IMPLEMENTATION OF FRACTIONAL DELAY WAVEGUIDE MODELS USING ALLPASS FILTERS

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ABSTRACT

This paper discusses a discrete-time modeling technique where the length of time delays can be arbitrarily adjusted. The new system is called a fractional delay waveguide model (FDWM). Formerly, FDWMs have only been implemented with FIR-type fractional delay filters. We show how an FDWM can be implemented using allpass filters. We use low-order allpass filters that are maximally-flat approximations of the ideal delay. The advantages of the allpass approach are computational efficiency and reduced approximation error. The proposed structure can be applied to discrete-time modeling of acoustic tubes, such as the human vocal tract or resonators of musical instruments.

1. INTRODUCTION

A discrete-time model of an acoustic tube system is traditionally constructed by approximating the profile of the tube using uniform sections of fixed length. The resulting tube system is then modeled by a digital lattice or ladder filter. This approach was first used for modeling the human vocal tract by Kelly and Lochbaum [1] and it has become a popular area of research (see, e.g., [2], [3]). Smith has recently generalized this approach to a class of techniques that he calls *digital waveguide modeling* [4]. A digital waveguide stands for a bidirectional delay line. Smith has shown that this methodology gives a way to directly simulate real-world systems using a discrete-time model. In addition to vocal tract modeling, the method is suitable to simulation of other one-dimensional resonators, such as musical wind instruments.

It has been noticed, however, that there is a severe limitation in the basic Kelly–Lochbaum scheme: the total length of the vocal tract is quantized according to the sampling interval and thus it cannot be changed smoothly. Strube [5] proposed that one of the unit delays of the lattice filter could be replaced by a *fractional delay* (FD) element, that is, a digital filter that approximates a delay smaller than a unit delay. Another possibility for accurate control of the total length is to change the sampling rate of the system by using interpolation [6], [7].

We have found that neither of the above methods is adequate for obtaining an accurately controllable vocal tract model. The remaining problem is that the length of individual tube sections cannot be controlled independently. In Strube's model, only one section is continuously controllable. The change of the sampling rate scales the length of all tube sections by the same factor.

Recently, we have proposed a method for changing the positions of the scattering junctions of a Kelly–Lochbaum (KL) model continuously [8], [9]. Consequently, the length of every tube section (as well as the total length of the tract) can be continuously varied. The model employs a nonrecursive interpolation technique together with a new technique that we call *deinterpolation* [8]. We call the resulting system a *fractional delay waveguide model* (FDWM) and it can be interpreted as an extension to Smith's framework.

In this work we study the possibility of applying *allpass* FD filters to the implementation of an FDWM. Earlier we have investigated the use of FIR interpolators for this task [8], [9]. The advantages of FD allpass filters over the FIR interpolators are that 1) the approximation error in magnitude is zero and 2) a good result is obtained with a low-order filter.

This paper is organized as follows. Section 2 discusses maximally-flat approximation of fractional delay using FIR and allpass filters. In Section 3 we review the FIR filter implementation of FD waveguide models. The new allpass filter FDWM is introduced in Section 4. Its performance is compared with those of an ideal waveguide model and an FIR implementation and the effects of the approximation errors are discussed.

2. MAXIMALLY-FLAT APPROXIMATION OF FRACTIONAL DELAY

The frequency response of a discrete-time delay element is

$$H(e^{j\omega}) = e^{-j\omega D} \tag{1}$$

where $\omega = 2\pi fT$ is the normalized radial frequency (with sampling interval T) and $D = \text{floor}(D) + d$ is the total delay with fractional delay d. The frequency response of the ideal discrete-time delay element given by (1) corresponds to a linear-phase allpass system. The impulse response of this system is a *shifted* (by D) *and sampled sinc function*, which is a two-sided infinite-length sequence. In practical applications some digital filter approximation for the fractional delay must be used. A tutorial on FIR and IIR FD filter design techniques has been written by Laakso *et al.* [10].

Audio signals have often more energy at low frequencies than at high frequencies. Thus, in audio signal processing it is recommendable to use an FD approximation that has smallest error at low frequencies. In maximally-flat (MF) approximation the error can be set equal to zero at $\omega = 0$. The MF FIR approximation to the FD is equivalent to the classical Lagrange interpolation [10]. Below we discuss the corresponding allpass filter approximation.

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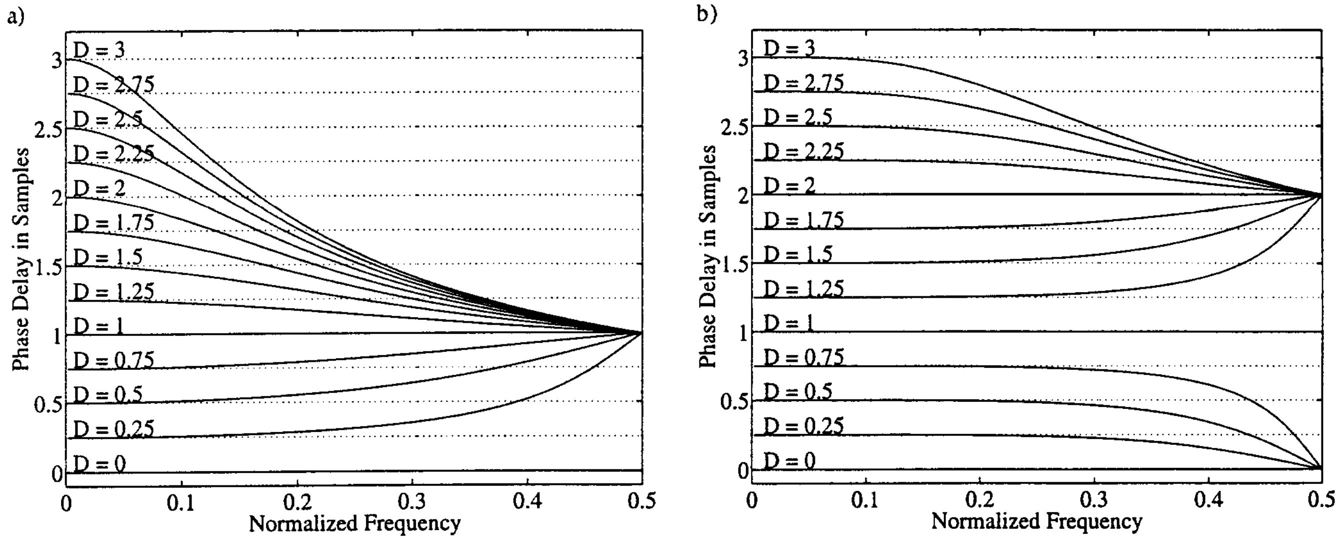

Fig. 1. *Phase delay (solid curve) of (a) the first-order and (b) the second-order MF allpass FD filter for several values of delay D. The dotted line shows the nominal phase delay for each case.*

2.1. Maximally-Flat Allpass FD Filter

An allpass filter is well-suited to FD approximation because its magnitude response is equal to unity. The transfer function $A(z)$ of an Nth-order discrete-time allpass filter is written as

$$A(z) = \frac{a_N + a_{N-1}z^{-1} + \ldots + a_1 z^{-(N-1)} + z^{-N}}{1 + a_1 z^{-1} + \ldots + a_{N-1} z^{-(N-1)} + a_N z^{-N}} \qquad (2)$$

A method for the design of MF FD allpass filters has been introduced in [11] and [10]. A remarkable feature of this design is that the filter coefficients can be given in a closed form, that is [12]

$$a_k = (-1)^k \binom{N}{k} \prod_{n=0}^{N} \frac{D-N+n}{D-N+k+n} \quad \text{for } k = 0,1,2,\ldots,N \qquad (3)$$

where N is the order of the allpass filter and D is the desired delay.

The coefficient of the first-order MF allpass FD filter is

$$a_1 = \frac{1-D}{1+D} \qquad (4)$$

The coefficients for the second-order filter are given by

$$a_1 = -2\frac{D-2}{D+1} \quad \text{and} \quad a_2 = \frac{(D-1)(D-2)}{(D+1)(D+2)} \qquad (5)$$

Figure 1 shows the phase delay response $\tau_p(\omega) = -\Theta(\omega)/\omega$ of these simplest allpass FD filters for several values of D.

In the first-order case (N = 1) the best approximation is obtained when the desired delay D is near to 1 (see Fig. 1a). When D = 0 the pole and the zero of the allpass filter are on the unit circle and the filter is asymptotically unstable—a situation which should be avoided. When N = 2, the approximation is most accurate near to D = 0 or D = 2.0 (see Fig. 1b). Also now D = 0 leads to potential stability problems.

3. FRACTIONAL DELAY WAVEGUIDE MODEL

Fractional delay waveguide modeling is a general framework for design and implementation of models of one-dimensional wave propagation in physical systems. It has been applied to computational modeling of the human vocal tract and woodwind instruments [9], [12]. The novel idea of this technique is the use of fractional delays for two purposes: to adjust the length of digital delay lines and to adjust the locations of junctions of waveguides.

Figure 2a illustrates scattering at the joint of two acoustic tubes of different diameter. The reflection coefficient r is determined by the cross-sectional areas A_k and A_{k+1} of the two tubes so that $r = (A_k - A_{k+1})/(A_k + A_{k+1})$. The scattering junction can be equivalently implemented using only one multiplier as depicted in Fig. 2b. In an FDWM, the scattering junction can be located at a noninteger point of a digital waveguide (i.e., bidirectional delay line). In FIR filter implementation of an FDWM, the interpolated scattering junction is based on the one-multiplier junction of Fig. 2b, but the allpass filter implementation which is presented in this paper is based on the four-multiplier junction of Fig. 2a.

Figure 3a illustrates a scattering junction between the sampling points of a digital waveguide. The four FD elements are

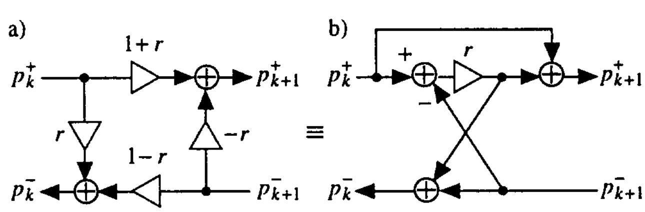

Fig. 2. *(a) Scattering junction between two acoustic tubes of different diameter and (b) its one-multiplier version.*

explicitly shown. The delay δ is defined by

$$\delta = 1 - d \tag{6}$$

and is called the *complementary fractional delay* (CFD) [12].

3.1. FIR Filter Implementation of FDWM

Formerly, FDWMs have been implemented using Lagrange interpolators [8], [9], [12]. The interpolated input into a digital delay line has been realized using deinterpolation [8], which is a new technique in signal processing. It is implemented using the transpose FIR filter structure with interpolating coefficients. Its impulse response, however, has to be time-reversed because a deinterpolator approximates the complementary FD δ. The two nonrecursive interpolators and the two deinterpolators can be combined, and thus only two interpolating FIR filters per FD junction are required (see [12] for details).

4. ALLPASS FILTER IMPLEMENTATION OF FRACTIONAL DELAY WAVEGUIDE MODEL

4.1. Derivation of the FD Junction Structure

Figure 3b shows a modified version of the junction depicted in Fig. 3a. Now the four FD elements have been pushed through the adders and branch nodes. This can be done because linear time-invariant systems commute. Now there are altogether eight FD elements. The FD elements z^{-d} and $z^{-\delta}$ can be combined into a single unit delay element (since $d + \delta = 1$) both in the upper and in the lower line (see Fig. 3c). The two FD elements z^{-d} on the left of Fig. 3b can be combined into a single block z^{-2d}, and the same is possible to do for the two CFD blocks on the right.

The block diagram presented in Fig. 3c is one possible allpass implementation of the FD junction. Other configurations can also be derived, but this one has some desirable properties. We realize the "double" fractional delay elements using allpass filters $A_D(z)$ and $A_\Delta(z)$. Consequently, the desired delay D and complementary total delay Δ are defined by

$$D = 2d \tag{7a}$$

$$\Delta = 2\delta = 2 - D \tag{7b}$$

This implies that when the noninteger part d of the position of the junction can have values on the interval [0, 1], the delays implemented by the two allpass filters will have values on the interval [0, 2].

4.2. Transmission Function

Let us consider the approximation error due to allpass interpolators. In the structure of Fig. 3c, there are no FD elements in the straight path in neither direction. Thus there is no approximation error in the transmission function through the FD junction. The only limitation that this solution brings about is that the point where the transmission occurs is not explicitly implemented and the signal value at that point cannot be directly obtained. At the open end of a tube this causes an error in the delay of the output signal. This does not affect the formant structure nor other important properties, but can be avoided—if desired—by incorporating an additional FD filter.

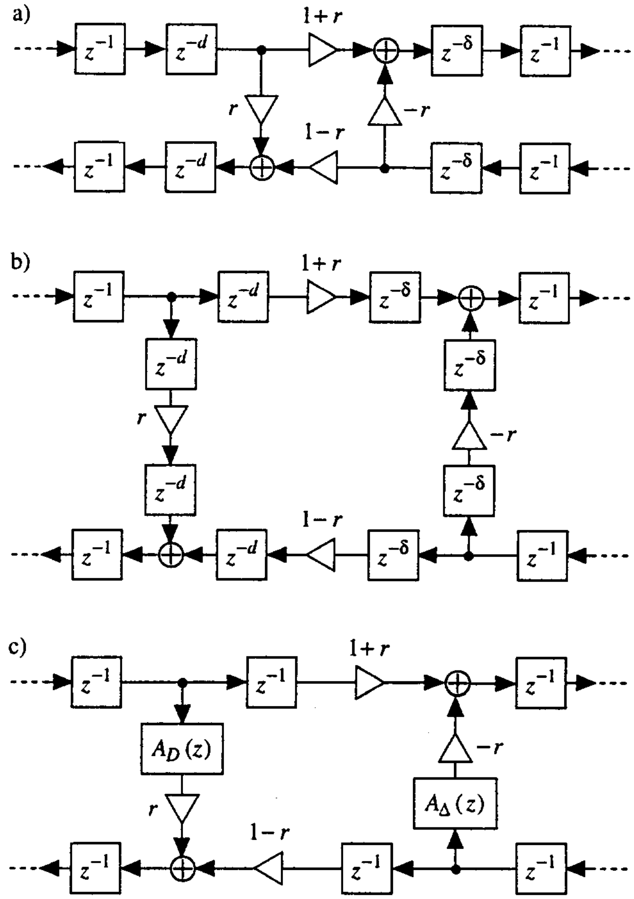

Fig. 3. *Derivation of the FD scattering junction employing commutation. (a) The original FD scattering junction. (b) The FD elements have been pushed through the branch nodes and adders. (c) The structure that is implemented.*

4.3. Reflection Function

The reflection from the FD junction suffers from an error since the "double" FDs are realized using digital filter approximations $A_D(z)$ and $A_\Delta(z)$. When MF allpass FD filters are used, the nature of this error can be predicted from Fig. 1. At very low frequencies the impedance discontinuity seems to be in the correct position, but with increasing frequency, the junction tends to move towards some other location. This is because the allpass filters approximate the delay accurately at low frequencies, but poorly at high frequencies. We may assume that at low frequencies the formant structure of a tube model implemented with these allpass filters is similar to the ideal one, but at higher frequencies the center frequencies of formants are incorrect.

4.4. Comparison of Implementation Techniques

In order to compare the properties of allpass and FIR implementations, we simulated a two-tube model with one FD junction. The total length of the tube system is 8 unit delays. The junction between the tubes is located between the 3rd and the 4th unit delay. The reflection coefficient is –0.5. One end of the

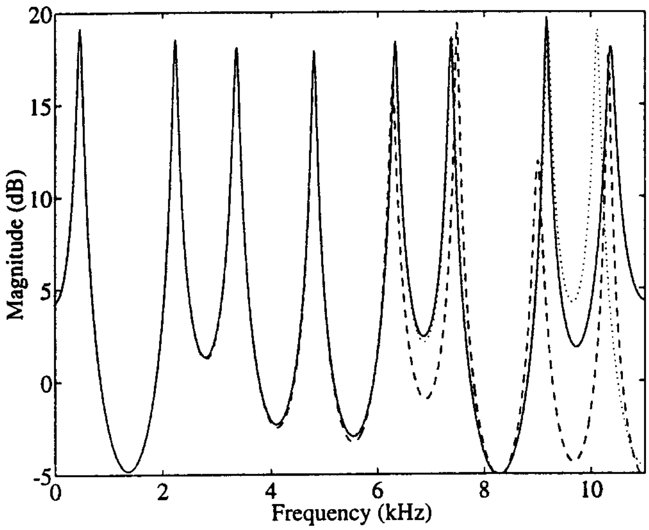

Fig. 4. *Transfer function of a two-tube system realized using second-order allpass filters (solid line), third-order Lagrange interpolation (dashed line), and ideal interpolation (dotted line).*

tube is assumed to be closed and the other one open. To simplify the example, the terminations have been approximated with purely resistive loads, $r_1 = 0.9$ and $r_2 = -0.9$. Second-order allpass FD filters were used because the approximation error obtained with first-order filters was considered too large.

Figure 4 shows the magnitude of the transfer function of this system. The solid line is obtained using second-order MF allpass FD filter and the dashed line using third-order Lagrange interpolation. The dotted line is the transfer function of a system that employs ideal interpolators. The sampling rate is 22 kHz. In this example $d = 0.4$ which is nearly the worst case for both the second-order allpass filter and Lagrange interpolation.

The transfer functions are nearly identical at low frequencies, but above 5 kHz the curve obtained using Lagrange interpolation (dashed line) deviates from the others. At highest frequencies all the curves go their own ways. The magnitudes of the formants of the allpass system do not, however, deviate from the ideal curve as much as for the Lagrange interpolator.

The computational complexity of the allpass filter implementation is less than that of the FIR filter implementation: a second-order allpass filter can be realized with 2 multiplications and 3 additions, whereas a third-order FIR filter takes 4 multiplications and 3 additions.

5. CONCLUSIONS AND FUTURE WORK

We have introduced a new method to realize a fractional delay waveguide model (FDWM) where the time delay between scattering junctions can be continuously controlled. In this approach the FD elements are implemented using low-order allpass filters that are maximally-flat approximations of ideal interpolation. This method is more efficient and more accurate than an FDWM implemented using Lagrange interpolation. In the future, the behavior of the allpass FDWM should be analyzed in the case when the positions of the junctions are slowly changing.

The allpass FDWM was shown to be suitable for constructing high-quality models of physical systems. This approach may be used, e.g., for implementing model-based sound synthesis of wind instruments or an articulatory speech synthesizer.

ACKNOWLEDGMENTS

This work was supported by the Academy of Finland. The authors wish to thank CARTES (Espoo, Finland) for the possibility to use their facilities.

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