{H}_i(k) + n(k) \qquad (4)$$

where

$$\mathcal{H}_i(k) \triangleq \sum_{m_1,\dots,m_i=-\infty}^{\infty} \underline{h}_i(m_1,m_2,\dots,m_i)\prod_{j=1}^{i} x(k-m_j)$$

We define the $j+1^{th}$ order moment of $r(k)$ as:

$$R_{j+1,r}(m_1,m_2,\dots,m_j) \triangleq E\left\{\prod_{i=1}^{j} r(k+m_i)r^*(k)\right\}$$

It can be verified that because of the circularly symmetry of the input, all moments of order greater than $N+1$ are zero. This fact can be used to estimate N. Also, the $N+1^{th}$ order moment depends only on the first and the N^{th} order term in equation 4 and is given by:

$$R_{N+1,r}(m_1,\dots,m_N) = E\{\prod_{j=1}^{N} \mathcal{H}_1(k+m_j)\mathcal{H}_N^*(k)\}$$
$$= \sigma_x^{2N}N!\sum_{l_1,\dots,l_N}\left[\prod_{j=1}^{N}\underline{h}_1(l_j+m_j)\right]\underline{h}_N^*(l_1,\dots,l_N) + R_{N+1,n}(m_1,\dots,m_N)$$

where $\sigma_x^2 = E\{|x(k)|^2\}$, $N!$ denotes the factorial of N, and $R_{N+1,n}(m_1,\dots,m_N)$ is the $N+1^{th}$ order moment of the noise. This term is clearly zero when $n(k)$ is circularly symmetric, so that the analysis is immune to such noise. In addition, the analysis is immune to noise with symmetric (asymmetric) probability density function when N is even (odd). We therefore drop this term in the sequel.

The polyspectrum is given by the N-dimensional Z-transform of the $N+1^{th}$ order moment (by a generalization of the Wiener-Kintchine theorem):

$$S_{N+1,r}(z_1,\dots,z_N) = C\prod_{i=1}^{N}\underline{H}_1(z_i)\underline{H}_N^*(z_1,\dots,z_N) \qquad (5)$$

where $C = \sigma_x^{2N}N!$. In what follows, we discuss identification methods for LTI-ZMNL, LTI-ZMNL-LTI and some Volterra models using the above equation.

3.1. LTI-ZMNL models

We make the following statement for a ZMNL nonlinearity:

Statement: The $N+1^{th}$ order polyspectrum of the ZMNL output $v(k)$ is given (to a constant factor) by the product of the power spectra of the ZMNL input $u(k)$:

$$S_{N+1,v}(z_1,z_2,\dots,z_N) = K\prod_{j=1}^{N} S_{2,u}(z_j) = K\prod_{j=1}^{N} |H_1(z_j)|^2$$

where $K = \sigma_x^{2N}N!C_N^*$, $S_{N+1,v}(z_1,z_2,\dots,z_N)$ is the $N+1^{th}$ order polyspectrum of $v(k)$, and $S_{2,u}(z)$ is the power spectrum of $u(k)$ (see figure 1). The above equation follows from equation 5 and equation 3 ($H_2(z)=1$). Clearly, $S_{N+1,v}(z_1,z_2,\dots,z_N)$ has no phase information. This is because the ZMNL input is Gaussian [3]. A 1-D slice of the above can then be used to identify $|H_1(z)|$ and therefore the impulse response (if it is of minimum phase) without knowledge of the ZMNL coefficients

3.2. LTI-ZMNL-LTI models

Using equations 5 and 3, we get:

$$S_{N+1,r}(z_1,\dots,z_N) = K\prod_{i=1}^{N}[|H_1(z_i)|^2H_2(z_i)]H_2^*\Big(\prod_{i=1}^{N} z_i\Big)$$

It is possible to identify $H_2(z)$ under some conditions using 1-D diagonal polyspectral slices [6]. In general however, we may use a 2-D slice of the type:

$$\begin{aligned}\tilde{S}_{N+1,r}(z_1,z_2) &\triangleq S_{N+1,r}(1,\ldots,z_1,\ldots,z_2,\ldots,1)\\ &= K_a\prod_{i=1}^{2}[|H_1(z_i)|^2H_2(z_i)]H_2^*(z_1z_2)\end{aligned}$$

where $K_a = K(|H_1(1)|^2H_2(1))^{N-2}$ (note that other slices can be used if $K_a = 0$).

3.2.1. Phase of $H_2(z)$

The phase $\Psi_{N+1,r}(z_1,z_2)$ of the 2-D slice is given by:

$$\begin{aligned}\Psi_{N+1,r}(z_1,z_2) &\triangleq Angle[\tilde{S}_{N+1,r}(z_1,z_2)]\\ &= \phi_K+\phi_2(z_1)+\phi_2(z_1)-\phi_2(z_1z_2)\\ &= \overline{\phi}_2(z_1)+\overline{\phi}_2(z_1)-\overline{\phi}_2(z_1z_2) \qquad (6)\end{aligned}$$

where $\phi_K = Angle[K_a]$, $\phi_2(z) = Angle[H_2(z)$, and $\overline{\phi}_2(z) = \phi_2(z)+\phi_K$. The above equation resembles the expression for the bispectrum of a linear time-series and methods developed for this purpose [1] can be used to estimate $\phi_2(z)$.

3.2.2. Cepstral Domain Identification

We discuss a parametric identification method here. With the ARMA model:

$$H_j(z) = I_j(z^{-1})O_j(z)\mathcal{A}_jz^{-d_j},\ \ j=1,2 \qquad (7)$$

where

$$\begin{aligned}I_j(z^{-1}) &= \frac{\prod_{i=1}^{L_{1j}}(1-a_{ij}z^{-1})}{\prod_{i=1}^{L_{3j}}(1-c_{ij}z^{-1})}\\ O_j(z) &= \prod_{i=1}^{L_{2j}}(1-b_{ij}z), j=1,2\end{aligned}$$

where $I_j(z^{-1})$ is the minimum phase component of the linear filter with transfer function $H_j(z)$, $\{a_{ij}\forall i,j\}$ and $\{c_{ij}\forall i,j\}$ are (respectively) its zeros and poles ($|a_{ij}|,|c_{ij}| < 1\ \forall i,j$), $O_j(z)$ is the maximum phase component with zeros $\{b_{ij}\}$ inside the unit circle ($|b_{ij}| < 1\ \forall i,j$), $\mathcal{A}_j$ and d_j are the gain and linear phase shifts of the transfer function. This important restriction that there be no zeroes on the unit circle can be relaxed [7] by using the polyspectral slice on a sphere of radius different from unity. The complex cepstrum $\tilde{c}(m,n)\triangleq Z^{-1}\{ln[\tilde{S}_{N+1,r}(z_1,z_2)]\}$ of the 2-D slice is given by:

$$\tilde{c}(m,n) = \begin{cases} ln[K_a|\mathcal{A}_1|^4|\mathcal{A}_1|^2\mathcal{A}_2] & m=0,n=0\\ \frac{-A_1(m)-B_1^*(m)}{m}-\frac{A_2(m)}{m} & m>0,n=0\\ \frac{A_1^*(-m)+B_1(-m)}{m}+\frac{B_2(-m)}{m} & m<0,n=0\\ \frac{-A_1(n)-B_1^*(n)}{n}-\frac{A_2(n)}{n} & n>0,m=0\\ \frac{A_1^*(n)+B_1(-n)}{n}+\frac{B_2(-n)}{n} & n<0,m=0\\ \frac{B_2^*(m)}{m} & m=n>0\\ \frac{A_2^*(-m)}{m} & m=n<0\\ 0 & \text{otherwise}\end{cases}$$

where $A_j(k)$ and $B_j(k)$ are the differential cepstra of the impulse response of the filter (and hence termed the differential cepstral parameters) defined as:

$$\begin{aligned}A_j(k) &\triangleq \sum_{i=1}^{L_{1j}}a_{ij}^k-\sum_{i=1}^{L_{3j}}c_{ij}^k\\ B_j(k) &\triangleq \sum_{i=1}^{L_{2j}}b_{ij}^k \qquad (8)\end{aligned}$$

Clearly, $A_2(k)$ and $B_2(k)$ can be readily identified from the above equation using estimates of the cepstral sequence. Procedures for estimation of cepstral sequences are discussed in [7]. The impulse response $h_2(k)$ can be recovered using the well-known recursive relation. The minimum and the maximum phase responses $i_2(k)$ and $o_2(k)$ of the response $h_2(k)$ are recovered using the following recursion with $i_2(0) = o_2(0) = 1$:

$$\begin{aligned}i_2(k) &= \frac{-1}{k}\sum_{n=2}^{k+1}A_2(n-1).i_2(k-n+1)\\ o_2(k) &= \frac{1}{k}\sum_{n=k+1}^{0}B_2(1-n).o_2(k-n+1) \qquad (9)\end{aligned}$$

Then $h_2(k) = i_2(k)\odot o_2(k)$ ($\odot$ denotes convolution). Since $A_1(k)$ and $B_1(k)$ (of the first filter) cannot be estimated uniquely for reasons discussed earlier, the magnitude response (and hence the impulse response if it is of minimum phase) are recovered using $A_1(k)+B_1^*(k)$ estimated using the cepstral sequence $\tilde{c}(m,n)$

3.3. Volterra models

Since the $N+1^{th}$ order polyspectrum depends only on the linear and the N^{th} order kernels, kernels of other orders cannot be estimated. Clearly, the N^{th} order kernel can be estimated from equation 5 when the linear kernel is known. Also, recovery of both the linear and the N^{th} order kernels may be possible by moment matching [4].

4. PERFORMANCE ANALYSIS

Performance of the cepstral domain identification method for LTI-ZMNL-LTI models is analyzed by means of Monte-Carlo simulations. A quadratic ZMNL with $C_1 = 1$ and $C_2 = 0.8+0.1i$ was used. The transfer function of the first linear subsystem was assumed to be:

$$H_1(z) = \frac{(1-a_1z^{-1})(1-b_1z)(1-b_2z)}{(1-c_1z^{-1})(1-c_2z^{-1})}, \qquad (10)$$

where $a_1 = -0.1$, $b_1 = (\sqrt{2}+i)/4\sqrt{2}$, $b_2 = 0.15.(\sqrt{2}+i)/\sqrt{2}$, $c_1 = 0.3.(1+\sqrt{3}i)$, and $c_2 = 0.35-0.25i$. The transfer function of the second linear subsystem was also assumed to be of the same form as $H_1(z)$ above, but with coefficients: $a_1 = -0.1$, $b_1 = (\sqrt{2}+i)/4\sqrt{2}$, $b_2 = 0.15.(\sqrt{2}+i)/\sqrt{2}$, $c_1 = 0.3.(1+\sqrt{3}i)$, and $c_2 = 0.35-0.25i$. The input was assumed to be circularly symmetric Gaussian sequence of unit variance. Circularly symmetric Gaussian noise was added to make the Signal to Noise Ratio (SNR) $10dB$ (SNR$\triangleq 10log_{10}[||r(k)||^2/||n(k)||^2]$). The FFT based procedure [7] was used to estimate the 2-D cepstral sequence. $A_2(k)$ and $B_2(k)$ were estimated using the cepstral equation, and the impulse response $h_2(k)$ using equation 9. Hundred realizations of the magnitude and phase of the transfer function $H_2(z)$ (estimated using 8000 data samples) are superimposed in figure 2 (top) and the average of these realizations is compared to the actual values in the same figure (bottom). The figure also compares the estimates obtained with 4000 and 16000 length data records.

5. CONCLUSIONS

A simple method was presented for blind identification of N^{th} order LTI-ZMNL-LTI models with circularly symmetric Gaussian input that bases its estimates of the impulse responses on a 2-D slice of the $N+1^{th}$ order polyspectrum of the output. The coefficients of the ZMNL are not required. The linear subsystems are allowed to be of *Non-Minimum Phase*, though the first subsystem can be completely identified only if it is of minimum phase. Identification of some restricted Volterra models is also possible. It was shown based on Monte-Carlo simulations that the methods perform effectively.

6. REFERENCES

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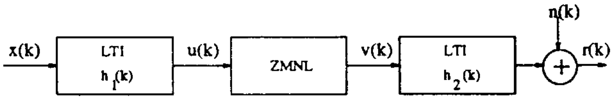

Figure 1: The discrete-time LTI-ZMNL-LTI model

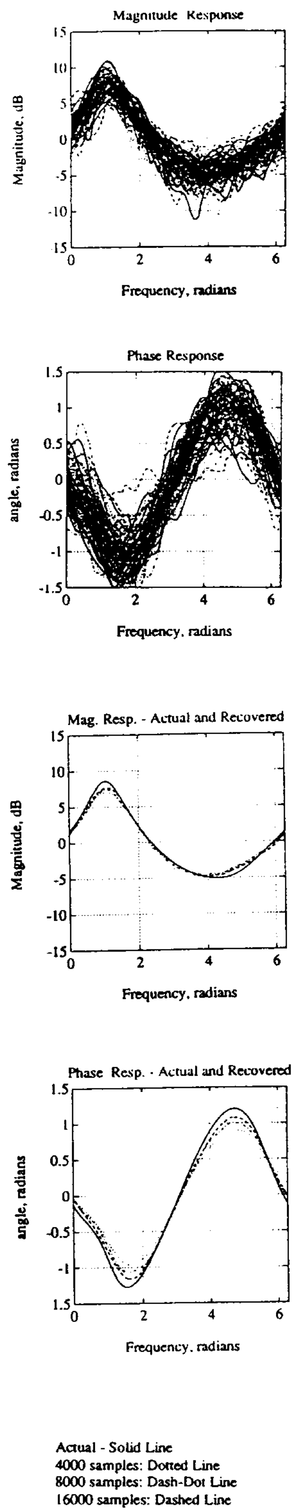

Figure 2: Recovered transfer function $H_2(z)$.