TUNABLE DOWNSAMPLING USING FRACTIONAL DELAY FILTERS WITH APPLICATIONS TO DIGITAL TV TRANSMISSION

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ABSTRACT

An efficient technique for sampling rate conversion for arbitrary (incommensurate) ratios is proposed. The technique is based on fractional delay filters that are efficient to implement and that can be controlled with a small number of arithmetic operations per output sample. We consider an application in digital television (DTV) transmission where, according to present standard proposals, conversions between several incommensurate sampling rates must be possible. Rather than trying to design separate fixed filters for each possible conversion, we outline a system which may be tuned for any possible downsampling ratio. A sampling rate conversion system based on the straightforward and simple Lagrange interpolation technique is illustrated, with a novel and highly efficient implementation structure. Various error sources involved are analyzed and a mean-square-error (MSE) type cost function is defined to aid in the system design.

I. INTRODUCTION

Sampling rate conversion is needed in several areas of digital signal processing. Conversion by a rational factor is easily done with a proper lowpass filter and a preceding insertion (in upsampling or interpolation) and subsequent cancellation (in downsampling or decimation) of samples [1], [2]. The rational-factor sampling rate conversion can be implemented efficiently with a polyphase structure [3]. A polyphase implementation can be viewed as a periodically time-varying linear filter, i.e., its transfer function is different at contiguous sampling instants, but there is a finite number of transfer functions that follow each other periodically.

However, the sampling rate alteration by an irrational factor is a much more challenging problem. A brute-force device with an extremely high upsampling before downsampling was introduced in [4]. An approach involving interpolation techniques for filter coefficients was introduced in [5], however involving analog interpolation devices or computationally intensive coefficient interpolation techniques. In [6], a block transform method based on the chirp-Z transform was proposed, with the inevitable block delay that is not acceptable in our application.

Recently Tarczynski *et al.* [7] proposed the use of fractional delay (FD) filters for incommensurate sampling rate conversion. They used a two-stage structure where fixed IIR filters estimate the signal value at certain time instants between the samples. The second stage includes a low-order Lagrange interpolator that employs both the original signal samples and the results given by the IIR filters. However, the order of the IIR filters is fairly high, which increases the computational complexity.

In this paper, we elaborate on the FD approach and develop a system which is suitable for the proposed European digital TV standard [8]. First, we introduce the environment with the requirements for several downsampling ratios. Second, we consider the implementation of a tunable FD filter suitable for the downsampling application. Employing Lagrange interpolation in the Farrow structure [9] results in a novel technique with an efficient implementation.

The main contributions of this paper are the following. A fractional delay-based sampling rate conversion technique based on a novel implementation of Lagrange interpolation is introduced. Detailed error analysis of the sampling rate conversion system is presented. According to these preliminary results, the proposed technique is a promising candidate to receiver realizations for the new TV standard proposal [8].

II. DTV PARAMETERS AND REQUIREMENTS

The European approach for digital TV standard proposes a multirate system for satellite and cable distribution [8]. This means that, depending on the available satellite transponder bandwidth, the operator may choose his transmitted QPSK symbol rate quite freely. The receiver must be capable to recognize the symbol rate (and coding) used and to adapt itself to the signal format. This sets a demanding problem for carrier and timing extraction as well as for adaptive filtering at the receiver.

The baseband part of a receiver for satellite digital TV signals uses QPSK modulation. The IF signal is a bandlimited passband signal with center frequency depending on the choice of the manufacturer, e.g., 70 MHz. The QPSK symbol rate may vary between 15 MBd and 30 MBd and be unknown to the receiver. The IF signal is demodulated into in-phase and quadrature components using mixers and cos/sin signals as conventional. The double frequency terms are eliminated using relatively loose analog filters which have only minor effect in the desired lowpass signal content. The resulting lowpass signals are then A/D converted using five or six bits and a fixed clock frequency f_s, which is assumed to be at least twice the symbol rate. For the values above, this would suggest a minimum clock frequency of 60 MHz.

Signal shaping is made using a raised-cosine half Nyquist filter with a roll-off factor $\alpha = 0.35$. This filter must adapt itself to the symbol rate. One way to arrange this is to use an FIR filter with fixed coefficients but having the clock signal to be an exact multiple of (preferably twice) the symbol rate of the incoming signal (sampling frequency f_s). This adaptation can be achieved by using a fractional delay filter. Also a method to adjust the effective sampling time to its optimum for the detec-

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tion process is needed.

In the demodulation scheme outlined above there remain other nontrivial tasks to obtain the correct carrier phase and derivation of the symbol rate and timing. In this paper it is assumed that information about symbol rate and relative timing delay D is available. We confine ourselves to the study of only one branch (in-phase or quadrature) of the receiver.

In summary, the specifications for our sampling rate conversion system are: input sampling rate $f_s = 60$ MHz, output sampling rate $f_{out} = 30...60$ MHz, conversion factor $r = 0.5...1.0$, and approximation bandwidth $[0, \ f_p]$ where $f_p = (1+\alpha)f_s/4 = 0.675 f_s/2$ since $\alpha = 0.35$. Note that we have to define the bandwidth for the widest possible case, $f_{out} = f_s$, because we do not know it beforehand. The approximation bandwidth is the frequency region where all deviations from the ideal system should be minimized.

III. TIME-VARYING FD FILTER FOR SAMPLING RATE CONVERSION

A. Problem Formulation

A sampling rate converter for a rational ratio conversion factor is a periodically time-varying linear system. When the ratio of sampling rates is irrational, a polyphase structure would include an infinite number of branches. One approach is to sample a finite polyphase grid and use linear interpolation to obtain filters in between [10]. In the fractional delay (FD) approach, we use the interpretation that each polyphase branch approximates a delay that is a fraction of the original sampling period. For any conversion factor r the sampling rate conversion can be implemented via time-varying FD filters whose coefficients are updated for every output sample. Hence, the sampling rate conversion problem can be turned into a task of implementing a tunable FD filter at the original sample rate f_s.

Let us now formulate the problem in more detail. Consider the design of an FD filter for one output sample. The input signal is $x(n)$ and the output signal of the filter $h(n)$ is $y(n)$ which is desired to approximate the ideal delayed sequence $y_{id}(n) = x(n-D)$ where D is the noninteger (fractional) delay value. The output of the filter is obtained via convolution as

$$y(n) = x(n)*h(n) = \sum_{k=0}^{\infty} h(k)x(n-k) \qquad (1)$$

We define the output error as the mean square error between the true and ideal response as

$$E_y = E\left\{\left|y(n) - y_{id}(n)\right|^2\right\} = E\left\{\left|x(n)*\left[h(n) - h_{id}(n)\right]\right|^2\right\} \qquad (2)$$

where $E\{\}$ denotes the expectation value. This error function can be turned into frequency domain via the Parseval theorem, i.e.,

$$E_y = \frac{1}{\pi}\int_0^{\pi} S_{xx}(\omega)\left|H(e^{j\omega}) - H_{id}(e^{j\omega})\right|^2 d\omega \qquad (3)$$

where $S_{xx}(\omega)$ is the power spectrum of the input signal and $H(e^{j\omega})$ is the frequency response of the FD filter to be designed, and

$$H_{id}(e^{j\omega}) = e^{-j\omega D} \qquad (4)$$

is the ideal frequency response of the FD filter. Now we will use the assumption made in Section II that the input signal $x(n)$ is bandlimited in the range $[0, \ \omega_p] = [0, 2\pi f_p]$. Let us further assume that the power spectrum $S_{xx}(\omega) = \sigma_x^2\pi/\omega_p$ is constant where σ_x^2 is the input signal power. Hence, (3) simplifies to

$$E_y = \frac{\sigma_x^2}{\omega_p}\int_0^{\omega_p}\left|H(e^{j\omega}) - H_{id}(e^{j\omega})\right|^2 d\omega \qquad (5)$$

To make the error criterion independent of the input power, let us define the normalized output error as

$$E_{norm} = \frac{E_y}{\sigma_x^2} = \frac{1}{\omega_p}\int_0^{\omega_p}\left|H(e^{j\omega}) - H_{id}(e^{j\omega})\right|^2 d\omega \qquad (6)$$

This is the final error measure to be used in designing the fractional delay filter in our application.

B. FD Approximation Using Lagrange Interpolator

A number of techniques for fractional delay approximation were reviewed in [11]. A well-known strategy to design FIR FD filters is Lagrange interpolation, which is a maximally-flat approximation to ideal interpolation. It is particularly desirable because of the closed-form solution of the coefficients and because it is readily suited for efficient implementation with the Farrow structure [9]. The filter coefficients $h(n)$ are computed as [11]

$$h(n) = \prod_{\substack{k=0 \\ k\neq n}}^{N} \frac{D-k}{n-k} \quad \text{for } n = 0,1,2,\ldots N \qquad (7)$$

where D is the desired delay and N is the order of the interpolator. Although Lagrange interpolation is not optimal with respect to the defined error measure (6) we will use it because of its simplicity and efficient implementation.

C. Novel Structure for Lagrange Interpolation

In the sampling rate conversion application the key point is to make the update of the filter coefficients fast enough while maintaining sufficient precision in the approximation. We propose the use of a novel time-varying interpolator structure. It was inspired by the idea of Farrow [9] that it is possible to write the transfer function of an interpolating filter as a function of the delay parameter D. This results in a structure that consists of a parallel connection of $(N+1)$ fixed transfer functions and N multiplications by D. This so-called *Farrow structure* is well suited for applications where frequent updates of the delay D are needed: the transfer function is directly controlled by D.

Välimäki has introduced a technique to design the Farrow structure for the Lagrange interpolator [12]. This structure is well suited for sampling rate conversion since, in addition to frequent updates of the coefficients, the accuracy of approximation is critical at low frequencies. Figure 1 shows the new struc-

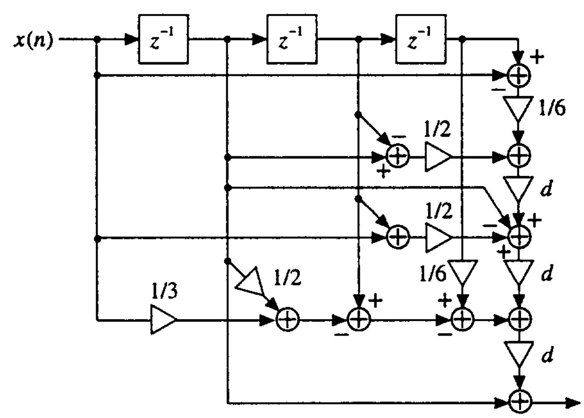

Fig. 1. *Novel structure for tunable 3rd-order Lagrange interpolator.*

ture for the third-order Lagrange interpolator. Three delay elements, 11 additions, 3 multiplications by d (i.e., fractional part of D), and 3 constant multiplications are needed. Multiplication by 1/2 can be implemented as a binary shift operation.

IV. ERROR ANALYSIS

In the proposed FD-based sampling rate conversion system, three main error sources can be identified:

1) *Errors in the delay parameter D* (constant bias error and randomly varying zero-mean component). We assume that the delay D is known, but in practice it has to be estimated from the received signal which inevitably introduces estimation errors. Furthermore, nonideal oscillators introduce jitter.

2) *Approximation error.* FD filter approximation introduces predictable distortion, which fortunately can be controlled by using appropriate techniques and high enough filter order.

3) *Quantization error.* Finite-precision signal presentation introduces quantization error in A/D conversion and subsequent processing. Here it is assumed that 5-6 bit input quantization is the main error source and further processing uses much longer wordlength so that errors can be neglected.

A. Delay Errors

Let us model the delay D as consisting of the ideal part and two additive terms:

$$D = D_{id} + \Delta + \delta(n) \tag{8}$$

where Δ is a constant bias error and $\delta(n)$ is a zero-mean stochastic error sequence with the variance σ_δ^2. We will discuss these two errors separately.

A1. Constant Bias Error in Delay

Let us first assume that delay error only consists of the bias term Δ. It is easily shown that, in the ideal case, this appears at the filter output as

$$y(n) = x(n - D_{id} - \Delta) = h_\Delta(n) * x(n - D_{id}) \tag{9}$$

where

$$h_\Delta(n) = \frac{\sin[\pi(n-\Delta)]}{\pi(n-\Delta)} \tag{10}$$

This shows in the error variance as

$$\begin{aligned} E_\Delta &= E\left\{\left|x(n-D) - x(n-D_{id})\right|^2\right\} = \frac{1}{\pi}\int_0^\pi S_{xx}(\omega)\left|1 - e^{-j\Delta\omega}\right|^2 d\omega \\ &= \frac{1}{\omega_p}\int_0^{\omega_p} \sigma_x^2 4\sin^2(\omega_p\Delta/2)d\omega = 2\sigma_x^2\left[1 - \frac{\sin(\omega_p\Delta)}{\omega_p\Delta}\right] \\ &\approx \frac{1}{3}\sigma_x^2\omega_p^2\Delta^2 \end{aligned} \tag{11}$$

where the last approximation holds for small values of Δ. The squared amplitude error thus grows approximately directly proportionally to the square of the delay error Δ in D.

A2. Stochastic Variation in Delay Error

Let us then assume purely stochastic zero-mean error process ($\Delta = 0$). Let us denote by $e_\delta(n)$ the amplitude error sequence caused by the delay error sequence $\delta(n)$. An equivalent problem of time jitter error of bandlimited sampled sequences has been considered by Papoulis [13]. According to those results, the error variance can be bounded by

$$E_\delta = E\left\{e_\delta^2(n)\right\} \le M_1^2\sigma_\delta^2 \quad \text{where} \quad M_1 = \frac{1}{\pi}\int_0^{\omega_p} \omega\left|X(e^{j\omega})\right| d\omega \tag{12}$$

In our case we obtain

$$M_1 = \frac{1}{\pi}\int_0^{\omega_p} \omega\sqrt{S_{xx}(\omega)}d\omega = \sqrt{\frac{\sigma_x^2\omega_p^3}{4\pi}} \tag{13}$$

which yields the stochastic delay error bound

$$E_\delta \le \frac{\sigma_x^2\sigma_\delta^2\omega_p^3}{4\pi} \tag{14}$$

Assuming that the bias term and the stochastic component are sufficiently small so that the corresponding error variances can be added (no coupling), the upper bound estimate for the composite error is obtained as

$$E_{tot} \approx E_\Delta + E_\delta \le \sigma_x^2\omega_p^2\left(\frac{\Delta^2}{3} + \frac{\sigma_x^2\omega_p}{4\pi}\right) \tag{15}$$

Hence, for wideband signals ($\omega_p \approx \pi$) the total amplitude error variance due to delay errors is roughly proportional to the total 'delay error power', i.e.,

$$E_{tot} \approx \frac{1}{3}\sigma_x^2\omega_p^2\left(\Delta^2 + \sigma_\delta^2\right) \tag{16}$$

Let us further normalize this error by the signal power.

$$E_D = \frac{E_{tot}}{\sigma_x^2} \approx \frac{1}{3}\omega_p^2\left(\Delta^2 + \sigma_\delta^2\right) \tag{17}$$

where subscript D stands for the delay error. Once the properties of the delay error are known, its effect on the amplitude error of the sampled signal is readily estimated via (17).

B. Approximation Errors

The approximation error as defined in (6) is a function of the defined bandwidth, filter order, and the fractional part of the delay. The delay dependence is of particular interest. As it can be assumed that in incommensurate-factor sampling rate conversions the delay is uniformly distributed in a sampling interval $[D_0, D_0+1]$ where D_0 is an appropriate integer delay, the average error is obtained from (6) via the formula

$$E_{ave} = \int_{D_0}^{D_0+1} E_{norm} dD \tag{18}$$

Let us further assume that the filter length L is even, i.e. the order $N = L-1$ is odd. In this case the worst-case normalized error is obtained when the delay to be approximated is $D = N/2$, i.e., it is exactly between the two middle sample filters of the approximating FIR filter. In this case the FIR filter has linear phase and a symmetric impulse response. Hence, an upper bound for the average error can be obtained by using the linear-phase response.

What is even better, we have experimentally observed that the error (6) depends on the fractional part d of the delay D approximately according to the following formula:

$$E_{norm}(D) = \sin^2(\pi d)E_{norm}(N/2) \tag{19}$$

This holds very well for odd-order filters, the better the higher the filter order. For linear interpolation ($N = 1$) the peak deviation is less than 7% and for $N = 3$ less than 3%, and it is practically independent of the bandwidth ω_p.

Furthermore, Eq. (19) implies that the average error is obtained directly from the worst-case error, that is

$$E_{ave} = \tfrac{1}{2}E_{norm}(N/2) \tag{20}$$

Figure 2 shows the average approximation error curves for Lagrange interpolators of order 1, 3, 5, and 7 (i.e., lengths 2, 4, 6, and 8) as a function of the approximation bandwidth ω_p. It is

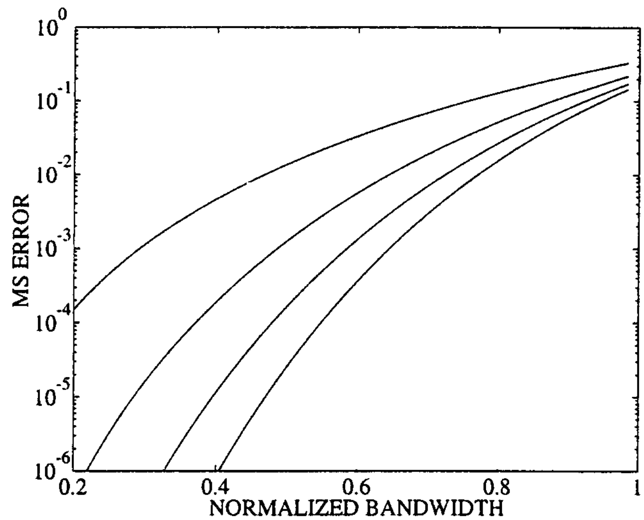

Fig. 2. *Mean square error of Lagrange interpolators of order N = 1, 3, 5, and 7 (from top to bottom) as a function of bandwidth.*

seen that the error is small at narrow bandwidths for all filters (due to their maximally flat characteristic at $\omega = 0$), whereas the error grows rapidly as the bandwidth is increased. At the bandwidth $f_p = 0.675 f_s/2$ (which is interesting for us) the mean square error is 5.9%, 1.4%, 0.48%, and 0.18% for 1st, 3rd, 5th, and 7th-order Lagrange interpolators, respectively.

C. Quantization Errors

The quantization errors include input quantization errors and those introduced in the subsequent processing. For fixed-point two's complement arithmetic, both of them can be modeled as additive white noise that is uniformly distributed in amplitude over a quantization step q, thus resulting in error variance

$$\sigma_q^2 = q^2 / 12 = 2^{-2b} / 12 \tag{21}$$

where $(1+b)$ is the wordlength at the particular quantization point. To enable comparison with other types of errors, we define the normalized quantization error as

$$E_Q = \frac{A_{max}^2 \sigma_q^2}{\sigma_x^2} = K \frac{2^{-2b}}{12} \tag{22}$$

where A_{max} is the peak signal amplitude corresponding to the largest representable number. In order to avoid overflow, a scaling of 1 or 2 bits is required so that the ratio $K = A_{max}^2 / \sigma_x^2$ is typically of the order 5-10.

V. EXAMPLE

Let us analyze the errors of a simplified sampling rate conversion system. We make the following assumptions:

- The approximation bandwidth is $\omega_p = 0.675\pi$.
- The bias delay error is 10% of the sampling interval and the random component has a standard deviation of 10%.
- The main quantization error is input quantization to 5 bits.

With these assumptions, the normalized delay error (17) is 3.2% of the input signal power and the input quantization error is 1.65% ($K = 5$). As discussed in Section IV.B, the 3rd-order Lagrange interpolator has an average error of 1.4%. This is thus of the same order as the input quantization error (22) and can be considered sufficiently small as it represents the worst case (i.e., conversion factor $r = 1$). When the conversion factor r is close to 0.5, the approximation bandwidth is reduced and the error will be considerably smaller (see Fig. 2). Ultimately, it would be desirable to have both delay and approximation errors clearly below the input quantization error level. This can, however, only be done when the system is specified in more detail.

VI. CONCLUSION

An efficient technique for sampling rate conversion for digital television transmission was proposed. The technique is based on fractional delay filters that are efficient to implement and can be controlled with a small number of arithmetic operations per output sample. The various error sources (delay errors, FD approximation, and input signal quantization) were analyzed in detail. Based on realistic system assumptions, it was shown that a 3rd-order Lagrange interpolator offers sufficient precision.

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