RECOVERY WITH LOST SUBBAND DATA IN OVERCOMPLETE IMAGE CODING.

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ABSTRACT

This paper deals with the recovery of an image when some partially redundant subband data has been irreversibly corrupted. The approach presented assumes an overcomplete subband decomposition and uses a local Optimal Recovery estimator that works as a block processing scheme. The method takes advantage of the inherent correlation and redundancy among the remaining uncorrupted subband data. We view this approach as an alternative or as a complementary method to be used with forward error correcting codes requiring only error detection capability. We test the performance using row/column separable processing on images.

1. INTRODUCTION

Subband Coding (SBC) of signals and images remains an important area of research related to multirate digital signal processing. There is a need to develop new *lossy* data compression schemes capable of competing or surpassing block transform methods (like the DCT used in JPEG) which often present *blocking* effects and other disadvantages.

Most of the work in SBC has focused on using *Maximally Decimated (MD)* systems since the amount of samples in the subbands is the same as in the original signal. Furthermore, most of the work on filterbanks assumes ideal transmission of the subbands and no errors from quantization.

In order to protect the subband data from losses during transmission, Forward Error Correcting (FEC) codes can be used. This implies the addition of redundant bits which reduces the amount of compression achieved with the SBC scheme. Nevertheless, there are instances where the noise is so high that the FEC decoder will fail to recover the correct binary sequence. Another possible source of data loss may be caused by buffer overflow due to processing delay at the receiver. In both cases, once the data is lost there is no way to recover it and the reconstructed signal will be distorted. The distortion will be more noticeable if data is lost in the low frequency subbands. An approach to this problem is to replace with zeros the corrupted samples. This is usually enough for the case of high frequency subbands, but it may lead to unacceptable reconstruction error for losses in the low frequency subbands.

In this article, we propose to add information redundancy so that the effect of data losses during transmission is minimized when an optimal signal recovery algorithm is used. The scheme consists of the use of Overcomplete (OVC) subband decompositions with an optimal linear estimation procedure. An OVC system is not an MD system since the number of channels N is greater than the decimation rate M, leading to the presence of extra information in the subbands. A method to design FIR synthesis filters for any M and N using Optimal Recovery (OR) theory has been obtained in [1] and [2]. With this design technique, we can always obtain the best constrained-length FIR synthesis filterbank for any subband decomposition.

2. OVERCOMPLETE SUBBAND DECOMPOSITIONS

The redundancy in OVC subband decompositions offers all types of possibilities when assigning the spectral coverage for each subband. To see this, consider the MD system with M channels. MD systems typically split the spectrum into M equal bands with filters having essential bandwidth $\frac{\pi}{M}$. Thus for the case in which $N > M$, we have more channels than necessary to cover the range $[-\pi, \pi]$. This requires some strategy to assign the spectral coverage of the subbands which will allow us to take advantage of the redundancy.

Let's consider the simplest OVC system which would have $M = 2$, $N = 3$ as in Fig. (1). We note that a system with a similar structure is obtained from a 2-level dyadic decomposition which gives 3 channels. If we adopt this spectral subband assignment for the OVC system, we notice that the low frequency channels are not maximally decimated. Therefore, this type of system adds redundancy at the low frequencies by leaving these subbands oversampled. This is an appealing feature in SBC since we are protecting the low frequency information.

For the system with $M = 2$, $N = 3$, we show in Fig. (2) the magnitude responses of the three analysis filters to be used in this paper. Using the design technique developed in [1] and [2], we can obtain corresponding FIR synthesis filters that give very accurate reconstruction. The applica-

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tion of these analysis and synthesis filters to images is easily achieved using the row-column separable approach.

3. OPTIMAL RECOVERY FROM SUBBAND DATA

In this section we describe a signal reconstruction approach based on a locally optimal linear estimation procedure that can be easily modified to take into account missing data. Our problem consists of finding the signal $\tilde{x}(n)$ that best fits the given subband data over a short time *frame*.

We start by defining a set $\mathbf{Y}$ which is formed by all the subband data received at the synthesis system. Each element in the set is denoted by $y_{k,i}$ or $y_k(i)$, where k denotes the subband index and i denotes the already decimated time index (decimated by a factor M). The approach in this paper uses a subset of subband data to estimate only a small number of values for $x(n)$ at a time, see Fig. (3). First we assume that there is no missing data, but in the next section, we make the required modifications so that the procedure can operate optimally despite missing data.

We define the *representors* for the undecimated subbands as

$$\phi_k(n) = \mathcal{F}^{-1}\{H_k^*(\omega)\} = h_k^*(-n) \qquad (1)$$

which allows us to express the scalar $y_k(i)$ in the time domain as

$$y_k(i) = \sum_{m=-\infty}^{\infty} x(m)\phi_k^*(m - Mi) = \langle x(m), \phi(m - Mi)\rangle_m . \qquad (2)$$

There are an infinite number of input signals that can produce a given finite subset of subband data from $\mathbf{Y}$. However, the minimum norm solution $\hat{x}$ always gives the best linear estimate for a given set of data [3]. The expression for this solution is [4]:

$$\hat{x}(n) = \sum_{k=1}^{N} \sum_{j=\ell-L_1}^{\ell+L_2} w_{k,j}\phi_k(n - Mj), \qquad (3)$$

where the L_1 and L_2 bounds determine the extent (of size $L = L_1 + L_2 + 1$) of the time window that defines the actual subset of subband data used in this case. We will see that only M values are to be computed from each subset of subband data.

To find the $\{w_{k,j}\}$ values using the available subband data, we enforce the constraint that $\hat{x}(n)$ produces this data, therefore

$$\begin{aligned} y_q(i) &= \langle \hat{x}(m), \phi_q(m - Mi)\rangle_m \\ &= \textstyle\sum_{k=1}^{N} \sum_{j=\ell-L_1}^{\ell+L_2} w_{k,j} \left\langle \phi_k(m - Mj), \phi_q(m - Mi)\right\rangle \\ &= \textstyle\sum_{k=1}^{N} \sum_{j=\ell-L_1}^{\ell+L_2} w_{k,j} g_{q,k}(M(j - i)), \end{aligned} \qquad (4)$$

for $-L_1 \le i \le L_2$, and $1 \le q \le N$, where $g_{q,k}(m)$ is a value of a cross-correlation between two impulse responses.

For a particular subband coefficient subset, we can form a vector $\mathbf{y}_\ell$ of $y_{q,i}$ values and form NL linear equations like (4). Using matrix notation we get

$$\mathbf{y}_\ell = \mathbf{G}_\ell \cdot \mathbf{w}_\ell \qquad (5)$$

and thus $\mathbf{G}_\ell$ must be inverted. To make the estimator practical, the values of L_1 and L_2 should not be very large in order to keep the inversion of $\mathbf{G}_\ell$ from being computationally expensive. In this paper, we implement the estimator using a *frame by frame*, block processing procedure to estimate M samples of $x(n)$ at a time. That is, each time we select a subset of subband data, we calculate only one frame of estimated samples denoted $\mathbf{z}_\ell^T = [\ \tilde{x}(n_0) \quad \tilde{x}(n_0+1) \quad \ldots \quad \tilde{x}(n_0+M-1)\ ]$ where $n_0 = M\ell$. The next step is to obtain another subset of NL coefficients from $\mathbf{Y}$, and to increment ℓ. Then, the corresponding $\mathbf{G}_\ell$ matrix is formed and the vector of weights $\mathbf{w}_\ell$ is calculated so that a new $\mathbf{z}_\ell$ can be computed.

Going further, we see that Eq. (3) can be expressed in vector notation and rewritten using (5) in the following way for a fixed value of n:

$$\hat{x}(n) = \mathbf{s}_n^T \cdot \mathbf{w}_\ell = \mathbf{s}_n^T \cdot \mathbf{G}^{-1} \cdot \mathbf{y} \triangleq \mathbf{c}_n^T \cdot \mathbf{y}_\ell. \qquad (6)$$

Since a frame λ_ℓ is estimated linearly to obtain $\mathbf{z}_\ell$, an expression for this estimation is

$$\mathbf{z}_\ell = \mathbf{C}_\ell \cdot \mathbf{y}_\ell, \qquad (7)$$

where the $\mathbf{C}_\ell$ estimator matrix is obtained by stacking the M $\mathbf{c}_n^T$ vectors. Clearly, to recover other frames, we need to repeat the process and re-calculate the estimator matrix. In the case where no data is missing, it is easy to see that the estimator matrix will not change and thus $\mathbf{C}_\ell = \mathbf{C}$. From this we note that (7) can be identified as the polyphase filtering process commonly used in multirate systems, and that we can obtain FIR synthesis filter coefficients from $\mathbf{C}$ for arbitrary M and N [1], [2].

The selection of parameters L_1 and L_2 is made according to the desired accuracy. In the no data loss case, the estimator and OR filters use the same quantity of subband data at a time, where the filter length is $M(L_1 + L_2 + 1)$ [2].

4. SIGNAL RECOVERY WHEN SUBBAND DATA IS LOST

Suppose that for a given frame we discard in the vector $\mathbf{y}_\ell$ some of the samples corresponding to missing data. In terms of equation (5) this translates into discarding the corresponding rows and columns of $\mathbf{G}_\ell$ so that a square matrix is preserved. The corresponding vector $\mathbf{w}_\ell$ is reduced in size as well, but we still obtain the best possible frame estimation from the available data. Therefore, an effective way to deal with data corrupted by burst errors is simply to discard it in the above process.

We can use a minimum amount of channel coding in the subband data so that we can detect only the occurrence of errors at the receiver. Let's define a set D_ℓ of ordered pairs which indicate the subband and time indices of the samples that have been corrupted for the recovery of frame λ_ℓ. Thus, our estimator equations are slightly changed to

$$\hat{x}(n) = \sum_{\substack{k=1 \\ (k,j) \notin D_\ell}}^{N} \sum_{j=\ell-L_1}^{\ell+L_2} w_{k,j}\phi_k(n - Mj), \qquad (8)$$

and

$$\mathbf{y}_\ell = \mathbf{G}_\ell \cdot \mathbf{w}_\ell, \qquad y_{q,i} \notin \mathbf{y}_\ell \Leftarrow (q,i) \in D_\ell. \qquad (9)$$

As we can see, the size of the estimator matrix $\mathbf{G}_\ell$ will vary from frame to frame, depending on the number of corrupted samples discarded from the subset. The matrix inverse has to be calculated continuously for each frame which can be computationally expensive and time consuming. To avoid this problem, we could pre-compute a set of matrices and store them to be used as needed.

5. APPLICATION TO IMAGE TRANSMISSION USING SUBBAND CODING

In this section we present the advantages of the combination of OVC decompositions and the OR frame by frame estimator when these are used to code images which are transmitted over lossy channels. A version of this problem was considered in [5] for the MD case. We assume a very simple SBC transmission scheme and reliable error detection to identify the corrupted data. We concentrate only on burst errors along subband rows (consecutive subband samples are lost) since the scheme is expected to work better when the lost data is far apart.

We assume that separable SBC is done by filtering first along rows and then along columns. For simplicity, the resulting subbands are scanned row-wise (but the estimator can adapt to other scanning patterns [6]) and transmitted. At the receiver the process takes place in reversed order. The illustrated system with $N = 3$ and $M = 2$ uses the analysis filters with magnitude responses shown in Fig. (2). In order to reconstruct the columns where there is no lost data, as well as all the rows of the image, a set of synthesis filters for this OVC system were obtained using the procedure in [2].

A block diagram of a "hybrid" reconstruction procedure is shown in Fig. (4). The recovery process at the receiver starts by detecting blocks of samples where errors have occurred. Then the subbands are reassembled and processed column wise in two manners: error free columns go through the normal filtering process, while columns in which at least one of the subband samples presents error, use the OR block processing estimator introduced in the previous section. Finally, the rows are processed in the usual way using synthesis filters. Hence, we obtained a procedure that tolerates large burst errors, since transmission is done row wise, while recovery from error is done column wise.

As an example, we use a 512×512 version of Lena, and introduce two blocks of dimension 3×6 pixels on the LL subband around the left eye and nose, as shown in Fig. (5). For the OR estimator, we used $L_1 = 7$, $L_2 = 8$ while for the filters we used $L_1 = 0, L_2 = 8$. Note that in our procedure we could easily vary the level of accuracy of the estimator by adaptively changing the L_1 and L_2 parameters. The reconstructed images for different vertical dimensions for the error blocks are presented in Fig. 5. As expected, the recovery capability of the estimator is inversely proportional to the size of the error gap [7].

6. DISCUSSION

Information theoretic redundancy schemes like channel coding have a limited ability to correct *random* errors. Requiring only error detection capability, the procedure presented here is fitted to a lossy compression scheme where only a good approximation is the main goal. We note that obtaining the estimator for each frame is computationally intensive due to the matrix inversion in Eq. (5). However, these matrices can be pre-computed and stored so that they can be retrieved as needed.

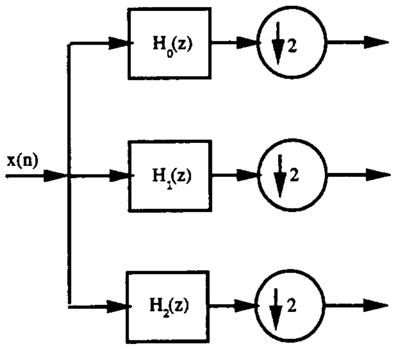

Figure 1: An Overcomplete (OVC) subband decomposition system.

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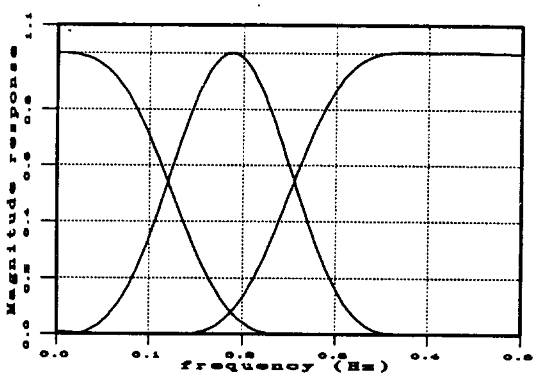

Figure 2: Frequency responses of the the 3 analysis filters suitable for OVC subband decomposition.

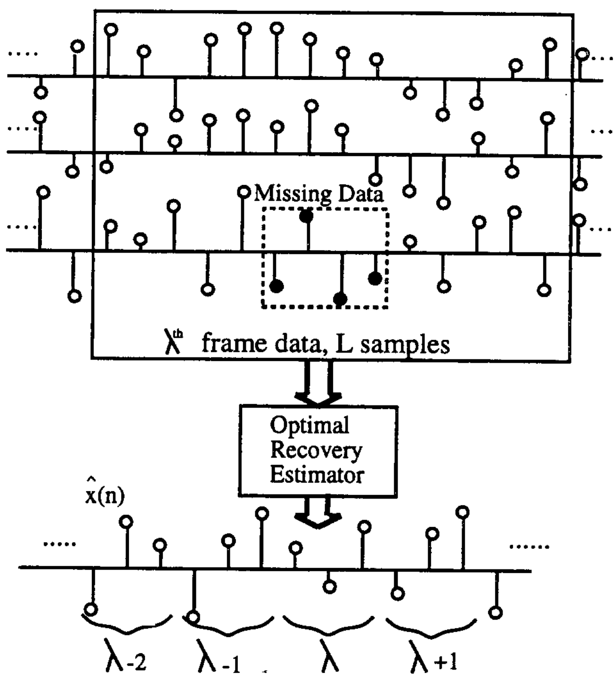

Figure 3: The frame by frame optimal recovery estimation procedure.

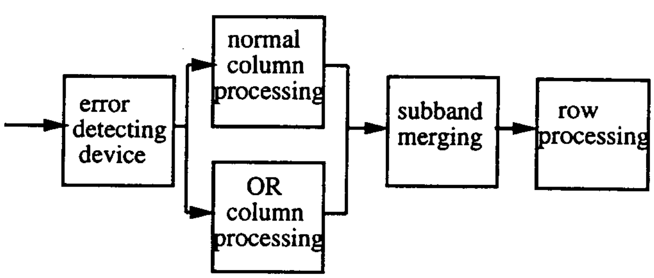

Figure 4: Use of OR estimator for reconstruction when errors are detected.

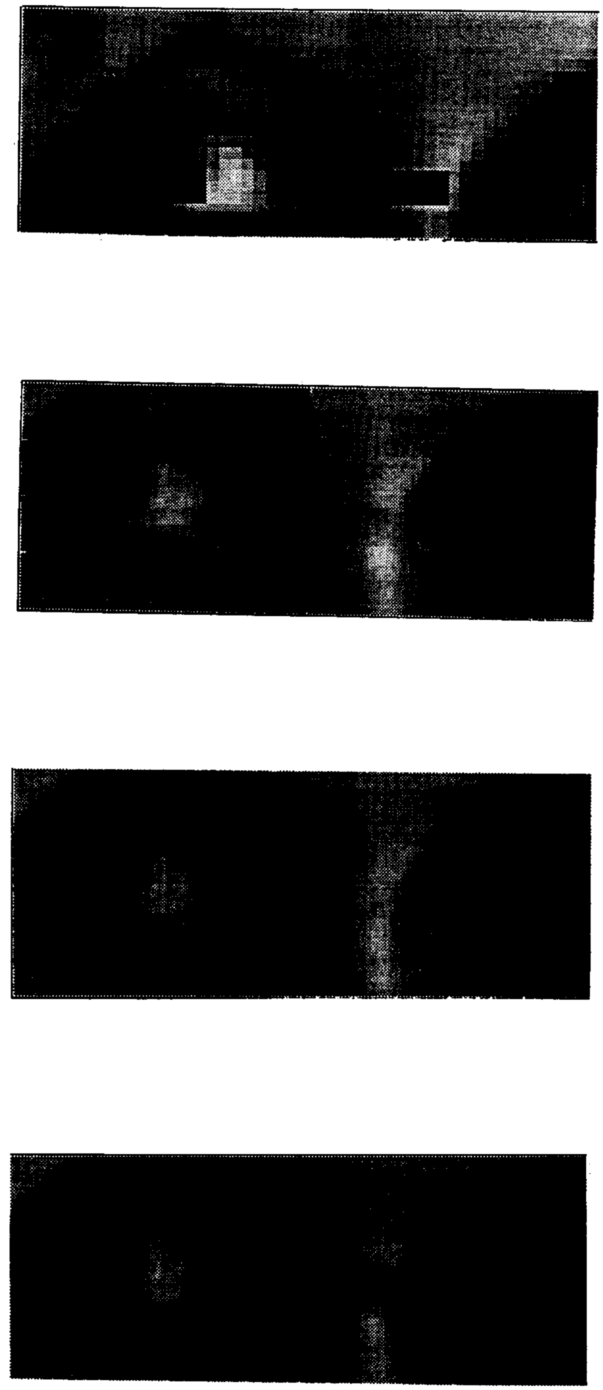

Figure 5: From top to bottom: the LL subband with the missing data (3 pixel vertical gaps); the reconstructed image with 3 pixel vertical gaps; the reconstructed image with 2 pixel vertical error; the reconstructed image with 4 pixel vertical error.