TIME-VARYING ORTHOGONAL FILTER BANKS WITHOUT TRANSIENT FILTERS

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ABSTRACT

We present a solution for the construction of orthogonal time-varying filter banks without transient filters. To reach this result the idea is the following: all the various filter banks used in the time-varying decomposition are not arbitrary, but are linked together and in fact are derived from an unique initial orthogonal filter bank. With this new technique, the PR property is always guaranteed even if we switch abruptly from one filter bank to an other without the use of transient filters.

We will explain, by taking an initial M-band orthogonal filter bank which performs a regular M-band frequency splitting, how to derive various mutually orthogonal filter banks with almost any arbitrary time/frequency resolution, even able to perform irregular frequency splitting like for example in a wavelet decomposition.

I. INTRODUCTION

Perfect Reconstruction subband transforms and more particularly orthogonal filter banks are used as general tools for many applications like coding of speech, audio and video, enabling HDTV/TV compatibility, image format conversion, performing adaptive filtering...

The various families of orthogonal transforms are now rather well studied and understood. In particular the cases of orthogonal 2-band filter banks linked with orthogonal wavelets and wavelets packets and the case of modulated M-band Orthogonal Transforms are widely used and described [1,2,3,4,5,6,7]. For example in [6] all the well known 2-band orthogonal solutions and M-band modulated solutions are merged into an unique formalism and theory, the Modulated Orthogonal Transforms.

A lot of research has been done to find the "optimum" filter bank, the "optimum" decomposition. But more and more it is thought that these optima are signal dependant. For example in the image coding field, we can try for each image to select the best wavelet packet decomposition according to an energy criterion or any other criterion. Nevertheless the content of one image is highly variable and the statistics of one part of an image can be very different to the statistics of another part of the same image. To match these changes in the image content, we can think about using time-varying filter banks. This means that in the analysis part there are some commutations and that various filter banks with different characteristics are used. But most of the time-varying filter banks approaches suffer from some defects: Even if each filter bank itself is orthogonal and then has the Perfect Reconstruction property, if we switch from one orthogonal filter bank to another in the analysis process and identically in the synthesis process, generally the Perfect Reconstruction property is lost. To recover the PR property one solution is to apply transient filters at the analysis or the synthesis stage to overcome the problem occurring in the transition area [7,8]. In that case the frequency characteristics of these transient filters can not really be controlled.

In this paper we propose a different way of performing orthogonal time-varying filter banks without transient filters, while maintaining by construction the PR property. Primarily the idea behind this paper is derived from [5,6].

II. THE TECHNIQUE

The idea to obtain the result is: All the various filter banks used in the time-varying decomposition are not arbitrary, but are linked together and in fact are derived from an unique initial orthogonal filter bank. For example by taking an initial M-band orthogonal filter bank which performs a regular M-band frequency splitting, we can derive various orthogonal filter banks but with almost arbitrary time/frequency resolution, as for example the irregular frequency splitting of a wavelet decomposition.

To understand the process, we have to make clearly the difference between the orthogonality property (which induces the PR property) of the filter bank, what we call M, i.e. the number M of channels and the time/frequency resolution of the filter bank. For example we can have a 8-band filter bank but with the time/frequency resolution of a classical 4 band filter bank as shown in a trivial example in Fig 1.

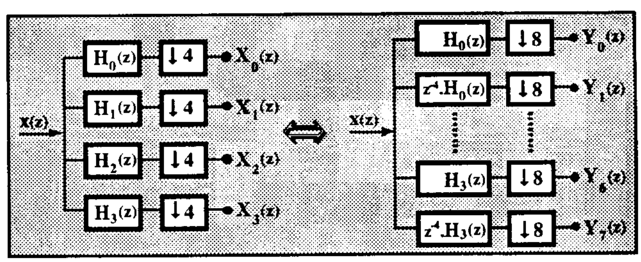

Fig 1: The same time/frequency decomposition but seen as a 4-band filter bank on the left side and a 8-band filter bank on the right side

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II.1 First approach

To explain briefly how to reach the result we will take the formalism of orthonormal basis rather than the classical signal processing approach with z-transform. (**Remark.** Normally there is a difference between the scalar product and the convolution product, nevertheless for simplicity reasons we do not make here this difference).

For a M-band normalised orthogonal filter bank, the M analysis filters $h_k(n)$ have to verify:

$$\forall k,l \in [0,M-1],\ \forall m,p \in \mathbb{Z},\ \langle h_{k,p}(n), h_{l,m}(n) \rangle = \delta_{k-l} \cdot \delta_{m-p}$$

<.,.> means the scalar product, $\delta_i=1$ if $i=0$ and 0 either, and we define $h_{k,p}(n)=h_k(n-pM)$.

This means that the filters and their shifted versions by a multiple of M are orthogonal each other. We can say that the filters $h_{k,p}(n)=h_k(n-pM)$ form an orthonormal basis, and that in fact the M-band filter bank performs an orthonormal basis change, a change of "coordinates".

In general we suppose that the initial filters $h_k(n)$ split regularly the frequency plane in M-bands and a lot of solutions exist to do so [1,2,3,4,5,6]. (see Fig 3a)

Now let us take U_p, an arbitrary unitary transform of size M, and let define $H_p(n)$ and $G_p(n)$ by:

$$H_p(n) = [h_{0,p}(n), h_{1,p}(n), \ldots, h_{M-1,p}(n)]^t$$

$$G_p(n) = U_p.H_p(n) = [g_{0,p}(n), g_{1,p}(n), \ldots, g_{M-1,p}(n)]^t$$

$H_p(n)$ are the set of analysis filters normally applied at position p. $G_p(n)$ will be an alternate set of filters to apply at position p. Because U_p is unitary, we can easily verify that:

$$\langle g_{k,p}(n), h_{l,m}(n) \rangle = 0, \text{ if } p \neq m$$

$$\langle g_{k,p}(n), g_{l,p}(n) \rangle = \delta_{k-l}$$

This means that if at position p, the filters $h_{k,p}(n)$ are replaced by $g_{k,p}(n)$ we still have a valid orthonormal basis change, and we still then have the Perfect Reconstruction property if at the synthesis process we perform the same change of filters at the same position. What was done only at one position p can be done for each position p by applying at each position p an unitary transform U_p.

With this idea if we apply the same unitary transform U at all positions p, we built a complete new orthogonal filter bank $g_{k,p}(n)$ starting from $h_{k,p}(n)$, and by doing so I will show in chapter III. that the $g_{k,p}(n)$ filter bank can simulate a lot of various time/frequency decompositions as for example wavelet packets.

To go further, by taking any arbitrary unitary transforms U_p of size M with $p \in \mathbb{Z}$, the filters $G_p(n)$ defined by $G_p(n) = U_p.H_p(n) = [g_{0,p}(n), g_{1,p}(n), \ldots, g_{M-1,p}(n)]^t$ will generate a valid orthonormal basis change. This means that the unitary transform U_p can change from one position p to an other.

> We obtain then a time-varying filter bank by taking different unitary transforms according to positions p. This time-varying filter bank does not need transient filters and the PR property is always verified by construction.

II.2 Simple implementation

This process is rather simple to realise. First perform the fixed original $h_{k,p}(n)$ filter bank to obtain the transformed coefficients $X_k(p)=<x(n),h_{k,p}(n)>$.

Now if you wanted in fact to apply at position p the filters $G_p(n) = U_p.H_p(n) = [g_{0,p}(n), g_{1,p}(n), \ldots, g_{M-1,p}(n)]^t$ instead of the initial filters $h_{k,p}(n)$ to obtain the transformed coefficients $Y_k(p)=<x(n),g_{k,p}(n)>$, the only thing to do is to apply the unitary transform U_p in the subband domain on values $X_k(p)$.

$$X(p) = [X_0(n), X_1(n), \ldots X_{M-1}(n)]^t$$

$$Y(p) = U_p.X(p) = [Y_0(n), Y_1(n), \ldots Y_{M-1}(n)]^t$$

This can then be seen as a reversible post-processing to be done in the subband domain.

II.3 Generalisation of the technique

In this example the unitary transform U_p is applied on subband coefficients $X_k(p)$, ie at the same spatial location, but this technique and the orthogonality of the resulting filters is also valid if an unitary transform of any arbitrary size N (not related to M) is applied to an arbitrary set of N filters from the possible $h_{k,p}(n)$ initial set. We can even apply different unitary transforms of different sizes on different initial sets, the orthogonality of the resulting filters is always guaranteed.

Furthermore we derived our explanation by taking as initial filter bank an orthogonal M-band solution with the same sub-sampling factor M for each filter $h_k(n)$. Nevertheless this techniques is also valid if the initial orthogonal filter bank $h_k(n)$ is for example a wavelet decomposition or a packet wavelet decomposition. In that case the sub-sampling factor M_k can be different for each filter $h_k(n)$, and the shifted versions of $h_k(n)$ are then $h_{k,p}(n)=h_k(n-pM_k)$.

Up to now, the explanation was performed on 1D filter banks, but obviously this can be generalised to multi-dimensionnal filter banks, and for example to 2D filter banks for image coding applications.

III. APPLICATIONS

III.1 Usefulness and practical use

Now we know the technique to modify an initial filter bank $h_{k,p}(n)$ to generate various $g_{k,p}(n)$ filters. Then we now how to build this kind of time-varying filters banks without the need of transient filters. But are we sure that we can simulate then almost any time/frequency resolution? Is this possibility useful?

The answer is yes, and generally we do not even have to choose complicated unitary transforms to do so. In a first approximation simple additions and subtractions (a simple butterfly) can most of the time be sufficient!

For example with M=8, a Perfect Reconstruction Cosine Modulated Filter bank solution with filter length 15 [4,5] gives filters $h_k(n)$. We can derive a 8-band filter bank with the time/frequency resolution of a classical 4-band filter bank by this simple following unitary transformation:

$$\boxed{G_p(n) = U.H_p(n)}$$

$$\text{with } U = \begin{pmatrix} H_2 & 0 & \cdots & 0 \\ 0 & H_2 & 0 & \vdots \\ \vdots & 0 & H_2 & 0 \\ 0 & \cdots & 0 & H_2 \end{pmatrix} \text{ and } H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

This is equivalent to say that filters $g_k(n)$ are given by:

$$\boxed{g_{2k}(n) = 1/\sqrt{2}\,[h_{2k}(n)+h_{2k+1}(n)]}$$

$$\boxed{g_{2k+1}(n) = 1/\sqrt{2}\,[h_{2k}(n)-h_{2k+1}(n)]}$$

Figure 2 gives the time and frequency responses of initial filters h_0, h_1 and the modified filters g_0, g_1.

It is easy to see that g_0 and g_1 are two times less localised in the frequency domain than h_0 and h_1 but are two times more localised in the time domain.

Remark: this new filter bank is not a "true" 4-band filter bank but the time/frequency resolution it performs is similar to the one of a "normal" 4-band filter bank. In practice, tested on synthetic images (like the zoneplate image) and natural images, this "false" 4-band filter bank solution is as efficient in term of frequency selectivity, PSNR and subjective quality than a classical 4-band solution with filters of similar number of taps.

For a more complicated and useful example we can try to simulate with a regular 8-band modulated filter bank the time/frequency resolution of the wavelet decomposition which goes up to level 4, as in fig 3.c

To do such a decomposition in a first and simple stage we can apply:

$$\boxed{G_p(n) = U.H_p(n)}$$

$$U = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \vdots \\ \vdots & 0 & H_2 & 0 \\ 0 & \cdots & 0 & H_4 \end{pmatrix}, H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, H_4 = \frac{1}{\sqrt{2}}\begin{pmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{pmatrix}$$

U_p to now the $g_{k,p}(n)$ filters have the time-frequency resolution as in Fig 3b. To obtain the decomposition as in Fig 3c we can modify the filters $g_{k,p}(n)$ to obtain the resulting filters $f_{k,p}(n)$ by:

$$\boxed{f_{k,p}(n) = g_{k,p}(n)} \quad \text{for } k\neq 0$$

$$\boxed{[f_{0,2p}(n), f_{0,2p+1}(n)]^t = H_2.[g_{0,2p}(n), g_{0,2p+1}(n)]^t} \quad k=0$$

III.2 Time Varying solutions

In the CCETT (our company, a France Telecom research center) we are beginning to experiment the technique developed in this paper to built Orthogonal Time Varying solutions applied to image coding. The idea beyond that is: For homogeneous areas we should be much selective in the frequency domain but less selective in the time domain, so that the mean value of a large area is only represented by one transformed coefficient (example: going from a 8*8 to a 16*16 like decomposition). For edges areas on the contrary we should have selective filters in the time domain so that few filters will have a strong correlation with the edge and the quantization error will be located very close to the edge (example: going from a 8*8 to a 4*4 like decomposition). For textured areas an intermediate solution should be applied as the one in Fig 4.

This selection can be done in the X direction, the Y direction, both direction and on a field or frame basis if the images are interlaced (see [5]). Up to now we developed a simple solution with around 8 possible candidate unitary transforms applied to the same sets of filters and selected according to a simple criterion. As the filter banks are orthogonal, the L2 norm is always kept whatever the candidate unitary transform selected. Consequently the L1 norm gives a simple way of knowing if the energy is concentrated on few transformed coefficients or highly spread in the transformed domain. We select then the unitary transform which gives the best energy concentration or equivalently the smallest L1 norm. Even in its immature stage this solution gives already some interesting results (0.5-1.0 dB PSNR improvement compared with classical 8*8 solution [5] for the same bitrate) but it is to be refined much further with more tuned unitary transforms, a better criterion, a optimised initial filter bank... For example instead of choosing predefined unitary transforms we could have a LBG like approach to built a dictionary of candidate unitary transforms.

CONCLUSION

We developed a simple and practical technique to design orthogonal Time Varying Filter Banks. In this approach, we do not have to care about Perfect Reconstruction problems. PR is always guaranteed. We do not have to calculate transient filters because we do not have transient filters. This approach can be done in a very efficient hardware saving manner. With this technique and an initial regular M-band filter bank, like the efficient M-band modulated solutions developed in [4,5,6], we can simulate almost any time/frequency resolution decomposition, and for example the irregular frequency decomposition of a wavelet or wavelet packet transform as in fig. 3 or 4

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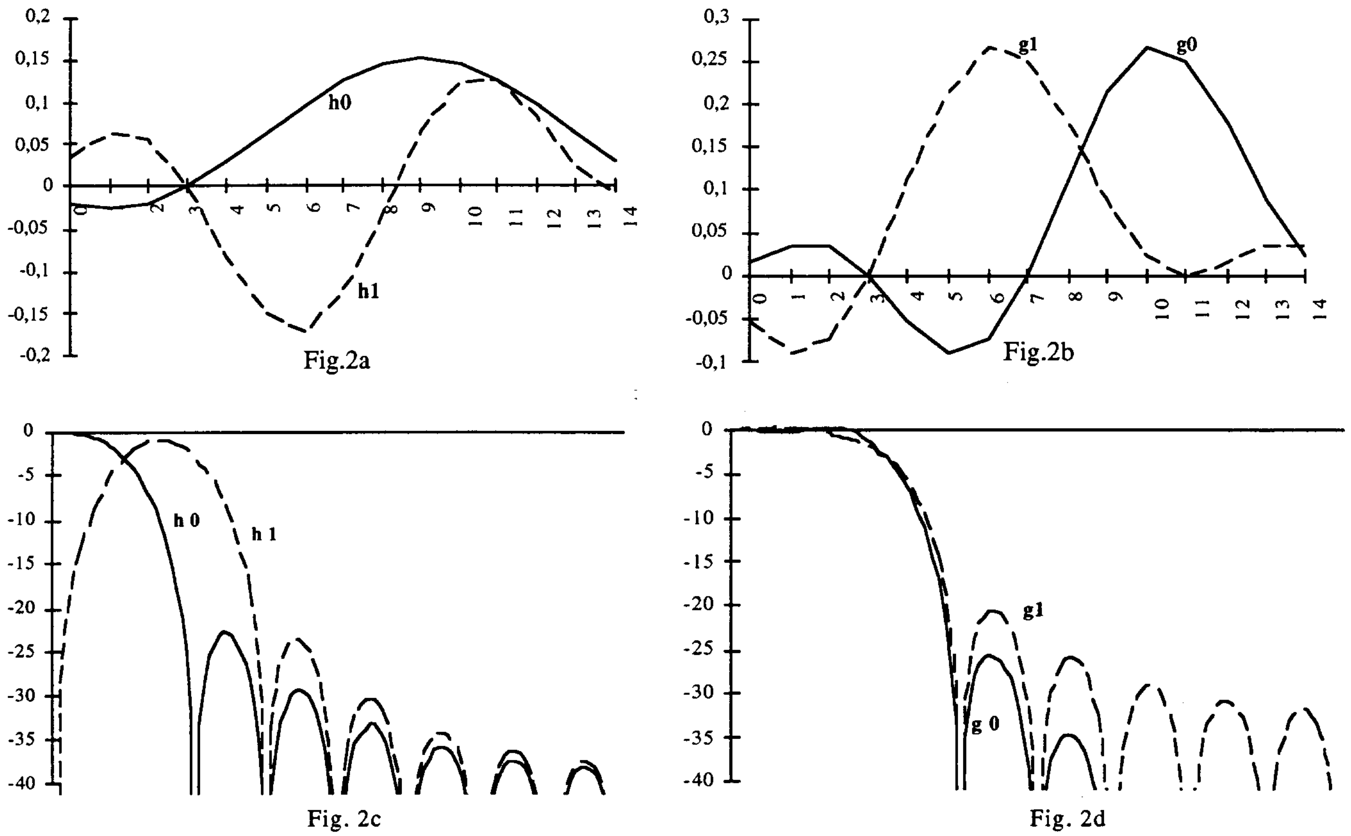

Fig2: Time responses (Fig. 2a and 2b) and frequency responses (Fig 2c and 2d) of filters h_0, h_1, g_0 and g_1

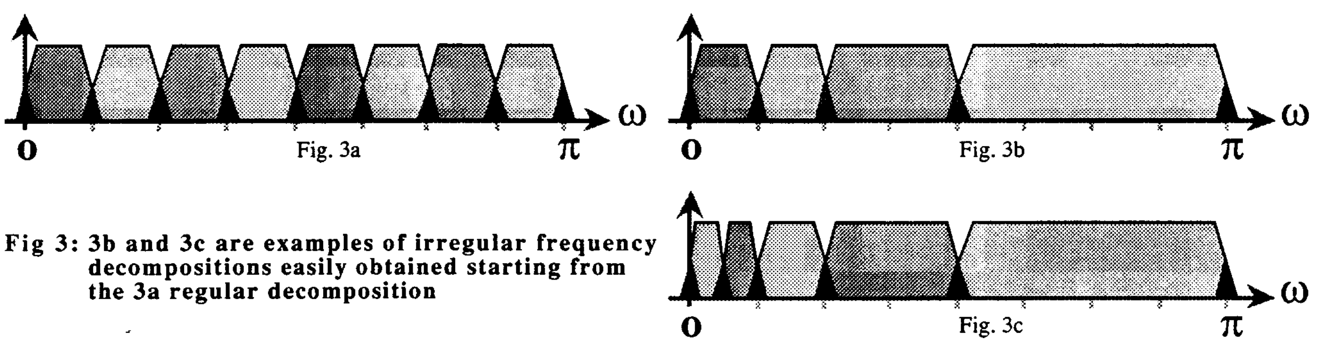

Fig 3: 3b and 3c are examples of irregular frequency decompositions easily obtained starting from the 3a regular decomposition

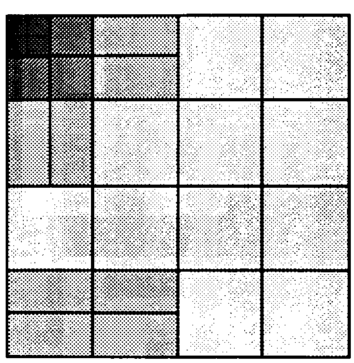

Fig 4: An irregular decomposition well suited for general interlaced image coding