A Generic Approach to the Design of M-Channel Uniform-Band Perfect-Reconstruction Linear-Phase FIR Filter Banks *

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ABSTRACT

A generic approach is presented for the design of uniform-band M-channel (M ≥ 2) perfect-reconstruction FIR filter banks employing linear-phase analysis and synthesis filters. The technique designs on the impulse responses of the analysis filters directly. The design problem is formulated as a quadratic programming problem. The perfect-reconstruction feature of the filter bank can either be implicitly enforced through a set of mathematical relationships among the analysis filters' coefficients, or through a set of constraints in the optimization program. The former approach results in a filter bank whose PR (perfect reconstruction) feature's dependency on hardware and software is eliminated or, at least minimized. The criterion for optimality is "least-squares." *

I. INTRODUCTION

The design of perfect-reconstruction linear-phase FIR filter banks has been an active area of research in recent years. Most works reported, though, have dealt with the two-channel case. The work done on the M-channel case is very limited. Novel lattice structures for M-channel ($M \geq 2$) perfect-reconstruction linear-phase FIR filter banks are reported in [1], where good-quality filters have been obtained. However, there are four drawbacks when implementing such lattice-structure filter banks: 1) Due to the existence of a delay-free path from the input to the output, the speed of the structure can be very slow unless heavy pipelining is employed; 2) The dynamic range of the optimized lattice coefficients is very wide; 3) The lattice method is not generic in two aspects. Firstly, there is no generic lattice structure; secondly, there is no generic technique to decompose the polyphase matrix in order to obtain the lattice structure; 4) The design technique is not user-friendly; the burden is on the designer to derive the lattice structure, which could be a very tedious task. Independent work in the direction of [1] has also been reported in [2]. In [3] another technique for the design of perfect-reconstruction linear-phase FIR filter banks is proposed. But this technique does not lead to a uniform-band filter bank. In addition, due to severe aliasing caused by this technique, coding gain is slightly lower than in the conventional filter banks and there could be an increase in bit allocation. In this paper a new technique is presented for the design of uniform band linear-phase perfect-reconstruction FIR filter banks. The technique is generic, and therefore results in a very user-friendly design procedure. Furthermore, it does not suffer from the drawbacks of the methods in [1] and [3].

II. DESIGN METHOD

A generic M-channel FIR filter bank is shown in Fig. 1. Assuming perfect transmission channels, the reconstructed signal $\hat{x}(n)$ is related to the input signal $x(n)$ by

$$\hat{X}(z) = \frac{1}{M}\sum_{l=0}^{M-1} X(zW^l)\sum_{k=0}^{M-1} H_k(zW^l)F_k(z) \tag{1}$$

where M is the number of channels, $W = \exp(-j2\pi/M)$, and $H_k(z)$ and $F_k(z)$ are the FIR analysis and synthesis filters,

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respectively. In order to cancel the aliasing for the full spectrum, we set all M-1 terms involving $X(zW^k)$ for $k \neq 0$ in the above equation to zero. We then treat M-1 of the synthesis filters as variables, solving them in terms of the analysis filters and the one remaining synthesis filter.This remaining synthesis filter is chosen in terms of the analysis filters in such a way as to guarantee the synthesis filters to be FIR.

In order to have a perfect reconstruction system, it is required that $\hat{X}(z) = T(z)X(z) = \alpha z^{-n_0}X(z)$ where $T(z)$ is the transfer function of the filter bank, and α is, in general, a complex constant. A reasonable requirement on $T(z)$ is that it be constrained to be symmetric and of odd length. Therefore, only the center coefficient of $T(z)$ is non-zero. The set of sufficient conditions summarized below for different categories of M guarantees $T(z)$ to be symmetric.

a) $M = 4k_M$ - An odd number of anti-symmetric analysis filters and
$$N_k = Ml_k + 1 \qquad 1 \le k \le M-1 \qquad (2)$$
where k_M and l_k are positive integers.

b) $M = 4k_M + 1$ - An even number of anti-symmetric analysis filters and (2).

c) $M = 4k_M + 2$ - An even number of anti-symmetric analysis filters and (2).

d) $M = 4k_M + 3$ - An odd number of anti-symmetric analysis filters and (2).

The above set of sufficient conditions also guarantees the synthesis filters to be symmetric for all M. Furthermore, the imposed requirement of symmetry on $T(z)$ results in the following condition on the analysis filters lengths.

$$\sum_{k=0}^{M-1} N_k = \begin{cases} (2k_N + 1)M & M \text{ odd} \\ 2k_N M & M \text{ even} \end{cases} \qquad (3)$$

where k_N is a positive integer.

The synthesis filters' coefficients are real for $M = 4k_M + 1$ or $M = 4k_M + 2$, and pure imaginary for $M = 4k_M$ or $M = 4k_M + 3$, in which case we can absorb the j into the scale factor α in $T(z) = \alpha z^{-n_0}$. Thus, we can implement the synthesis filters with real coefficients for all M.

III. PROBLEM FORMULATION

For a given set of filter lengths and given stopband and passband cutoff frequencies for the analysis filters, nonlinear optimization is used to optimize the analysis filters' coefficients. The optimality criterion is "least-squares." The objective function to be minimized is as follows:

$$f = \frac{1}{2\pi} \sum_{k=0}^{M-1} \left\{ \alpha_k \left[\int_{\omega_{pk1}}^{\omega_{pk2}} (1 - |H_k|)^2 d\omega + \right.\right. \qquad (4)$$
$$\left.\left. \alpha_{s_k} \left(\int_0^{\omega_{sk1}} |H_k|^2 d\omega + \int_{\omega_{sk2}}^{\pi} |H_k|^2 d\omega \right) \right] \right\}.$$

Of course, for lowpass and highpass filters the last two integrals are reduced into one integral. One can also design some of the filters separately and then use their coefficients as known inputs to the final program; it is just required that there are enough degrees of freedom to enforce the PR constraints as well as to satisfactorily optimize the remaining filters.

The PR feature of the filter bank can either be implicitly enforced through a set of mathematical relationships among the analysis filters' coefficients, or through a set of constraints in the optimization program. In the latter case, considering that all constrained optimization programs have a tolerance level on how well the constraint equations are met, the accuracy of the PR property becomes heavily software and hardware dependent. Namely, it significantly depends on the optimization program and type of arithmetic and the precision used for number representation. As a result, we propose, when true PR is crucial, to implicitly enforce the PR property through a set of mathematical relationships between the filters' coefficients, independent of the optimization program. This method ensures that the dependency of the PR property on hardware and software is eliminated, or at least minimized.

The set of equations that we propose to enforce mathematically guarantees the PR feature. We choose as many analysis filters' coefficients as variables as we have constraints. These variables are selected such that the constraint equations form a set of linear equations in those variables of the form $AX = b \neq 0$. We then solve these equations mathematically (preferably symbolically, not numerically) for the chosen variables. Since the equations are independent, and the number of variables equals the number of equations, the solution set is unique. The solution set guarantees the PR property through a set of implicit mathematical relationships. The solution set is then used to substitute for the chosen variables in (4). Subsequently, a nonlinear unconstrained optimization program is used to optimize the independent filters' coefficients. The objective function to be minimized is given in (4). We then need to check that for the optimized coefficients the center coefficient of the transfer function is nonzero, which is most likely the case. Alternatively, we can use that as a constraint in the optimization program. Notice that in this case there is only one constraint equation, and it is an extremely loose constraint. The proposed technique of enforcing the constraint equations independent of the optimization program also provides significant savings in design time. The saving occurs in the time required for convergence of the optimization algorithm.

IV. DESIGN EXAMPLE

A four-channel uniform-band perfect-reconstruction linear-phase FIR filter bank with $N0 = 65$ and $N1 = N2 = N3 = 45$ was designed. $H_0(z)$, $H_1(z)$, and $H_3(z)$ are symmetric and $H_2(z)$, anti-symmetric. The transition bandwidth is 0.035π. To enforce the PR feature the first 24 coefficients of $H_0(z)$ were used as variables in the set of linear equations of the form $AX = b$. The Sequential Quadratic Programming (SQP) method of nonlinear optimization was used to optimize the remaining analysis filters' coefficients [7]. The magnitude response plots of the resulting analysis and synthesis filters are shown in Fig. 2 and Fig. 3, respectively.

V. CONCLUSION

In this paper we have presented a new approach to the design of M-channel perfect-reconstruction linear-phase FIR filter banks. We have formulated the design problem as a quadratic programming problem with at most one constraint. The perfect-reconstruction feature of the filter bank is ensured implicitly through a set of mathematical relationships among the analysis filters' coefficients, independent of the optimization algorithm. Our proposed method results in a very user-friendly and generic technique of designing uniform-band M-channel linear-phase PR FIR filter banks.

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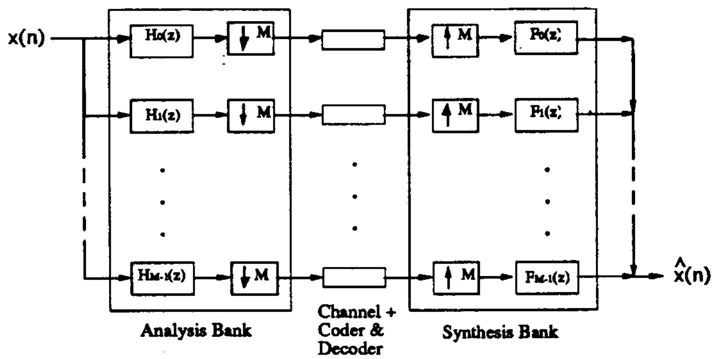

Fig. 1. M-channel analysis and synthesis filter bank.

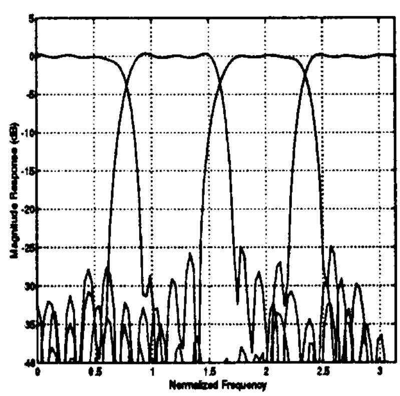

Fig. 2. Magnitude response plots of the analysis filters.

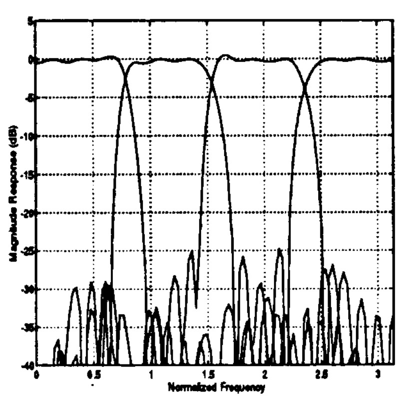

Fig. 3. Magnitude response plots of the synthesis filters.