TIME-VARYING FILTERING VIA MULTIRESOLUTION PARAMETRIC SPECTRAL ESTIMATION

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ABSTRACT

We aim to recover a multi-frequency component nonstationary signal from its broadband noise-corrupted measurements using a time-varying optimal Wiener filter. A new method for realizing the Wiener filter is proposed, based on our multiresolution parametric spectral estimator (MPSE). Conventional estimators for contaminated AR processes are all fixed resolution based methods, which are mostly suitable for stationary situations. In nonstationary applications, the estimator must not only locate the signal components in frequency but also in time. MPSE offers better time resolution than conventional fixed resolution parametric estimators. The MPSE frequency band splitting reduces necessary model orders and improves SNR. The Wiener filter is given in terms of the MPSE parameters. Experiments show that the performance of the MPSE Wiener filter lies much closer to the ideally possible performance than for a Wiener filter based on fixed resolution AR modeling.

1. INTRODUCTION

The optimal linear filter commonly known as the Wiener filter is the best estimator based on the minimum mean-square error criterion. Given the spectral characteristics of signal and noise, it provides the optimal estimate of the signal. Optimal filtering has played an important role in SNR enhancement, signal detection and tracking, spectral estimation and system identification.

Several difficulties arise in applying optimal filtering theory to signal estimation. The major difficulty is that the design of a Wiener filter requires *a priori* knowledge of the statistics of the data to be processed, such as spectral density or correlation functions. Such statistical characteristics of signal and noise are usually unknown, and it becomes impossible in many practical cases to design and implement the optimal Wiener filter. Moreover, the performance of the filter depends greatly on the accuracy of the information on which the design of the filter is based. The filter is optimum only when the statistical characteristics of the signal and noise match the information used for design. Estimation errors may bring about serious distortion in the estimated signal. Therefore, the key to successful optimal filter design is to properly estimate the spectral or statistical characteristics of the input data.

The theory of Wiener filtering is not restricted to stationary environments. If the signal or noise is nonstationary, a time-varying optimal filter can be derived according to the time-varying characteristics of the input data [1]. The nonstationarity will make the spectral estimation more difficult, since most spectral estimation techniques are based on the assumption that the process is (wide sense) stationary. The solution to overcoming the nonstationarity is to apply a proper time window. A trade-off between estimation variance and nonstationarity inference is required when the window length is selected.

We aim to recover a nonstationary signal via a time-varying Wiener filter. Here the signal is considered to be a multi-frequency component nonstationary process which has been corrupted by broadband white noise. Thus, the signal can be modeled as a time-varying multi-pole (high order) AR(p) process. Contaminated with white noise, the observed data can be modeled as a nonstationary ARMA(p, p) process [2, 3].

Several techniques have been proposed to estimate the model parameters for a contaminated AR process, such as the noise compensation method [2], general ARMA estimators, high order AR modeling [3], or prefiltering of the data to reduce the observation noise [4].

A new approach to time-varying Wiener filtering is presented in this paper. The process consists of a two-stage procedure. The first stage is to estimate the statistical characteristics of the nonstationary data using our Multiresolution Parametric Spectral Estimator (MPSE) [5] in conjunction with a noise compensation method. The second stage is to design the Wiener filter based on the estimated MPSE parameters, for subsequent filtering of the contaminated data. These two stages can run concurrently, so that an adaptive Wiener filtering operation is established.

2. WIENER FILTERING

Consider a real AR(p) process s_n, represented as:

$$s_n = -\sum_{i=1}^{p} a_i s_{n-i} + x_n \tag{1}$$

where $\{a_i: i = 1,2,\ldots,p\}$ are the AR parameters, and x_n is the normally distributed white noise excitation with zero mean and variance σ_x^2. The Power Spectral Density function (PSD) of the time series s_n is defined by:

$$P_s(z) = \frac{\sigma_x^2}{A(z)A^*(z^{-*})}; \quad A(z) = \sum_{i=0}^{p} a_i z^{-i}; \quad a_0 \equiv 1 \tag{2}$$

Complex conjugation is denoted by *, and - indicates taking the reciprocal. Since s_n is real, the AR parameters are also real, and the roots of $A(z)$ are either real or occur in complex conjugate pairs. We assume that the signal process is stable, with the roots of $A(z)$ located inside the unit circle.

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The observed time series y_n can be written as:

$$y_n = s_n + e_n \tag{3}$$

where e_n is the observation noise, which is a normally distributed white process with zero mean and variance σ_e^2. Under the assumption that the processes e_n and x_n are independent, the PSD of y_n is defined by:

$$P_y(z) = P_s(z) + \sigma_e^2 = \frac{\sigma_x^2 + \sigma_e^2 A(z)A^*(z^{-*})}{A(z)A^*(z^{-*})} \tag{4}$$

Using the optimal Wiener filter [1] we can write

$$H_{opt}(z) = \frac{P_{ys}(z)}{P_y(z)} = \frac{P_s(z)}{P_y(z)} \tag{5}$$

where $P_{ys}(z)$ is the cross-PSD between y_n and s_n. The second equality results from e_n and x_n being independent, so that $P_{ys}(z)$ equals $P_s(z)$.

Substituting (2) and (4) into (5), the optimal Wiener filter can be written as:

$$H_{opt}(z) = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_e^2 A(z)A^*(z^{-*})} \tag{6}$$

Note from (4) that the denominator of (6) can be spectrally factored.

$$\sigma_x^2 + \sigma_e^2 A(z)A^*(z^{-*}) = B(z)B^*(z^{-*}) \tag{7}$$

Using (7), the optimal Wiener filter can be rewritten as:

$$H_{opt}(z) = \frac{\sigma_x^2}{B(z)B^*(z^{-*})} = \left[\frac{\sigma_x}{B(z)}\right]\left[\frac{\sigma_x}{B^*(z^{-*})}\right] \tag{8}$$

Equation (8) is the ideal optimal Wiener filter for recovering a white noise contaminated AR(p) process. Since the filter is a noncausal IIR filter, it is not physically realizable. However, if a block of observations $\{y_n : n = 0,1,\cdots,N-1\}$ is available, and the block length N is much longer than the transient of $\sigma_x/B(z)$, the noncausal IIR filter in (8) can be approximated well by an off-line procedure.

The procedure consists of two steps. First the observations $\{y_n : n = 0,1,\cdots,N-1\}$ are filtered by the causal filter $\sigma_x/B(z)$, and the output denoted v_n. Next, the anticausal filter $\sigma_x/B^*(z^{-*})$ is implemented by filtering $\tilde{v}_n$, the time-reverse of the sequence v_n, with $\sigma_x/B(z)$, creating $\tilde{s}_n$, and finally time reversing $\tilde{s}_n$. The latter creates $\hat{s}_n$, the optimal Wiener filter estimate. Approximation errors are caused by the transients at both ends of the filtered data. Using (7) the filter parameters of $\sigma_x/B(z)$ can be computed from $A(z)$, σ_x^2, and σ_e^2, or their estimates.

When the signal s_n is a nonstationary process, its AR parameters and spectrum are time-varying. This requires the continuous availability of the time-varying information. This makes the design of the time-varying Wiener filter difficult. However, the time-varying characteristics of the signal can be tracked by a spectral estimator with a proper time window. Within the window, the signal is considered to be pseudo-stationary, such that the noncausal IIR Wiener filter can be used directly. The time-varying characteristics of the signal are updated in each sample interval while the time window slides along the time axis. The window should be short enough to ensure pseudo-stationarity.

3. SUMMARY OF MPSE

An optimal Wiener filter or its approximation can be fully determined from the signal AR parameters, the power of the excitation process σ_x^2 and the power of the observation noise σ_e^2, or their estimates. This information can be obtained, for example, with the Burg or Recursive Least Squares algorithms (RLS). When the signal is time-varying, a time window needs to be introduced in order to preserve pseudo-stationarity. There are two major difficulties when the signal consists of multi-frequency components. First, a multi-frequency component signal corresponds to a high order AR process. To estimate the parameters, the window width needs to be at least three times the AR model order for the signal [3]. Second, since the signal contains several frequency components, each signal component can have different time-varying rates and patterns. It is hard to find a fixed time window suitable for all frequency components. MPSE was developed to overcome the window problems in estimating the model parameters of a time-varying signal.

The MPSE process starts by splitting the signal frequency band into lower and upper halfbands using lowpass and highpass filters. The output of the highpass filter is forwarded to an AR spectral estimator with a sliding window. The result is the first octave band spectrum. The output of the lowpass filter is decimated by a factor of two, and sent to the next processing block. The processor recursively divides the entire frequency band from high to low frequency into multiple octave bands [5, 6].

The output rate of each subband signal is different. The first octave band has the highest rate, equal to the sample rate of the original signal. The second octave band has an output rate one half that of the first octave band, and its time scale doubles compared with the first octave band.

In practice, the signal often occupies only a few of the octave bands. The 'empty' subband signal components can be treated as zero-th order processes and used to estimate $\hat{\sigma}_e^2$, the variance of the observation noise. Note that the bandwidth of the i-th octave band is a fraction (2^{-i}) of the entire frequency band, so that the estimated noise power $\sigma_e^{(i)\,2}$ in the i-th octave band needs to be re-scaled, i.e. $\hat{\sigma}_e^2 = 2^i \sigma_e^{(i)\,2}$.

An advantage of MPSE is that a high order AR process can be decomposed into a number of lower order AR processes; this improves the statistical performance of the estimation since there is less cross-talk between the component processes. For the time-varying case, MPSE provides much better frequency tracking capability than the conventional AR estimator. It also improves frequency resolution, by increasing SNR in each subband, and time resolution, since lower order processes require shorter data window widths.

4. TIME-VARYING MPSE WIENER FILTER

The MPSE parameters are the parameters of the noise-contaminated observation y_n. To improve the corrupted estimates, a noise compensation technique [2] can be used in conjunction with MPSE. The noise compensation method is based on the relation between the autocorrelation functions of y_n and s_n. That is, the estimated autocorrelation function of the signal can be determined using $\hat{R}_s = \hat{R}_y - \hat{\sigma}_e^2 \mathrm{I}$. The autocorrelation functions can be used either directly, to construct an FIR type Wiener filter, or indirectly, to determine the signal component parameters for constructing an IIR type Wiener filter or to estimate the time-varying spectrum of the signal component.

In situations where the SNR is extremely low, a two step estimation stage can help to reduce the noise level in the final results. The first step is to prefilter the noise-contaminated signal component in each non-empty octave band using an FIR type Wiener filter at the subband level. This estimation-step optimal filter is constructed from the autocorrelation functions estimated by MPSE with noise compensation. The output from the first step, $\hat{s}_n^{(i)}$, an estimate of the signal component, is then used to re-estimate the AR parameters of the subband signal component. Since the SNR condition has been improved by the prefiltering step, the re-estimated signal parameters are less corrupted by noise than without the prefiltering step. However, extra distortion can be caused by prefiltering, which usually produces over-smoothed estimates. The choice of a one or two step estimation stage is based on the trade-off between further noise reduction and extra distortion. Under high SNR conditions, a one step estimation stage yields better results than a two step estimation stage. The experimental results presented in the next section were produced by a two step estimation stage.

If the waveform of the signal s_n is of major interest, the optimal filtering has to be performed over the entire frequency band to prevent waveform distortion caused by different group delays. After the signal has been decomposed into the multi-frequency components in each octave band, the AR polynomial $A_j(z)$, for the j-th octave band can be represented as :

$$A_j(z) = \prod_{i=1}^{p_j}(1-\rho_{ij}z^{-1}) \qquad j = 1,\ldots,M \tag{9}$$

where p_j is the order of the signal component in the j-th octave band, and M is the total number of octave bands.

The collection of all roots ρ_{ij} is exactly the set of roots for the overall AR polynomial $A(z)$, except that the root positions have been changed due to decimation. To reconstruct $A(z)$, the roots in each octave band need to be projected from its octave band to the entire frequency band. Writing the roots in (9) in polar form:

$$\rho_{ij} = \gamma_{ij}\exp(j\theta_{ij}) \tag{10}$$

the projection from the octave band to the entire frequency band is reflected in

$$\rho'_{ij} = \gamma_{ij}\exp(j\frac{\theta_{ij}}{2^{j-1}}) \tag{11}$$

$\hat{A}(z)$ can then be approximated as

$$\hat{A}(z) = \prod_{j=1}^{M}\prod_{i=1}^{p_j}(1-\rho'_{ij}z^{-1}) \tag{12}$$

From (1) and (3) we see

$$\sum_{i=0}^{p} a_i y_{n-i} = x_n + \sum_{i=0}^{p} a_i e_{n-i} \tag{13}$$

Taking the variance on both sides of (13) yields

$$E\left(\left|\sum_{i=0}^{p} a_i y_{n-i}\right|^2\right) = \sigma_x^2 + \sigma_n^2 \sum_{i=0}^{p}|a_i|^2 \tag{14}$$

where we used the condition that the excitation process and the observation noise are zero mean white and independent processes. Finally, the excitation noise of the signal process can be approximated as

$$\hat{\sigma}_x^2 = \frac{1}{N}\left(\sum_{i=p}^{N}\left|\sum_{l=0}^{p}\hat{a}_l y_{i-l}\right|^2\right) - \hat{\sigma}_n^2 \sum_{i=0}^{p}|\hat{a}_i|^2 \tag{15}$$

where N is the number of data points in the time window. After $\hat{A}(z), \hat{\sigma}_e^2$, and $\hat{\sigma}_n^2$ have been estimated, the Wiener filter for the entire frequency band can be constructed.

Equations (12) and (15) provide approximations for constructing the second stage optimal Wiener filter for the entire frequency band, based on the estimates in each subband, as obtained in the first stage. When the poles of the signal components are located close to the unit circle, the approximation is a good representation of the actual signal model; otherwise, some error in the estimation of excitation noise power may result. The performance of the Wiener filter depends much more on correct pole position than on correct noise powers. If the spectral estimator can track the changes of the signal components and provide correct pole positions over time, the signal components will pass the filter. A filter constructed from incorrect pole information can cause serious distortion, even filter out the actual signal. The gains and bandwidths centered at each pole are controlled largely by the excitation noise and the observation noise. With noise compensation, over-estimation of the observation noise power will produce over-smoothed results (narrower passband). It may also cause $\hat{r}_y(0) - \hat{\sigma}_e^2 < 0$ (negative PSD).

5. SIMULATION EXPERIMENTS

Performance is illustrated using a time-varying AR(4) process in wideband noise, with $SNR = 0$ dB (Fig. 1). The poles of the AR(4) process vary according to

$$\rho_{1,2}(n) = 0.99\exp\left[\pm j2\pi(0.063+0.038\cos(0.011n))\right]$$
$$\rho_{3,4}(n) = 0.99\exp\left[\pm j2\pi(0.031-0.019\cos(0.011n))\right] \quad (16)$$

in (cycles/sample). The excitation noise of the AR process is a normally distributed process with zero mean and variance $\sigma_x^2 = 10^{-4}$. The global Wiener filter realizations used are all noncausal IIR approximations. Given the true time-varying parameters, the ideal Wiener filter (designed from perfect knowledge) provides 10.46 dB of noise reduction (Fig. 2), thereby setting the upper limit to performance. The Wiener filter updated from a conventional fixed resolution windowed RLS algorithm [7] achieved 4.09 dB of noise reduction (Fig. 3), whereas the MPSE based Wiener filter realized 8.73 dB of noise reduction (Fig. 4). The MPSE Wiener filter is constructed by projecting the estimated parameters for each sub-band process from its octave band to the global frequency band.

This experiment shows that the performance of the MPSE based Wiener filter lies much closer to the ideally possible performance than for one based on conventional fixed resolution AR modeling.

6. CONCLUSION

Compared with conventional fixed resolution parametric estimators, the MPSE method offers better time resolution. The frequency band splitting procedure used in MPSE reduces necessary model orders and improves SNR, and thus reduces estimation errors. The MPSE parameters can be used to directly update the time-varying Wiener filter. Experimental results show that the performance of the MPSE based Wiener filter lies much closer to the ideally possible performance than for one based on the usual AR modeling.

7. REFERENCES

1. Charles W. Therrien, *Discrete Random Signals and Statistical Signal Processing*, Prentice Hall, 1992.
2. J. S. Lim and A. V. Oppenheim, "Enhancement and Bandwidth Compression of Noisy Speech," Proc. IEEE, Vol. 67, pp. 1586-1604, Dec. 1979.
3. S. M. Kay, *Modern Spectral Estimation: Theory and Application*, Prentice Hall, 1987.
4. J. S. Lim, "All Pole Modeling of Degraded Speech," IEEE Trans. Acoustics, Speech, and Signal Processing, Vol. ASSP-26, pp. 197-209, June 1978.
5. M. Xie and A. A. (Louis) Beex, "Multiresolution Parametric Spectral Estimation," 26th Southeastern Symposium on System Theory, Athens OH, 10-12 March 1994, pp. 432-436.
6. M. Xie and A. A. (Louis) Beex, "Multiresolution Parametric Spectral Estimation," IEEE Trans. on Signal Processing, submitted July 1994.
7. B.-Y. Choi and Z. Bien, "Sliding-Windowed Weighted Recursive Least-Squares Method for Parameter Estimation," Electronics Letters, Vol. 25, No. 20, pp. 1381-1382, 1989.

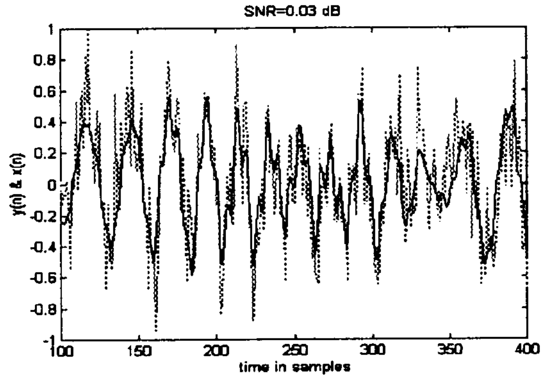

Fig. 1. Signal x(n) (solid) and observation y(n) (dotted).

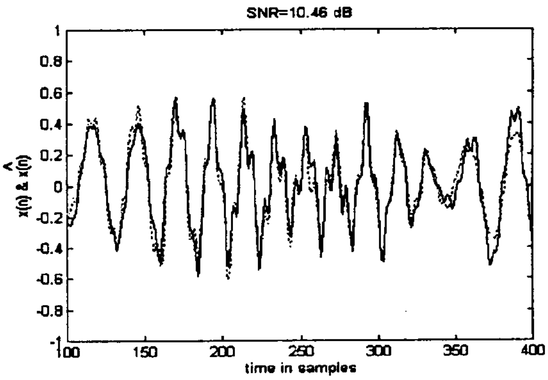

Fig. 2. Signal (solid) and ideal Wiener filter estimate (dotted).

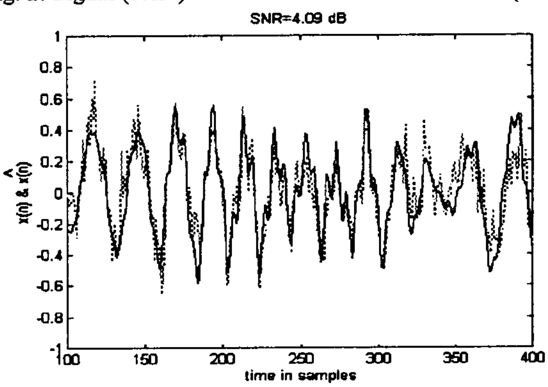

Fig. 3. Signal (solid) and fixed resolution AR Wiener filter estimate (dotted).

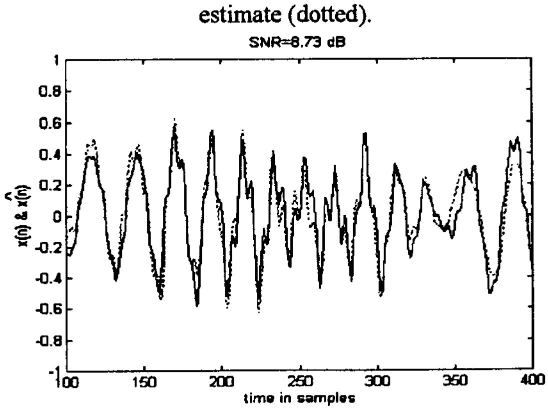

Fig. 4. Signal (solid) and MPSE Wiener filter estimate (dotted).