OFFSET WINDOWING FOR FIR FRACTIONAL-SAMPLE DELAY

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ABSTRACT

Non-optimal FIR filters used for fractional-sample delay, despite their wideband nature, are shown to benefit significantly from application of windowing. Here simple raised-cosine windows prove to be very effective, particularly if they are cast as asymmetric modifications of their conventional forms. The offset von Hann window is surprisingly potent when the number of coefficients is large and window offset coincides with the fractional delay required of the overall filter.

1. INTRODUCTION

Fractional-sample delay is a topic of increasing interest in various application sectors of digital signal processing. Symbol timing recovery [1], precision beamforming and prediction in speech coding are just a few of the areas where the ability to have controlled offset of sampled signals is of great benefit. Although multirate systems can in principle perform interpolation, delay and subsequent re-decimation in order to gain access to intra-sample data, their use is sometimes an "overkill" solution for dynamic situations like adaptive delay estimation. Frequently the need is for a monorate system that can perform selectable interpolation by means of standard digital filters.

Growing interest has been shown in stand-alone fractional delay subsystems (realized by both FIR and IIR filters [2], [3]) as well as in filters which combine fractional delay along with other functionality (e.g. fractional-sample-delayed differentiation [4]) thereby promoting rendezvous goals concurrent with processing tasks. While some excellent fractional delayors can be delivered by powerful optimization algorithms, in real-time data communication there is considerable need for simple (usually small FIR) delayors which can be rapidly tuned to different delay values. Several closed-form expressions for obtaining filter coefficients are in use, though their suboptimal nature partially negates their implementation appeal. The objective of this paper is to explore ways for improving the performance of such simple fractional-sample delay filters through the expedient of windowing.

We set out to approximate well the ideal desired transfer function.

$$D(\nu) = e^{-j2\pi\beta\nu} \tag{1}$$

where ν is normalized frequency and

$$\beta = \alpha + r \tag{2}$$

Here, for an FIR filter having N coefficients, α equals $(N-1)/2$ as customarily seen in expositions of linear-phase design. It is the added delay factor "r" that offsets the filter impulse response centre of symmetry and takes us out of the realm of linear-phase design in the traditional sense. Indeed, due to restriction to only N coefficients, there is no true impulse response symmetry, even if (wastefully) r should happen to be an integer in the range $[-\alpha, \alpha]$, since coefficients demanded for symmetry "drop off one end". Our concern is with the more subtle impulse response lopsidedness incurred when r is non-integer. Restriction to $r \in [-0.5, 0.5]$ is of principal interest, since bulk integer delays are readily available to "take up the slack" in any overall delay requirement.

Our task is to design a practical z-domain transfer function H(z) such that the total error magnitude

$$|\varepsilon(\nu)| = |H(e^{j2\pi\nu}) - D(\nu)| \tag{3}$$

is kept acceptably small. Complete success (for arbitrary r in our basic range) is easy at d.c. and impossible at Nyquist frequency, where the Tarczynski Bound

$$|\varepsilon(0.5)| \geq |\sin\beta\pi| \quad (4)$$

must always apply [2], [3]. Thus we must take a baseband-oriented view of approximation quality. Full-band approximation fidelity is a pointless aspiration. The delay filter sought will normally not have a cutoff frequency in the sense that a lowpass filter does; we must instead decree a measurement cutoff frequency. In our work we have chosen to focus on two regimes when assessing quality of a fractional delayor: d.c. up to ν = 0.25 (which we call "halfband designs" occupying 50% of the available band) and d.c. up to ν = 0.45 ("wideband designs" which handle 90% of the full band).

2. WINDOWING SINC FRACTIONAL DELAYORS

The most popular way of practically implementing fractional delay is by utilizing the SINC filter:

$$h(k) = \sin\pi(k - \beta)/\pi(k - \beta) \quad (5)$$

where k is the "time" index ranging between 0 and N-1 (inclusive) and h(k) is the impulse response sequence corresponding to $H(e^{j2\pi\nu})$. We have shown in [2] that despite its widespread usage such a sinc filter performs poorly in the fractional-sample delay role. Superior performance is achievable with Frequency Sampling designs, which come in four standard variants depending upon the evenness or oddness of N and whether transfer function samples are anchored at d.c. or at Nyquist frequency. Such designs can be put in closed form, and prove to be valuable alternative non-optimal candidates for windowing. Our paper will be confined to just the sinc filter (badly in need of improvement) and, for brevity, only to odd values of filter size N. Our work in [2] has shown that N odd will always be preferable to even N when $r \in [-0.25, 0.25]$, so we have selected only the specimen case r = 0.2 here to illustrate how windows might be employed to make a favourable situation even better.

Figure 1 shows what happens when wideband measurements are made on a sinc filter before and after multiplication by two raised-cosine windows:

$$w(k) = (1 - C) + C \cos(2\pi/N)(k - \alpha - r_w) \quad (6)$$

when r_w = 0 and C = 0.46, as it must be if the "classic" linear-phase Hamming window is in use. We can also clearly see the (low-frequency) benefit the von Hann window (C = 0.5) imparts.

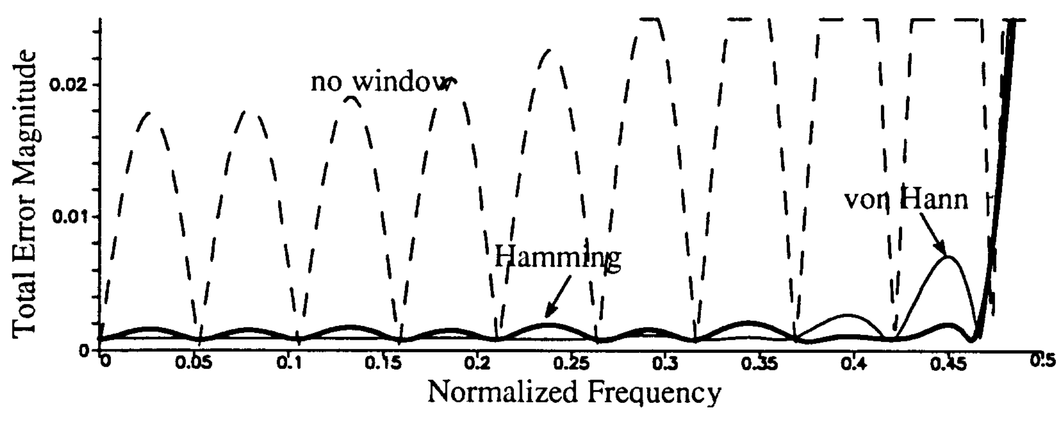

Figure 1 Effect of Hamming and von Hann Windowing on Wideband Error Profiles
($r = 0.2$; $r_w = 0$; $N = 21$; Tarczynski Bound ≈ 0.588)

It is germane to wonder whether some departure from the traditional linear-phase window would be even more successful. Different choices of r_w in (6) turn out to have little effect on wideband performance. However, as Figure 2 shows, the peak error (measured in dB) seen in the halfband measurement frame is dramatically reduced. The lower cluster of curves suggests that optimal effect from this window is obtained when the window offset is harmonized with the filter's delay (that is, $r_w = r$). This choice eliminates the error bias evident in Figure 1.

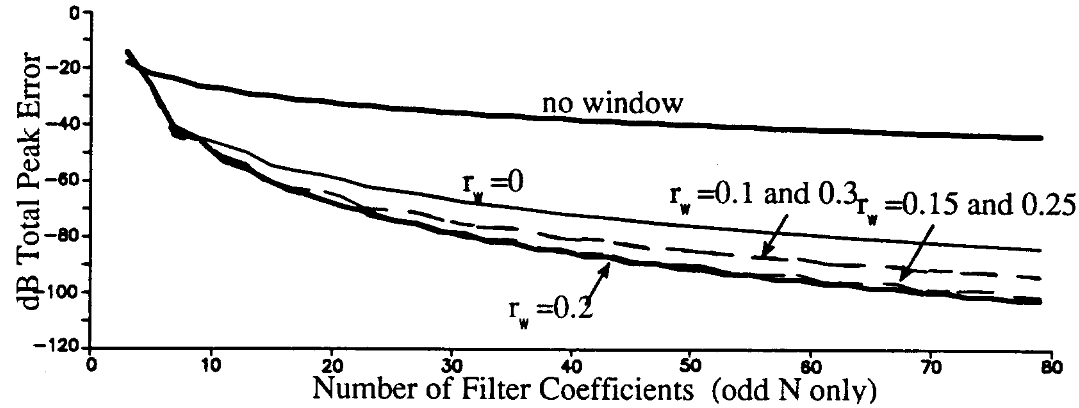

Figure 2 Halfband Comparison for Hamming Window Offset
(r = 0.2 and odd N Sinc Delayors)

Contrasting the two heavy solid traces in Figure 2, it is obvious that this type of windowing is always worth doing, and that as much as 50 dB enhancement margin is obtained thereby for large N!

Results for several window types are given in Figure 3. Notice that wideband results are being shown, where even the von Hann window can create an improvement margin (over no windowing) of 20-30 dB for large N. It is prudent to consider other aspects of error distributions before becoming too enthusiastic about abatement only of peak error; happily rms error also shows a similar tendency, suggesting that the error characteristics, more broadly, are amenable to containment and potent reduction through windowing.

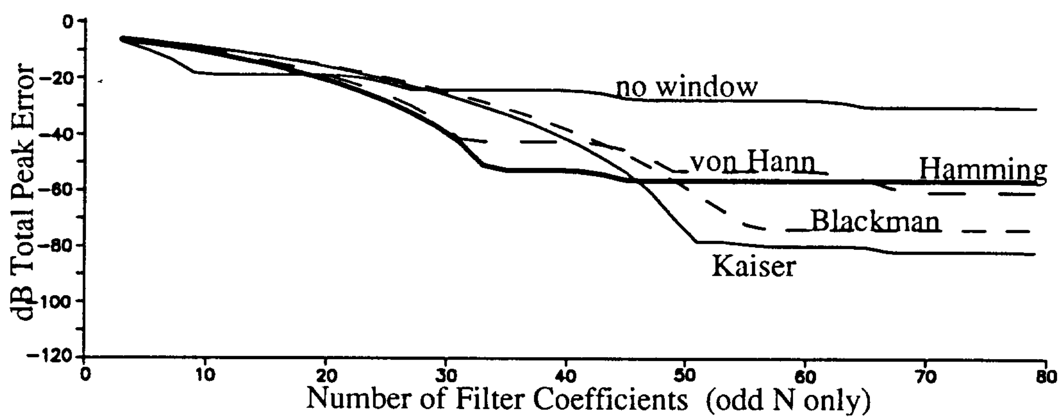

Figure 3 Wideband Comparison of Several Offset Window Types
($r=r_w=0.2$; odd N Sinc Delayors)

3. REMARKS

Figure 3 indicates that windowing is not worth the effort for filters below N = 19. However, this is a harsh view; if halfband judgements can instead be tolerated then, as exhibited in Figure 4, windowing shows its worth. Notice the very rapid improvement as filter sizes go above about 7. Again, von Hann performance for its manifest simplicity is surprisingly good!

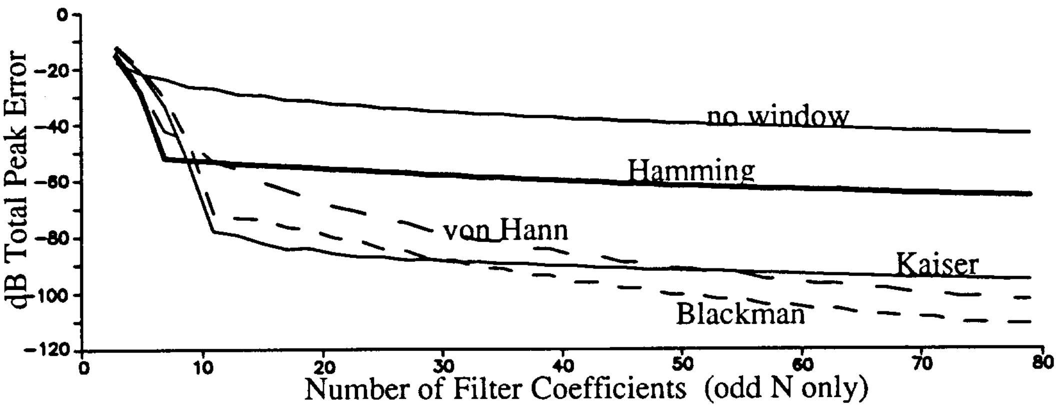

Figure 4 Halfband Comparison for the Same Conditions used in Figure 3

The small cross-section of results given here merely shows the surface of the complicated tradeoff patterns which characterize this method of FIR fractional-sample design. Although it is not meaningful to make sweeping pronouncements about relative merits of various window schemes, we are at least able to say that, if error over somewhat less than 90% of full band is of interest, then windowing is nearly always attractive, and its potency grows with filter size. Windowing can do much to make the sinc filter more worthy of its popularity. Comparable attractions are also found for Frequency Sampling designs.

Acknowledgement: Our interest in the windowing approach to this problem arose from valuable discussions with Dr. Les Sabel of DSTO, Australia, who seems to be one of the few investigators to have employed windowing in a system which harnesses fractional delay [5].

4. REFERENCES

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