THE GENERALIZED LAPPED TRANSFORM (GLT) FOR SUBBAND CODING APPLICATIONS

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ABSTRACT

This paper proposes a new family of perfect reconstruction (PR) linear phase filter banks called the generalized lapped transform (GLT). The GLT differs from the traditional lapped orthogonal transform (LOT) [1] in that it is nonorthogonal and hence offers more freedom to avoid blocking effects and improve the coding gain. Since the GLT can also be viewed as a generalization of the traditional discrete cosine transform (DCT), fast algorithms [2-4] for their implementation are also available.

1. INTRODUCTION

Perfect reconstruction (PR) maximally decimated quadrature mirror filter banks (QMF) has important applications in audio and image coding. It is known that very efficient PR FIR filter banks, the modulated filter banks (MFB), can be obtained by modulating a linear phase prototype filter [5-7]. However, the resulting analysis and synthesis filters are in general not linear phase. In [8], it was found that the simplicity of the modulated filter banks is closely related to the structural constraints imposed on the polyphase matrix. By exploiting the concept of structural PR, general linear phase orthogonal and nonorthogonal cascade PR systems were obtained. In particular, structures for orthogonal and nonorthogonal LOT with greater overlap are also proposed. However, satisfactory design for image coding applications had not yet been obtained. Complete factorization of the orthogonal linear phase PR filter banks has also been obtained by Soman et al [9]. More recently, Queiroz et al [10] has made use of this factorization to extend the LOT to length greater than 2M, where M is the number of channels. It was observed by Aase and Ramstad [11] that for image coding, the functions of the analysis filters and synthesis filters are quite different. The analysis filter bank should maximize the energy compaction, while the synthesis filter bank should provide blocking-free reconstruction. In addition, the synthesis filters should be short to avoid excessive ringing and should be smooth enough to reduce blocking effects. Since the analysis and synthesis filters of an orthogonal system must be time-reverse of each other, it is very difficult to achieve these objectives simultaneously. On the other hand, nonorthogonal filter banks do not suffer from this restriction and is more appropriate for this application. In this paper, we shall introduce a new family of nonorthogonal filter banks, called the generalized lapped transform (GLT), to serve these purposes. It is based on the nonorthogonal version of the LOT introduced in [8]. Like the LOT, the GLT has relatively low complexity of implementation and is based on the well-known discrete cosine transform (DCT).

2. THE LINEAR-PHASE PR CASCADE

It was found in [8] that linear phase PR filter banks can be obtained with a lattice type cascade structure of the polyphase matrix, $E(z)$, as follows:

$$E(z)=\left[\prod_{l=0}^{K-1}\begin{bmatrix}U_{00}^l & 0\\ 0 & U_{11}^l\end{bmatrix}\begin{bmatrix}I_{M/2} & I_{M/2}\\ I_{M/2} & -I_{M/2}\end{bmatrix}\begin{bmatrix}I_{M/2} & A_l\\ A_l & I_{M/2}\end{bmatrix}\right.$$

$$\left.\cdot\begin{bmatrix}I_{M/2} & 0\\ 0 & z^{-1}I_{M/2}\end{bmatrix}\right]\begin{bmatrix}I_{M/2} & I_{M/2}\\ I_{M/2} & -I_{M/2}\end{bmatrix}\begin{bmatrix}I_{M/2} & J_{M/2}\\ I_{M/2} & -J_{M/2}\end{bmatrix}R \quad (1)$$

where

A_l are diagonal matrices,

$I_{M/2}$ and $J_{M/2}$ are $(M/2)\times(M/2)$ identity and antidiagonal matrices respectively,

U_{ii}^l are $(M/2)\times(M/2)$ invertiable matrices,

R is $(M\times M)$ persymmetric matrix, and

M is the number of channels and is even

It can be shown that the matrix $\begin{bmatrix}I_{M/2} & A_l\\ A_l & I_{M/2}\end{bmatrix}$ can be absorbed into the block diagonal matrix $diag\{U_{00}^l, U_{11}^l\}$ and can be ignored. Furthermore, if U_{ii}^l are unitary (orthogonal), the filter bank will be unitary (orthogonal) as well.

3. THE GENERALIZED LAPPED TRANSFORM

An example of the orthogonal representation is the Lapped Orthogonal Transform (LOT) which is of length 2M. The corresponding polyphase matrix is:

$$E(z)=0.5\cdot P\begin{bmatrix} I_{M/2} & 0 \\ 0 & (C^{II}_{M/2}S^{IV}_{M/2})^T \end{bmatrix}$$

$$\cdot\left\{\begin{bmatrix} I_{M/2} & I_{M/2} \\ I_{M/2} & -I_{M/2} \end{bmatrix}\begin{bmatrix} I_{M/2} & 0 \\ 0 & z^{-1}I_{M/2} \end{bmatrix}\right\}R \qquad (2)$$

$$R=P'diag\{B_2,\ldots,B_2\}C^{II}_M J_M \text{ and } B_2=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Here,

C^k_M, S^k_M denote the type k length-N discrete cosine and sine transform,

P is a permutation matrix which permutes the k and k+M/2 rows to 2k and 2k+1 (k = 0,..,M/2-1) rows respectively,

P' is a permutation matrix which permute the 2k and 2k+1 rows to k and k+M/2 (k = 0,..,M/2-1) rows respectively.

We propose a class of nonorthogonal PR filter bank, the GLT of length 2M:

$$E(z)=0.5\cdot PU\begin{bmatrix} I_{M/2} & 0 \\ 0 & (C^{II}_{M/2}S^{IV}_{M/2})^T \end{bmatrix}$$

$$\cdot\left\{\begin{bmatrix} I_{M/2} & I_{M/2} \\ I_{M/2} & -I_{M/2} \end{bmatrix}\begin{bmatrix} I_{M/2} & 0 \\ 0 & z^{-1}I_{M/2} \end{bmatrix}\right\}\tilde{R} \qquad (3)$$

where

$U=diag\{U_{00},U_{11}\}$ is a block diagonal invertiable matrix and,

$\tilde{R}=P'diag\{B_2,\ldots,B_2\}DC^{II}_M J_M$ with D a diagonal matrix

We parameterize U_{ii} by products of block diagonal (2x2) invertiable matrices:

$$U_{ii}=V^i_3V^i_2V^i_1 \qquad (4)$$

where V^i_k is obtained by replacing the (k,k), (k,k+1), (k+1,k), (k+1,k+1) entries of the identity matrix by the elementary invertiable matrix $\begin{bmatrix} x & y \\ y & x \end{bmatrix}$:

$$V^i_k=\begin{bmatrix} I_{k-1} & & & 0 \\ & x^i_k & y^i_k & \\ & y^i_k & x^i_k & \\ 0 & & & I_{M-k-1} \end{bmatrix} \qquad (5)$$

with inverse

$$(V^i_k)^{-1}=\frac{1}{(x^i_k)^2-(y^i_k)^2}\begin{bmatrix} I_{k-1} & & & 0 \\ & x^i_k & -y^i_k & \\ & -y^i_k & x^i_k & \\ 0 & & & I_{M-k-1} \end{bmatrix} \qquad (6)$$

Other product forms for U_{ii} can be used but the present choice has the advantage that U_{ii} will be diagonal dominance. Longer filter can be defined similar. However, short filters are preferred in image coding applications to avoid excessive ringing. Parameters x^i_k, y^i_k, d_{ii} are used to minimize the following objective function:

$$E=\alpha_1E_G+\alpha_2E_A+\alpha_3E_s \qquad (7)$$

where

$$E_G=M\left[\prod_{k=1}^{M}A_kB_k\right]^{1/M},$$

$$\text{with } B_k=\frac{1}{M}\sum_{n=0}^{L-1}g_k^2(n),\quad A_k=\sum_{i=0}^{L-1}\sum_{j=0}^{L-1}h_k(i)h_k(j)\rho^{|i-j|} \qquad (8)$$

$$\text{and}\qquad E_s=\sum_{k=0}^{M-1}(B_k-\frac{1}{M})^2 \qquad (9)$$

E_G is the inverse of the coding gain of the nonunitary filter banks for an AR(1) process with correlation coefficient ρ [12-13]. $h_k(n)$ and $g_k(n)$ are the impulse response of the k-th analysis and synthesis filters, respectively. E_A is the sum of stopband energies of the synthesis lowpass filter, $g_0(n)$, with an interval of $\pm I$ at frequencies i/M, i = 1,...,M/2. This helps to reduce the blocking effects. E_s is used to avoid poor scaling of the synthesis filters. Also the synthesis filters are forced to have small values at the ends of the impulse response. α_i's are constants to tradeoff between the three objectives.

The signal flowgraph for a 8-channels length-16 GLT optimized with $\rho=0.95$ is shown in figure 1. Figure 2 shows the impulse responses of the first and second analysis and synthesis filters of the GLT. Notice that the synthesis filters are no longer the time reverse of the corresponding analysis filters. Also the synthesis filters are much smoother than the analysis filters and decay at both ends to reduce visual artifacts during reconstruction after signal quantization. Figure 3 and figure 4 show the frequency responses of the analysis and synthesis filter banks. Here, I is chosen as 0.002. We have made use of the NCONF program in the IMSL library to perform the constrained optimization. The resulting coding gain of the filter bank is 9.58dB. Future investigations will be concentrated on better filter design techniques and parameterization of the nonorthogonal matrices to obtain filter with larger number of channels and filter lengths.

CONCLUSION

A new family of perfect reconstruction linear phase nonorthogonal filter banks called the generalized lapped transform (GLT) is presented. It offers more freedom for tradeoffs between the competing objectives in coding applications and has fast form of implementation based on the discrete cosine transform.

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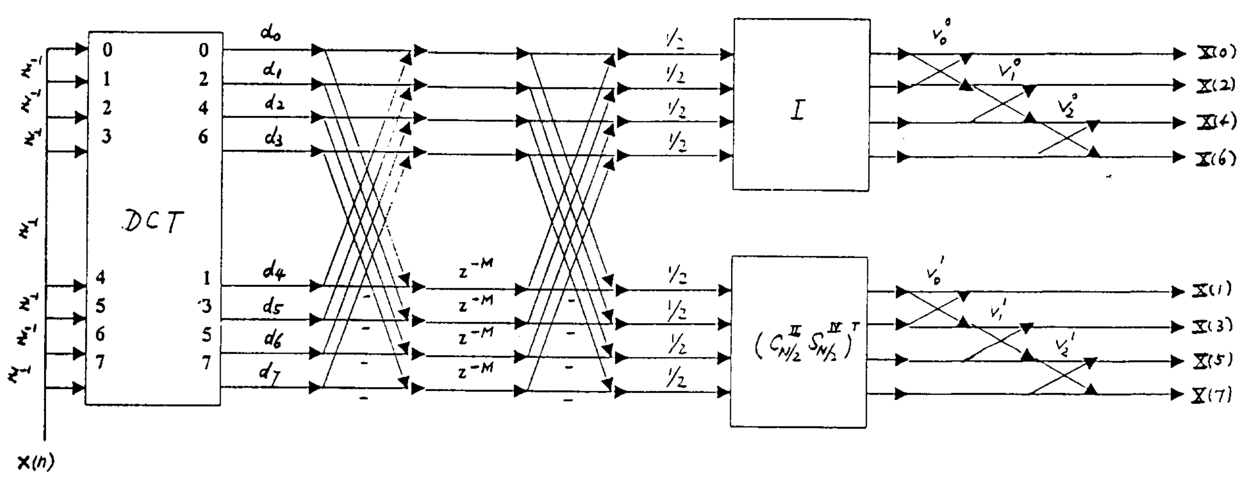

Figure 1. Signal Flow graph of the 8-channel length-16 generalized lapped transform (GLT)

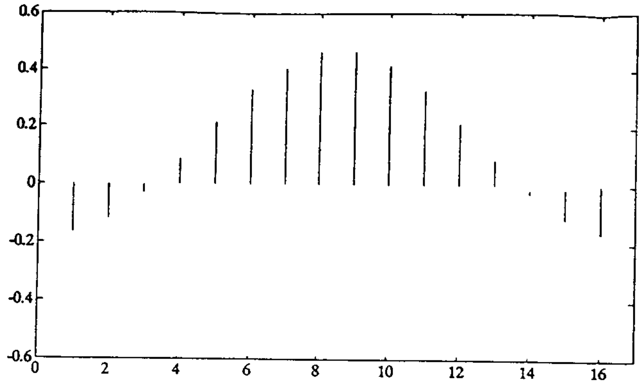

Fig. 2a. Impulse response of analysis filter, $h_0(n)$

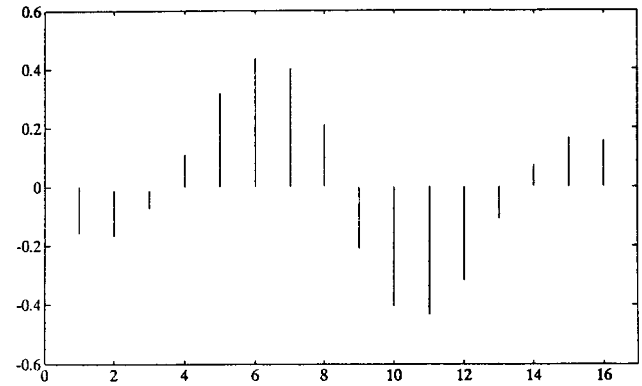

Fig. 2b. Impulse response of analysis filter, $h_1(n)$

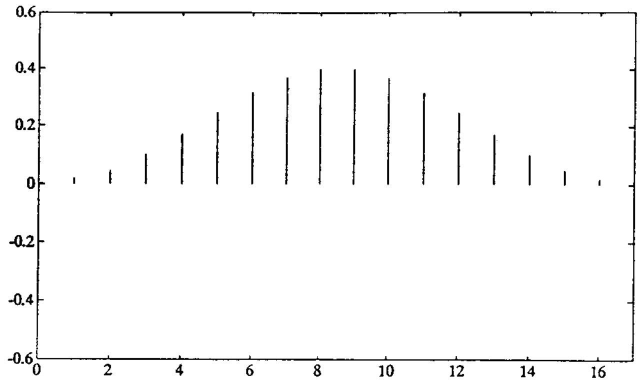

Fig. 2c. Impulse response of synthesis filter, $g_0(n)$

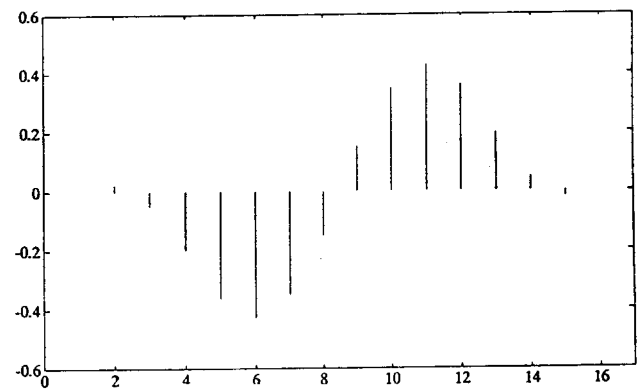

Fig. 2d. Impulse response of synthesis filter, $g_1(n)$

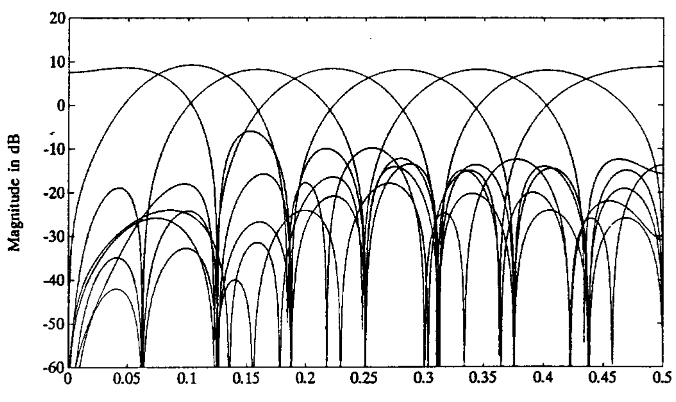

Fig. 3. Frequency response of analysis filters

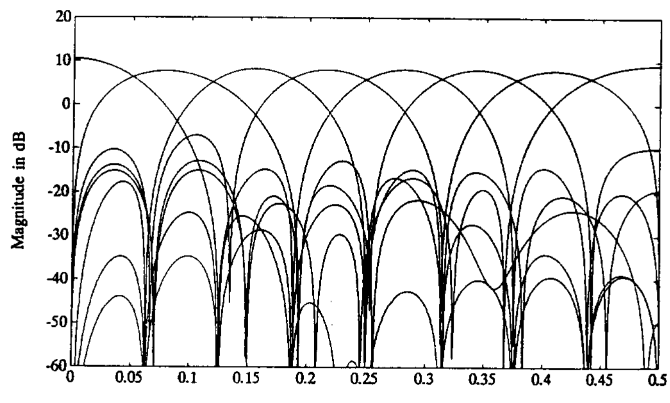

Figure 4. Frequency response of synthesis filters