Subband Filters Optimized for Lost Coefficient Reconstruction

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Abstract

Packet-based transmission of subband coded images over lossy networks presents a reconstruction problem at the decoder. Accurate reconstruction of the high-energy low frequency subband coefficients is imperative in providing consumer-grade image quality. This paper introduces a family of one-dimensional quadrature mirror filters (QMFs) designed to minimize the mean-squared error of reconstructed low frequency coefficients for a given reconstruction algorithm to be implemented at the decoder. Mean-reconstruction, in which a missing coefficient is replaced with the average of its neighbors either horizontally or vertically, is selected for its simplicity and implementation ease. The resulting filters perform well as QMFs and provide the desired reconstruction properties in the event of loss. While the filters are developed using mean-reconstruction, the filter design algorithm can be used with more sophisticated reconstruction techniques, providing that the mean-squared error can be expressed in the appropriate quadratic form.

1. Introduction

Transmission of digitally coded still images over lossy packet networks presents a reconstruction problem at the decoder. Standard techniques for data recovery such as forward error correction and automatic retransmission query protocols become unwieldy at the high data rates required by image transmission [1]. However, visual data contains much redundancy which can be exploited by the decoder to reconstruct lost information using signal processing techniques. A simple reconstruction algorithm minimizes decoder complexity, so the inclusion of source coding techniques designed to optimize the reconstruction performance can improve the quality of the reconstructed image.

Previous work on subband reconstruction has been entirely decoder-based, and has not considered the encoding process [2, 3]. The resulting algorithms provide good visual quality in the synthesized images, but require computation at the decoder that is best suited to a DSP or algorithm-specific hardware. In cases where minimizing both complexity and additional hardware at the decoder is required, a simple algorithm is preferable. Simplifying the reconstruction algorithm and decoder suggests that the encoding procedure should be modified to achieve high quality reconstructed images. Using subband filters that have been optimized to achieve the best reconstruction performance for a given reconstruction algorithm is such a modification.

This paper introduces a family of filters optimized for hierarchical subband coded image reconstruction. In a hierarchical subband decomposition, analysis is recursively performed on the low frequency subband, yielding a hierarchy of subbands which lend themselves to applications such as progressive transmission and multi-rate broadcast. In this paper, the two-dimensional (2-D) subband filters are separable and each is formed as the outer product of a one-dimensional (1-D) filter with itself. Because the 2-D filtering process can be decomposed into two sequential 1-D filtering processes, the design of 1-D filters is considered.

For a pre-selected reconstruction algorithm, a family of even-length quadrature mirror filters (QMFs) is designed to minimize the mean-squared error (MSE) of reconstructed subband coefficients in the low frequency band by trading off the quality of the overall transfer function of the analysis/synthesis system with the reconstruction performance. The resulting filters perform well as subband filters, outperform a commonly used QMF when the subband data is quantized, and provide the desired reconstruction performance on both unquantized and quantized data.

The organization of this paper is as follows. Section 2 formulates the reconstruction criterion to be used in the filter design. The filter design algorithm is presented in Section 3, and Section 4 describes design of the filter family. Section 5 outlines performance on images and reconstruction results, and the paper is concluded in Section 6.

2. Reconstruction Criterion Formulation

2.1 Coefficient Domain Error Measurement

Because the low frequency subband typically contains over 95% of the image energy, errors in low frequency coefficients have a much greater effect than high frequency reconstruction errors, so consideration of reconstruction errors is limited to this component. Furthermore, high frequency subbands can be reconstructed using linear interpolation with negligible effect on the reconstructed image quality [3].

The reconstruction criterion, which is minimized when designing the filters, is defined as the MSE of the reconstructed low frequency coefficients. Thus the MSE in the coefficient domain is considered, rather than the MSE in the subsequent synthesized image. Error measured in the reconstructed synthesized signal is caused by two factors: error in reconstructed coefficients, and error induced by the overall transfer function (OTF) of the analysis/synthesis system when near-perfect reconstruction filters are used. Of the two error components, OTF error is preferable because it is distributed over the entire image, whereas reconstruction errors are localized to small areas and hence are more visually distracting. It can be shown that providing that the filters

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offer near-perfect reconstruction and therefore that the reconstruction error is significantly greater than the OTF error, measuring error in the two domains is equivalent.

2.2 Low Frequency Subband Correlation Model

In order to obtain an expression for the MSE of the reconstructed coefficients, a correlation model for the low frequency subband coefficients is required. Because the low frequency band is simply a low-pass filtered and subsampled version of the original image, it resembles the original and can be considered to be an image itself.

Consider a low frequency band in which the final stage of 2-D analysis is performed separably by sequential 1-D filtering and subsampling in the horizontal and vertical directions on individual rows and columns, respectively. The directions can be processed in either order and, for clarity, this description assumes horizontal followed by vertical processing. After the horizontal processing stage, the intermediate signal consists of columns whose values have been averaged over a window of several columns by the low-pass filtering operation. The averaging does not significantly alter the appearance of the column signals when compared to the original column signals before the horizontal processing stage. Hence the correlation of the 1-D intermediate signal $x(n)$ is modelled using a traditional technique for images, as an exponential function of a correlation parameter ρ and the separation k [4, 5], i.e., $E\{x(n)x(n+k)\} = \sigma^2\rho^k$, where the variance σ^2 is normalized to 1 in the following.

2.3 Reconstruction Criterion

The reconstruction criterion in the coefficient domain is formulated as follows. Let $x_L(n)$ represent the low frequency subband signal from a 1-D filter bank, obtained by analyzing an original signal $x(m)$, and let h represent the even-length symmetric low-pass filter. In the case of images filtered with 2-D separable filters, $x(m)$ can be considered as either the rows or columns obtained after 1-D analysis in the vertical or horizontal directions, respectively, when generating the low frequency band. Then $x_L(n)$ represents the 1-D rows or columns after the second 1-D analysis. To provide the minimum decoder complexity, consider 1-D mean-reconstruction in the direction of the second analysis, i.e. $\hat{x}_L(n) = 1/2(x_L(n-1) + x_L(n+1))$, in which it is assumed that $x_L(n-1)$ and $x_L(n+1)$ are known. Then the MSE of the reconstructed low frequency coefficients can be written as a quadratic function of the filter coefficients h as $MSE = \boldsymbol{h}'_{half}M_{rec}\boldsymbol{h}_{half}$, where $\boldsymbol{h}_{half}$ represents the $N/2$ unique filter coefficients of the filter h and M_{rec} is an $N/2 \times N/2$ matrix that is a function of ρ.

3. QMF Design Incorporating the Reconstruction Criterion

3.1 The Reference Filter Design Algorithm

The filter design algorithm is a modified version of an algorithm presented in [6]. In the reference algorithm, two criteria are used in filter design:

C1. Stopband energy: minimize

$$E_{sb} = \frac{1}{\pi}\int_{\omega_{sb}}^{\pi}\left|H(e^{j\omega})\right|^2 d\omega = 2\boldsymbol{h}'_{half}M_{sb}\boldsymbol{h}_{half}$$

where M_{sb} is a function of ω_{sb}.

C2. Overall transfer function: minimize the ripple energy

$$E_{rip} = \sum_{\substack{n=-(N/2-1) \\ n \neq 0}}^{N/2-1} ((h*h)(2n+1))^2 = 2\boldsymbol{h}'_{half}M_{rec}\boldsymbol{h}_{half}$$

where M_{rip} is a function of $\boldsymbol{h}_{half}$, and meet the constraint $2\boldsymbol{h}'_{half}\boldsymbol{h}_{half} = 1$.

The first criterion is often used in filter design; the second criterion results because the overall transfer function of an analysis/synthesis system using even-length QMFs is given by $h*h(2n+1)$, which is ideally a delayed Kronecker delta function, but has non-zero values at off-center locations for QMFs with length $N > 2$.

In C2, E_{rip} is a quartic function of $\boldsymbol{h}_{half}$. This quartic is reduced to a quadratic by introducing a filter η with coefficients η_{half}. An initial filter η is selected, and a cost function $J(\boldsymbol{h}_{half})$ that includes C1 and C2 is minimized subject to the constraint in C2 to solve for $\boldsymbol{h}_{half}$. Then $\eta \leftarrow h$, and the iteration proceeds until a stopping condition is reached.

3.2 Inclusion of the Reconstruction Criterion

The reference algorithm formulates an objective function as a quadratic function of the filter coefficients. The reconstruction criterion developed in Section 2 is also a quadratic function of the filter coefficients, and can therefore be incorporated directly into the algorithm:

C3. Reconstruction criterion: minimize

$$MSE = \boldsymbol{h}'_{half}M_{rec}\boldsymbol{h}_{half}$$

where M_{rec} is a function of ρ.

It can be shown that the reconstruction criterion behaves approximately like a stopband criterion, and hence has a "natural stopband frequency" associated with it [7]. For $\rho \geq 0.90$, the natural stopband frequencies are approximately constant at $0.20(2\pi)$, and the natural stopband frequency is less than $\pi/2$ for all values of $\rho < 1$ and decreases with ρ. In order to achieve near-perfect reconstruction, however, QMFs require that $\left|H(e^{j(\pi/2)})\right| = \left|H(e^{j\omega})\right|/\sqrt{2}$ for ω in the passband. Thus, in order to prevent the reconstruction criterion from causing undesirable effects in the passband and hence in the OTF, the reconstruction criterion must be weighted appropriately with respect to the stopband criterion. Furthermore, to minimize the effects of the natural stopband frequency outside of the stopband as specified by ω_{sb}, two more filter design criteria are added to the algorithm:

C4. Passband ripple energy: minimize

$$E_{pb} = \frac{1}{\pi}\int_{0}^{\omega_{pb}}(\left|H(e^{j\omega})\right| - \sqrt{2})^2 d\omega$$

$$= 2\boldsymbol{h}'_{half}M_{pb}\boldsymbol{h}_{half} + \boldsymbol{h}'_{half}g + k$$

where g and k are constants and M_{pb} is a function of ω_{pb}.

C5. Transition band constraint:

$$\left|H(e^{j(\pi/2)})\right| = 1 \quad \rightarrow \quad 2\boldsymbol{h}'_{half}M_T\boldsymbol{h}_{half} = 1$$

where M_T is a constant.

The cost function is then defined as

$$J(\boldsymbol{h}_{half}) = \boldsymbol{h}'_{half}(2M_{rip} + 2M_{pb} + \alpha_{sb}2M_{sb} + \alpha_r 2M_{rec})\boldsymbol{h}_{half}$$
$$+ \boldsymbol{h}'_{half}g + k + \lambda_1(2\boldsymbol{h}'_{half}\boldsymbol{h}_{half} - 1)$$
$$+ \lambda_2(2\boldsymbol{h}'_{half}M_T\boldsymbol{h}_{half} - 1)$$

where λ_1 and λ_2 are Lagrange multipliers, and the weighting constants α_{sb} and α_r are varied to exchange reconstruction performance and the OTF performance. As in the reference algorithm, a filter η is introduced and the quartic function for E_{rip} is replaced with a quadratic. Additionally, the constraints are linearized using η so that $J(\boldsymbol{h}_{half})$ may be minimized, and $\boldsymbol{h}_{half}$ is iteratively solved for until a stopping condition is reached.

4. Filter Family Design

This section describes the selection of the required parameters (N, ρ, ω_{pb}, ω_{sb}, α_r, α_{sb}) when designing the QMF family. While the use of an automated algorithm avoids manual intervention and tweaking during the filter design stage, care is required is choosing the parameters, and some trial-and-error is required to select parameters that yield good filters. The designed filters were initially evaluated by examining the minimum attenuation in the stopband and the maximum ripple in the OTF. The six filter parameters were selected as follow.

Filter Length N. The filter length was chosen to be $N = 32$, so the filters could be directly compared with a generally recognized filter with good performance, Johnston's 32d filter [8].

Markov Parameter ρ. The value $\rho = 0.95$ was selected after examining empirical correlation coefficients for randomly selected rows and columns of several 512×512 luminance images from the USC database analyzed using Johnston's 32d filter (the correlation coefficients do not vary greatly with different filters).

Cutoff Frequencies ω_{pb} **and** ω_{sb}. The passband and stopband frequencies were selected as $\omega_{pb} = 0.23(2\pi)$ and $\omega_{sb} = 0.30(2\pi)$. First, the passband frequency was selected to be "close" to $\omega = \pi/2$, so that the passband constraint would be enforced over as large a range as possible, while still allowing a transition band. Trial and error then demonstrated that stopband frequencies larger than $0.30(2\pi)$ gave poor stopband attenuation when the reconstruction criterion was included

Stopband and Reconstruction Criteria Weightings α_{sb} **and** α_r. These values were selected to be $\alpha_{sb} = \{1/8, 1/4, 1/2\}$ and $\alpha_r = \{0, 1/8, 1/4, 1/2\}$, which were selected after comparing the stopband attenuation and OTF ripple for filters designed with α_{sb} and α_r ranging over powers of 2 from 2^{-6} to 2^3. The final ranges selected represent filters in which performance was dominated by neither the stopband nor the reconstruction criteria.

The value $\alpha_r = 0$ was included to gauge the quality of non-reconstruction-optimized filters designed using the algorithm, for direct comparison with Johnston's 32d filter.

5. Filter Performance and Reconstruction

5.1 Overall Transfer Function Quality

To measure OTF quality, the *couple* image was hierarchically decomposed to one, two, and three levels (4, 7, and 10 bands) and then synthesized without loss. The twelve filters were compared with a benchmark filter (Johnston's 32d, from [8]) using both PSNR and visual inspection of the resulting filtered images.

The filter performance on the unquantized *couple* image for one decomposition level is plotted for the filter family in Figure 1. In all cases, the unquantized synthesized images are indistinguishable from each other. However, the PSNRs exhibit several trends. Excepting $\alpha_{sb} = 1/8$, as the reconstruction weighting α_r increases, the PSNR of the synthesized image decreases for constant stopband weighting α_{sb}, indicating that the quality of the overall transfer function decreases. When quantized, images generated using the filters with $\alpha_r = 0$ and $\alpha_r = 1/8$ exhibit higher PSNRs than those quantized with the benchmark filter.

5.2 Reconstruction Performance

Reconstruction performance was evaluated by reconstructing a 10% regularly spaced loss pattern on the low frequency band of the *couple* image and measuring the MSE of the reconstructed low frequency coefficients. As desired, the MSEs of reconstructed coefficients for both quantized and unquantized data indeed decrease as the reconstruction weighting α_r increases. These results are plotted in Figure 2 for $\alpha_{sb} = 1/8$ and one decomposition level for both unquantized and quantized data. Similar results hold for two and three decomposition levels, and for $\alpha_{sb} = 1/4$ and $\alpha_{sb} = 1/2$.

The PSNRs for the unquantized reconstructed images are plotted in Figure 3. The PSNR spread has been reduced to less than 0.5 dB when 10% of the low frequency coefficients have been reconstructed, compared to a spread of 5-8 dB with no loss. The distortion measured by the PSNR consists of two components; namely, error caused by the OTF of the analysis/synthesis system, which increases as α_r increases, and error caused by coefficient reconstruction, which decreases as α_r increases. At a 10% loss rate, the overall PSNRs are approximately equal for the designed filter family and the benchmark, but the relative amounts of reconstruction and OTF error are different. The smaller reconstruction error in the designed filters is enough to compensate for the lower quality overall transfer functions, and results in smaller localized errors caused by reconstruction. The designed filters produce more visually pleasing reconstructed images than the benchmark filter, whose images have higher PSNR due to smaller OTF error but has higher localized reconstruction errors. An image suffering 10% loss and reconstructed is shown in Figure 4.

6. Summary

This paper has introduced a quadrature mirror filter family designed to minimize the mean-squared reconstruction error in the low-frequency subband coefficients for a specified reconstruction algorithm to be implemented at the decoder. Given a simple intra-band reconstruction algorithm for the low frequency band, the reconstruction criterion is formulated based on a correlation model and incorporated into a filter design algorithm, with appropriate modifications introduced to provide good quality filters. The resulting filters designed for mean-reconstruction perform well as QMFs as well as provide the desired mean-squared error behavior when the data is both unquantized and quantized. While the filters were developed using mean-reconstruction, the filter design algorithm is flexible and can be used with more sophisticated reconstruction techniques, providing that the mean-squared error can be expressed in the appropriate quadratic form.

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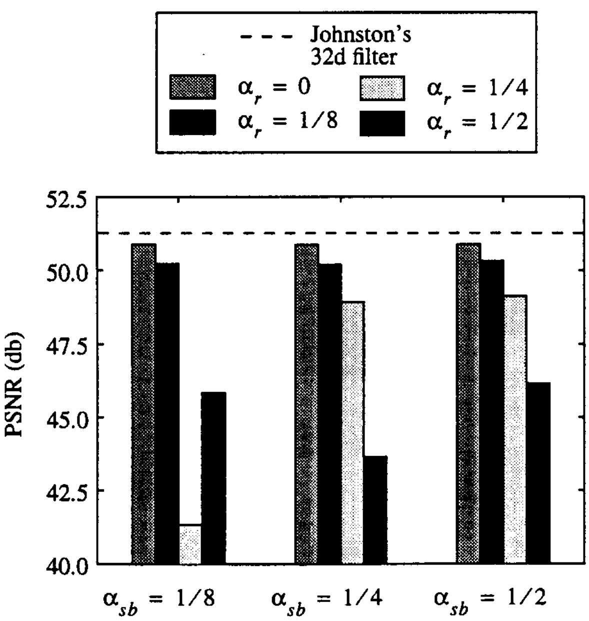

Figure 1 Filter performance of the QMF family for unquantized *couple*, 1 decomposition level, compared with Johnston's 32d.

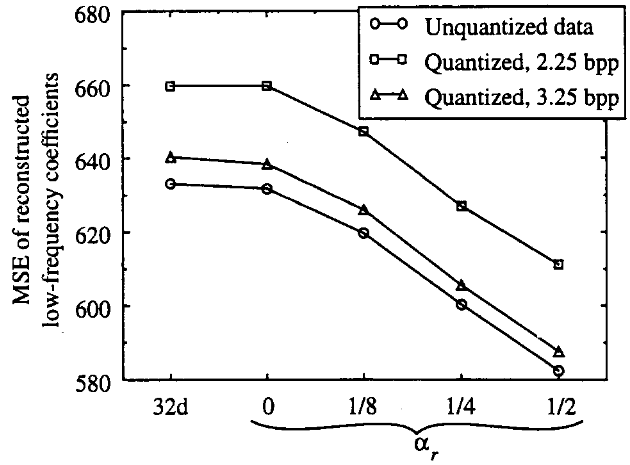

Figure 2 MSE performance for $\alpha_{sb} = 1/8$, evaluated on *couple* with 10% regular loss, 1 decomposition level.

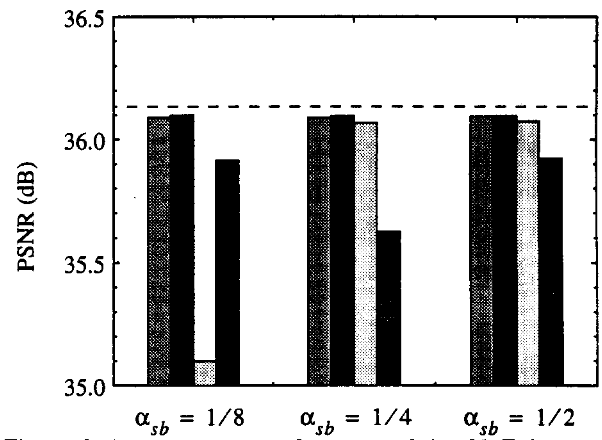

Figure 3 Reconstruction performance of the QMF family for *couple*, compared with Johnston's 32d, 1 decomposition level, for 10% regular loss in the low frequency band.

Figure 4 Reconstructed *couple*, $\alpha_{sb} = 1/8$, $\alpha_r = 1/2$ with 10% regular loss in the low frequency band, 1 decomposition level, PSNR = 35.9 dB.