EVOLUTIONARY SPECTRAL ANALYSIS AND THE GENERALIZED GABOR EXPANSION

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ABSTRACT

In this paper, we present a connection between the discrete Gabor expansion and the evolutionary spectral theory. Including a scale parameter in the Gabor expansion, we obtain a new representation for deterministic signals that is analogous to the Wold-Cramer decomposition for non-stationary processes. The energy distribution resulting from the expansion is easily calculated from the Gabor coefficients. By choosing gaussian windows and appropriate scales, the expansion can represent narrow-band and wide-band signals, as well as their combination. As an application, we consider the masking of signals in the time-frequency space and provide an approximate implementation using the new Gabor expansion. Examples illustrating the time-frequency analysis and the masking are given.

1. INTRODUCTION

The time-frequency analysis of deterministic signals, and the estimation of time-varying spectra of non-stationary processes are topics of theoretical and practical interest in the processing of speech, seismic and biological signals. The Gabor expansion has been widely used in these areas [1, 2, 3], and it can be viewed as an extention of the short-time Fourier transform. Recently, a discrete representation was developed and used in the filtering of signals [4, 5]. On the other hand, the evolutionary spectral theory [6] deals with the spectral representation of non–stationary processes and the estimation of their time-varying spectra. The Wold–Cramer decomposition [7] is the spectral representation of a non-stationary process $x(n)$,

$$x(n) = \int_{-\pi}^{\pi} A(n,\omega)e^{j\omega n}dZ(\omega) \qquad (1)$$

Using this decomposition, the evolutionary spectrum is defined as $S_{WC}(n,\omega) = |A(n,\omega)|^2$.

Our aim in this paper is to show that an analogous representation is possible for deterministic signals. We develop a new version of the Gabor expansion, including scaled non-orthogonal windows, to obtain such a representation. The resulting time-frequency energy distribution is shown to be related to the Gabor coefficients. Whenever the windows are gaussian and the scales are chosen appropriately, the generalized Gabor expansion is able to represent narrow-band and wide-band signals, and their combination. To illustrate the application of the proposed Gabor expansion, we consider the time-frequency masking of signals and propose an efficient implementation of it. Examples of the time-frequency analysis, and the masking are given.

2. GENERALIZED DISCRETE GABOR EXPANSION

2.1. Discrete Gabor Expansion

Given a finite–support or periodic signal $x(n)$, its discrete Gabor expansion is [4]

$$x(n) = \sum_{m=0}^{M-1}\sum_{k=0}^{K-1} a_{m,k}h_{m,k}(n) \qquad 0 \le n \le N-1 \qquad (2)$$

where the synthesis window function is defined as

$$h_{m,k}(n) = h(n-mL)e^{j\omega_k n}$$

and $\omega_k = 2\pi k/K$. To evaluate the expansion coefficients $\{a_{mk}\}$, a set of functions $\{\gamma(.)\}$ orthogonal to $\{h(.)\}$ are needed. To find them the frequency step, $2\pi/K$, and the time step, $L = N/M$, must be such that $2\pi L/K \le 2\pi$, or equivalently $L \le K$. In the critically sampled case, $L = K$, a unique set of functions $\{\gamma(.)\}$ can be obtained, whereas in the oversampled case, $L < K$, the set of functions is not unique but can still be obtained.

The biorthonormality between $h(.)$ and the analysis windows

$$\gamma_{m,k}(n) = \gamma(n-mL)e^{j\omega_k n}$$

is given by

$$\sum_{n=0}^{N-1} h_{m,k}(n)\gamma_{s,t}(n) = \delta_{s-m}\delta_{t-k} \qquad (3)$$

for $0 \le m,s \le M-1$ and $0 \le k,t \le K-1$. The Gabor coefficients are then evaluated according to

$$a_{m,k} = \sum_{n=0}^{N-1} x(n)\gamma^*_{m,k}(n) \qquad (4)$$

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2.2. Evolutionary Energy Distribution

The Gabor coefficients $\{a_{m,k}\}$ have no significance in the time–frequency space. One can, however, use them to calculate an evolutionary energy distribution. Consider the following representation, analogous to the Wold–Cramer decomposition, for a deterministic signal $x(n)$

$$x(n) = \sum_{k=0}^{K-1} A(n,\omega_k) e^{j\omega_k n} \qquad 0 \le n \le N-1 \qquad (5)$$

Comparing the above with the Gabor expansion in (2), we obtain

$$\begin{aligned} A(n,\omega_k) &= \sum_{m=0}^{M-1} a_{m,k}\ h(n-mL) \\ &= \sum_{m,l} x(l)\ \gamma^*(l-mL) h(n-mL) e^{-j\omega_k l} \end{aligned}$$

after the $\{a_{m,k}\}$ in (4) are replaced.

If we reorder the above equation we get

$$A(n,\omega_k) = \sum_m h(n-mL) \sum_l x(l)\gamma^*(l-mL) e^{-j\omega_k l}$$

which is similar to the evolutionary periodogram (EP) in [8], except we have now non-orthogonal window functions, whereas in the EP we used orthogonal polynomials.

Let us then show that $|A(n,w_k)|^2$ is an energy distribution. Since the Gabor expansion of $x(n)$ can be done with either $h(.)$ or $\gamma(.)$, we thus have that for a fixed k,

$$\begin{aligned} A(n,\omega_k) &= \sum_{m=0}^{M-1} a_{m,k}\ h(n-mL) \\ &= \sum_{l=0}^{M-1} b_{l,k}\ \gamma(n-lL) \end{aligned}$$

If we use the above expressions to calculate $|A(n,w_k)|^2$ and then use the biorthonormality we get

$$\begin{aligned} \sum_n |A(n,\omega_k)|^2 &= \sum_{m,l} a_{m,k}\ b^*_{l,k} \sum_n h_{m,k}(n)\gamma^*_{l,k}(n) \\ &= \sum_m a_{m,k}\ b^*_{m,k} \end{aligned}$$

Now, since the total energy of the signal is

$$\begin{aligned} \sum_n |x(n)|^2 &= \sum_{m,k}\sum_{s,t} a_{m,k}\ b^*_{s,t} \sum_n h_{m,k}(n)\gamma^*_{s,t}(n) \\ &= \sum_{m,k} a_{m,k}\ b^*_{m,k} \end{aligned}$$

we then have that

$$\sum_n |x(n)|^2 = \sum_n \sum_k |A(n,\omega_k)|^2 \qquad (6)$$

indicating that $|A(n,\omega_k)|^2$ is the energy distribution of $x(n)$ in the time–frequency space.

2.3. Generalized Gabor Expansion

The time and frequency resolution of $|A(n,\omega_k)|^2$ depends on the width of the analysis window used. To improve the resolution one could average the energy distributions obtained using different scaled windows. For each of P scales we obtain a Gabor representation of $x(n)$, and thus

$$\begin{aligned} x(n) &= \frac{1}{P}\sum_{i=0}^{P-1}\sum_{m=0}^{M-1}\sum_{k=0}^{K-1} a_{i,m,k} h_{i,m,k}(n) \\ &= \frac{1}{P}\sum_{i=0}^{P-1}\sum_{k=0}^{K-1} A_i(n,\omega_k) e^{j\omega_k n} \qquad (7) \end{aligned}$$

where

$$A_i(n,\omega_k) = \sum_{m=0}^{M-1} a_{i,m,k}\ \frac{1}{\sqrt{s_i}} h(\frac{n-mL}{s_i}) \qquad (8)$$

and the synthesis window, for a scale s_i, is given by

$$h_{i,m,k}(n) = \frac{1}{\sqrt{s_i}} h(\frac{n-mL}{s_i}) e^{j\omega_k n} \qquad (9)$$

and similarly for the analysis windows $\{\gamma_{i,m,k}(.)\}$.

Once the $A_i(n,\omega_k)$ are available, there are many averaging procedures [9] to improve the resolution of the evolutionary distribution. In the examples, we will illustrate the arithmetic and geometric average-based calculations.

3. TIME–FREQUENCY MASKING

Masking in the time–frequency space consists in changing the energy distribution of a given signal in the way specified by a masking function $|H(n,\omega_k)|^2$. In the evolutionary case, the energy distribution $|A(n,\omega_k)|^2$ is simply multiplied by the given masking function $|H(n,\omega_k)|^2$ to obtain the energy distribution of the masked signal.

In the following, we propose an approximate implementation that breaks up the input signal into non-overlapping frames where the filtering is done by linear time–invariant (LTI) filters. This method is computationally more efficient than the one proposed in [10], where the processing is done line by line. It should be mentioned that equivalent linear time–varying filters can be obtained for the above procedures. The advantage of our procedure is that we obtain the masked signal without implementing LTI filters.

The input signal is broken into J non-overlapping frames, that is

$$x(n) = \sum_{j=0}^{J-1} x(n) w_j(n) = \sum_{j=0}^{J-1} y_j(n) \qquad (10)$$

where

$$w_j(n) = \begin{cases} 1 & Nj/J \le n < N(j+1)/J \\ 0 & \text{otherwise} \end{cases}$$

for $j = 0, \cdots, J-1$.

Using the Gabor representation of $x(n)$ in (7), then each $y_j(n)$ has a non-orthogonal expansion

$$y_j(n) = \frac{1}{P} \sum_{i=0}^{P-1} \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} a_{i,m,k} f_{i,m,k}^{j}(n) \qquad (11)$$

based on the window functions

$$f_{i,m,k}^{j}(n) = h_{i,m,k}(n) w_j(n)$$

which are by construction zero outside the analysis frame. We then filter the signal $y_j(n)$ to eliminate frequency components outside the average band $[\omega_{j1}, \omega_{j2}]$:

$$\begin{aligned} z_j(n) &= \sum_{l} y_j(l) g_j(n-l) \\ &= \frac{1}{P} \sum_{i=0}^{P-1} \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} a_{i,m,k} [f_{i,m,k}^{j}(n) * g_j(n)] \end{aligned}$$

where $g_j(n)$ is the impulse response of the LTI filter in the jth frame. Taking the Fourier transform of the above equation we get

$$Z_j(\omega) = G_j(\omega) \frac{1}{P} \sum_{i=0}^{P-1} \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} a_{i,m,k} F_{i,m,k}^{j}(\omega)$$

where $G_j(\omega)$ is the frequency response of an ideal bandpass filter of unity magnitude in the band $[\omega_{j1}, \omega_{j2}]$ or equivalently for k in $[k_{j1}, k_{j2}]$. We then find that

$$z_j(n) = \frac{1}{P} \sum_{i=0}^{P-1} \sum_{m=0}^{M-1} \sum_{k \in [k_{j1}, k_{j2}]} a_{i,m,k} f_{i,m,k}^{j}(n) \qquad (12)$$

Hence, the combination of all windowed $z_j(n)$ will result in the masked signal

$$z(n) = \sum_{j=0}^{J-1} z_j(n) w_j(n) \qquad (13)$$

The above procedure basically considers the input signal stationary inside the analysis frame. The mask is thus approximated by a concatenation in time of LTI filters operating on the analysis frames. Overlapping the windows, one can improve the results but at a higher computational cost.

4. EXPERIMENTAL RESULTS

Example 1. The signal in this example is a combination of a sinusoid, a delta function and two chirps with time-varying amplitudes. Using gaussian windows with 9 different scales, the arithmetic average of the obtained energy distributions is displayed in Fig. 1. Notice that the scales permit good representation of narrow-band and wide-band signals.

Example 2. In this example, we illustrate the geometric average of the energy distributions of a signal which is a periodic sequence of delta functions with time-varying period. Figure 2 shows the distribution obtained using a narrow gaussian window, and Fig. 3 shows the distribution when a wide window is used. The geometric average is shown in Fig. 4. Notice that this figure clearly indicates the change in periodicity and the energy concentration of the signal.

Example 3. The signal in this case consists of sinusoids, occurring intermittently in time, and a chirp that connects two of them as shown in Fig. 5. We wish to mask the chirp, and so an appropriate definition of the mask is used and the energy distribution of the masked signal is shown in Fig. 6.

5. CONCLUSIONS

In this paper, we have obtained the connection between the discrete Gabor representation and the evolutionary spectral theory. A generalized Gabor expansion including scale parameters was developed. Averaging the energy distributions, obtained for different scales, permits us to improve the time-frequency resolution. Finally, we proposed a practical implementation of masking in the time-frequency space using the new Gabor representation.

6. REFERENCES

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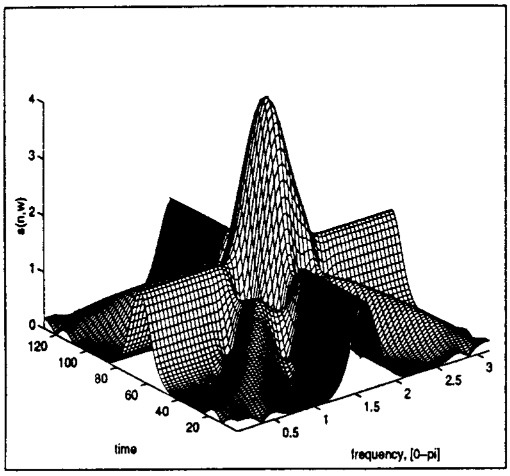

Figure 1: Arithmetic average energy distribution

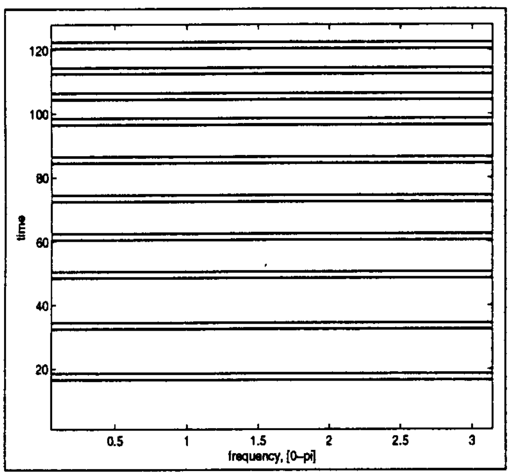

Figure 2: Narrow-window energy distribution

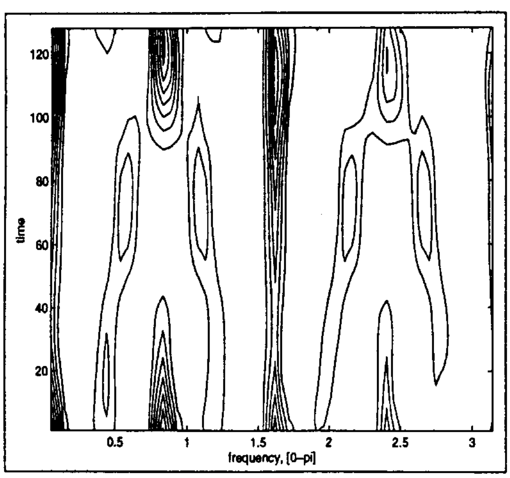

Figure 3: Wide-window energy distribution

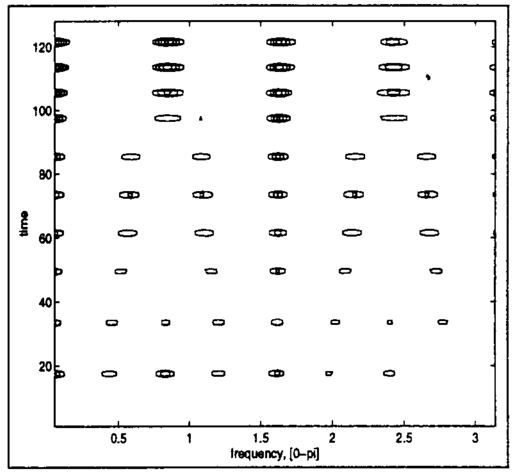

Figure 4: Geometric average energy distribution

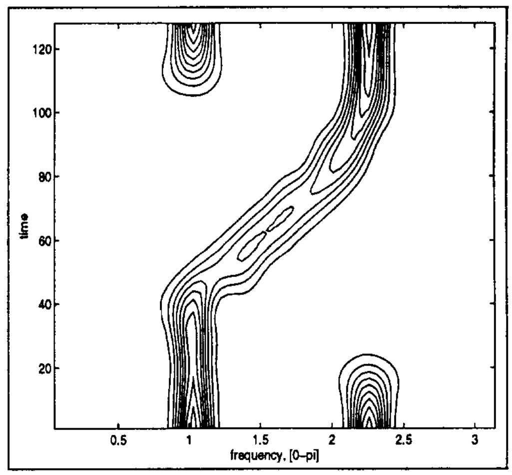

Figure 5: Energy distribution of the given signal

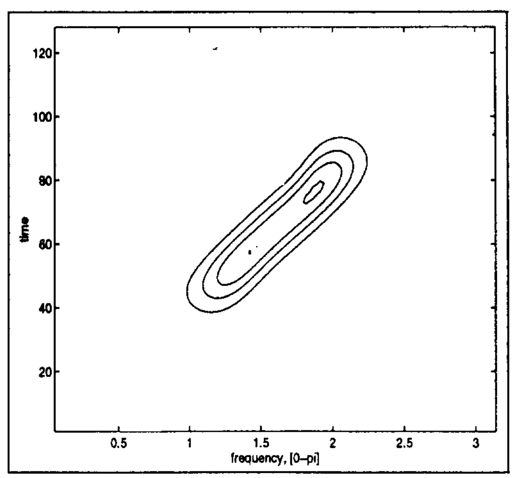

Figure 6: Energy distribution of the masked signal