ESTIMATION OF NONSTATIONARY AR MODEL USING THE WEIGHTED RECURSIVE LEAST SQUARE ALGORITHM

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ABSTRACT

In this paper a new method of estimating time-varying AR models using weighted recursive least square algorithm with variable forgetting factor is described. The variable forgetting factor is adapted to a nonstationary signal by a generalized likelihood ratio algorithm through so-called discrimination function which gives good measure of nonstationarity. In this way we connect results from nonstationary signal estimation and jump detection area and obtain algorithm which exhibits good tracking performance together with parameter estimation accuracy. The feasibility of the approach is demonstrated with both simulation data and real speech signals.

1. INTRODUCTION

We propose a new method of estimating time-varying auto-regressive (AR) signal model based on weighted recursive least squares algorithm with a variable forgetting factor (WRLS-VFF). The strategy of choosing forgetting factor have long been recognized as the most important stage of estimation procedures for time varying signals [3]. The most frequently, forgetting factor is chosen in a heuristic way, based on fact that its small value correspond to small memory of whole estimation scheme, and vice verse [4]. Therefore, for the stationary or near stationary parts of the signal the forgetting factor (FF) λ should be near unity, allowing the adaptive algorithm to use most of the previous information in the signal, and, for the nonstationary or transient parts of the signal a small λ will shortening the effective memory length of the estimation process.

We propose a new scheme for λ, based on the fact that λ must be time-varying within intervals $[\lambda_{min}, \lambda_{max}]$ and at each step proportional to the degree of nonstationarity of the signal. The degree of nonstationarity we obtain from so called discrimination function of modified generalized likelihood ratio (MGLR) algorithm [2], developed for automatic detection of abrupt changes in stationarity of signal.

In order to get insight into algorithm performance we compare it with Cho and all. algorithm [1]. Results show better adaptability to the nonstationary parts of tested signal, and gives lower bias and lower variance at the stationary signal segments. Eventually, we show on real speech signal example, that our technique can accurately estimate and track time-varying AR parameters of natural speech, which illustrate variety of possible applications - from speech coding and recognition, image analysis, failure detection in measurement and control to seismology.

2. ALGORITHM DESCRIPTION

A nonstationary AR process $y(t)$ under consideration is represented by the following recursive equation

$$y(t) = -\sum_{i=1}^{p} a_i(t)y(t-i) + u(t) \qquad (1)$$

where $u(t)$ is a zero mean stationary white noise process, with variance σ_e^2, and time-varying parameters $\{a_i(t), i = 1, \ldots, p\}$. Under local stationary conditions (1) can be rewritten in the form of ordinary linear regression

$$y(t) = Z^T(t)\theta + u(t) \qquad (2)$$

where $Z^T(t)$ is the observation vector $Z^T(t) = [-y(t-1) \cdots -y(t-p)]$ and θ^T is the vector of AR parameters $\theta^T = [a_1 \cdots a_p]$.

WRLS approach requires finding the set of AR parameters minimizing the weighted cumulative squared error $\epsilon(t)$,

$$J_k(\hat{\theta}) = \frac{1}{k}\sum_{t=1}^{k} \lambda^{k-t}|\epsilon(t)|^2 \qquad (3)$$

which gives estimation scheme [3]:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\epsilon(t), \qquad (4)$$

This work was supported by Ministry of Sciences and Technology of Republic Serbia, project No.0403, through Institute of Mathematics SANU, Belgrade and project 1007 through Faculty of Electrical Engineering, Belgrade

$$K(t) = \frac{P(t-1)Z(t)}{\lambda + Z^T(t)P(t-1)Z(t)}, \quad (5)$$

$$P(t) = \frac{1}{\lambda}[P(t-1) - K(t)Z^T(t)P(t-1)], \quad (6)$$

$$\epsilon(t) = y(t) - Z^T(t)\hat{\theta}(t-1). \quad (7)$$

The FF λ, where $0 < \lambda \leq 1$, is a data weighting factor that may be used to weight recent data more heavily in a WRLS computation, and thus to permit tracking slowly varying signal parameters. The speed of adaptation is determined by the asymptotic memory length

$$N_m = \frac{1}{1-\lambda}, \quad (8)$$

which implies that the information dies away with N_m. If a stationary signal is composed of subsignals with different memory lengths varying between a minimum value N_{min} and a maximum value N_{max}, the AR parameters of the signal model can be estimated by using the FF λ varying between λ_{min} and λ_{max} to each subsignal. MGLR algorithm provides for localization of the boundaries of stationary parts of a signal, or, equivalently, for detecting the instants of abrupt changes of the signal stationarity [2]. Namely, MGLR algorithm uses three models of the same structure and order, whose parameters are estimated on fixed length intervals (windows) of signals. Assume that the so-called reference and test windows both have length I, while the third one is their union and has length $2I$. Reference, test and union windows cover the intervals $[i-I+1, i]$, $[i+1, i+I]$ and $[i-I+1, i+I]$, respectively, and move one sample forward with each new sample. The first step of the MGLR algorithm is a calculation of the discrimination function

$$D(i, I) = L(i-I+1, i+I) - L(i-I+1, i) - L(i+1, i+I) \quad (9)$$

where

$$L(c, d) = (d - c + 1)\ln\left\{\frac{1}{d-c+1}\sum_{j=c}^{d}\epsilon_j^2\right\} \quad (10)$$

denotes the maximum of the logarithmic likelihood function calculated on the basis of the model estimated from interval (c, d). Sequence $\{\epsilon_j\}$ in (10) is the residual (7) of the estimated AR model. It is easy to see that (9) is GLR for hypothesis that change in signal model is occured at the instant i against hypothesis that signal remain unchanged on the interval $[i-I+1, i+I]$. The basic advantage of the MGLR algorithm relates to the discrimination function (9) which allows us to perform a posteriori analysis since it appears in a closed form independent of the previously detected changes. Generaly speaking, two major factors influence the value of the discrimination function: how fast the signal is changed and how large the magnitude of a change is. Therefore a strategy for choosing the VFF may now be defined by letting $\lambda = \lambda_{max}$ when $D = D_{min}$ and $\lambda = \lambda_{min}$ when $D = D_{max}$, as well as by taking the linear interpolation between these values. In this way, at the beginning one has to adopt a priori D_{min} and D_{max}. It was found experimentally that $D_{min} = 0$ and $D_{max} = 100$ give satisfactory results, provided 12 bits A/D conversion [5], [6] . In addition, one has to set up D_{max} values during the WRLS algorithm iterations, whenever the calculated D-function becomes greater than the current D_{max} value. In such situations one has to replace D_{max} with the calculated $D(i, I)$ value in (9), with i being the current iteration number, and to use this new D_{max} in determining the VFF λ. Moreover, it is found experimentally that $\lambda_{min} = 0.95$ and $\lambda_{max} = 0.99$ give reasonably good values in practice.

3. EXPERIMENTAL RESULTS

To demonstrate main features of proposed algorithm we apply it to test signals consisting of sinusoids with time-varying frequency, and different SNR. Note that the same test signal is used by Cho and all. in [1]. Fig. 1 shows the true variation of the frequency of test signal 1 (dashed line) and estimated time-frequency trajectory obtained by FFF $\lambda = 1$ and $\lambda = 0.75$ for SNR = 10 dB. Booth results exhibits poor tracking ability for fixed FF. For $\lambda = 1$ adaptation time is very long and for $\lambda = 0.75$ veriance of the estimation is large. For this reasons we applied algorithm with VFF where based on the signal stationarity characteristics, corresponding FF is generated.

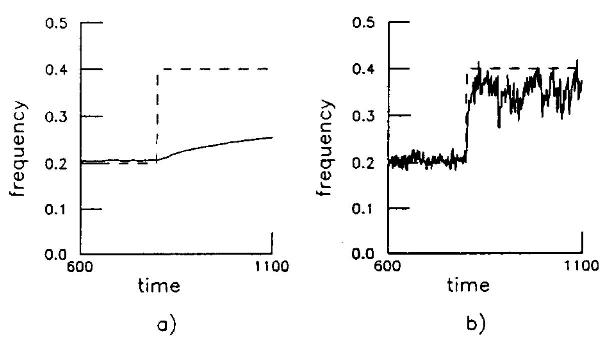

Fig. 1: Results for test signal 1 (SNR = 10 dB) by FFF λ **(a)** $\lambda = 1$; **(b)** $\lambda = 0.75$

Fig. 2 shows the true variation of the frequency of test signal 1 (dashed line) and estimated time-frequency trajectory obtained by method of Cho at all. and by our algorithm for SNR = 10 dB. Algorithm proposed in [1] gives good VFF for simple cases. Besides, for SNR < 20 dB it gives poor results. Namely, VFF is generated on the basis of only last five samples of the residuals, and in presence of high level noise, errors appear. We compared our algorithm with algorithm from [1] for SNR = 10 dB. VFF obtained with our algorithm gives significantly better results than one from [1]. In Fig. 3 the results for test signal 2 is presented. In

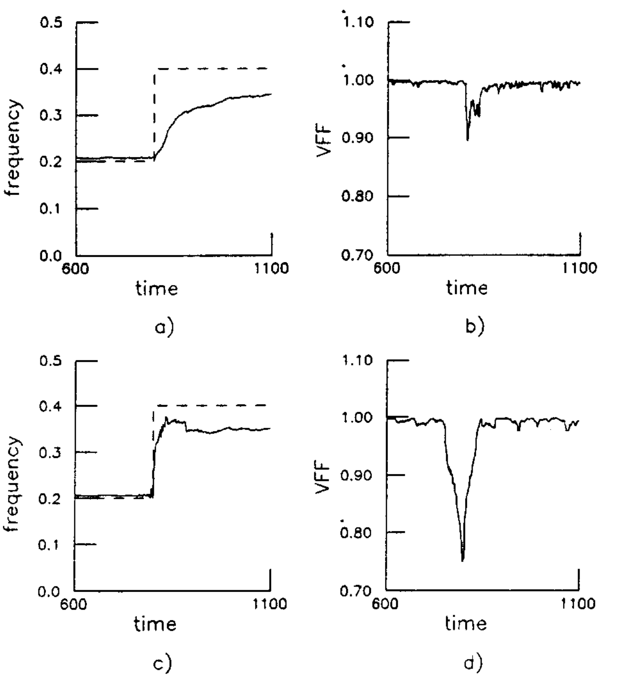

Fig. 2: Results for test signal 1 (SNR = 10 dB): **(a)** time-frequency representation, Cho and all. WRLS and corresponding VFF **(b)**; **(c)** time-frequency representation, WRLS-MGLR and corresponding VFF **(d)**

the case of a natural human speech, the true values of the vocal tract parameters are unknown. However, the AR parameter estimates obtained by the non-recursive LS algorithm with a sliding window shorter than the pitch period were used as the most accurate or reference parameter estimates [7]. In order to compare performance of the algorithms at the non-stationary parts of a speech signal, ten isolately digits (i.e. 0 - 9) are used. All signals were low-pass filtered with an upper limit frequency $F_g = 4kHz$, and digitized by 12 bit A/D conversion with a sampling rate of 10 kHz. In addition, the preemphasis of the speech signal was also performed. Fig. 4 depicts the trajectory of AR parameter a_1 obtained from the Serbian digit 'sedam' (i.e. '7'), which is characteristic for the algorithm's behavior. The results show that WRLS reduces the influence of voice excitation periodicity on the parameter estimation, producing less estimate bias and less estimate variability over time. On the other hand, standard LS analysis gives greater estimate bias, as well as more local estimate variations, due to the high sensitivity to the pulse excitation during voiced portion of

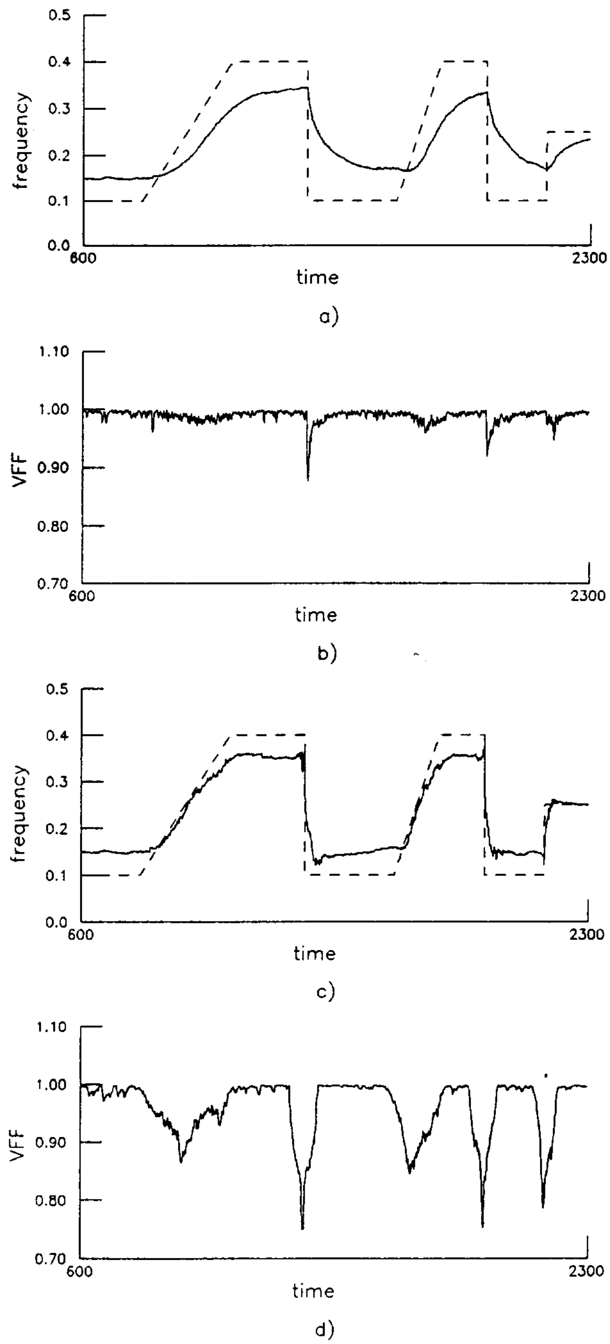

Fig. 3: Results for test signal 2 (SNR = 10 dB): **(a)** time-frequency representation, Cho and all. WRLS and corresponding VFF **(b)**; **(c)** time-frequency representation, WRLS-MGLR and corresponding VFF **(d)**

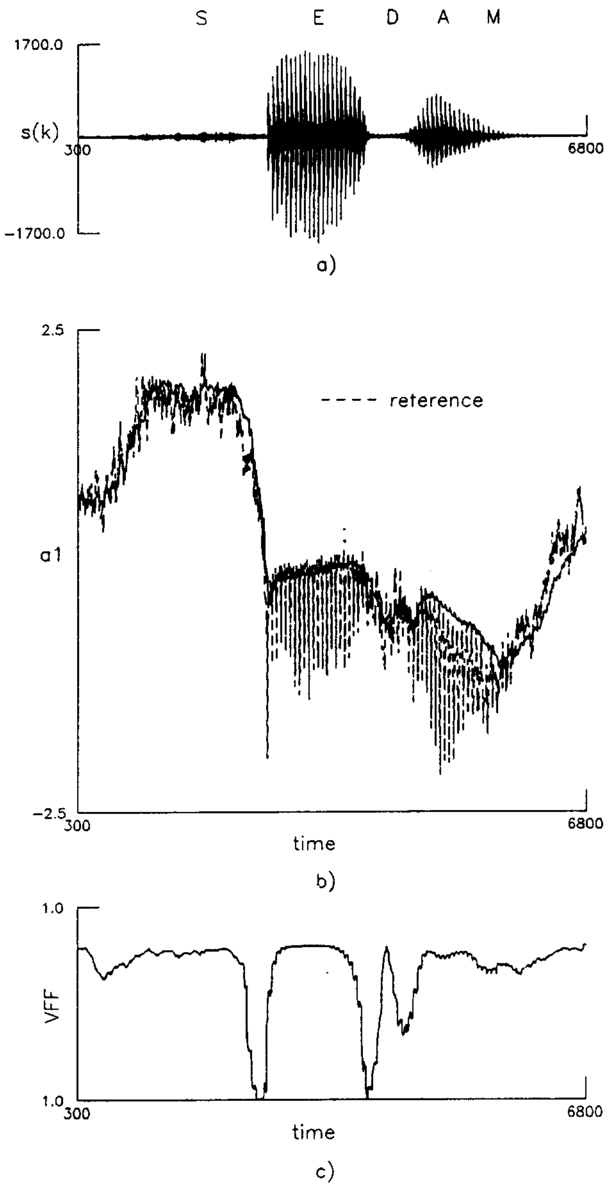

Fig. 4: Results for real data. Performance of WRLS MGLR algorithm on the isolately Serbian digit 'sedam' ('7'): (a) isolated digit after preemphasis; (c) comparison of reference trajectory with WRLS estimates based on VFF λ; (c) VFF λ

speech. Good behavior of WRLS steems from accurate nonstationary tracking ability of scheme for FF assesement based on MGLR discrimination function. It is not surprising, because MGLR algorithm is approved to be one of the best procedures for nonstationary signal segmentation [2].

4. CONCLUSION

In this paper, a method for estimation of the nonstationary AR model by the weighted recursive least square algorithm with the variable forgetting factor (WRLS-VFF) is described. A new VFF generating procedure is proposed, which is based on calculating of the discrimination MGLR-algorithm function [2]. The generated VFF ensures good adaptability of the WRLS algorithm in the nonstationary situations, as well as small variance of the estimations in the stationary cases.

5. REFERENCES

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