EFFICIENT GENETIC ALGORITHM DESIGN FOR POWER-OF-TWO FIR FILTERS

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ABSTRACT

This paper presents an efficient genetic approach to the design of digital finite impulse response (FIR) filters with coefficients constrained to be sums of power-of-two terms. To obtain such efficiency, i.e. a reduction of computational costs and an improvement in performance, a specific filter coefficient coding scheme has been studied and implemented. The resulting genetic algorithm (GA) is explained and compared experimentally with other state-of-the-art design techniques on several power-of-two FIR filter design cases. It can be seen that the proposed genetic technique is able to attain results as good as or better than the other methods. Moreover it can be easily implemented on parallel hardware.

1. INTRODUCTION

The high-speed specific DSP chips designed for FIR filtering are usually implemented using fast fixed point arithmetic. In these cases, the number of bits used to represent the input data and the filter coefficients must be small for efficiency and for limiting the cost of the needed digital hardware. Consequently, filters with very coarsely quantized coefficients are valuable. Although several optimal design procedures are available for the unquantized filters [1], the design of finite word length FIR filters is a difficult optimization problem which usually requires an execution time very high (see for example [2]). Among the possible quantizations, the case when the filter coefficients are constrained to be powers of two or simple sums of them, receives a particular interest [3-7], in fact, in binary representation, the multiplication between two integer numbers can be substituted by a shift if one of them is a power-of-two. However, depending on the particular filter design, the available design techniques may produce sub optimal filters with poor performance.

It is well known that GAs [8] are search algorithms based on the genetic and natural selection paradigm and that they can be successfully employed for minimizing or maximizing a cost function. The design of quantized digital FIR filters, being an optimization problem over a discrete coefficient space, can therefore be faced using a genetic approach. In [9], a classical quantization problem (without the power-of-two constrain) is solved using a simple evolutionary algorithm, where a population of several filter design realizations (there called "filtomorphs") was allowed to evolve and find a sub optimal solution by the mutation mechanism. A genetic approach to design power-of-two FIR filters is proposed in [10]; in this paper, the filter coefficients are directly coded by the binary Gray code. Such direct coding of the filter coefficients can severely impair the GA performance when more than 4 bits per coefficient are used [10].

In this paper we present an efficient design method for digital FIR filters with power-of-two coefficients, which effectively uses the genetic approach. In order to reduce the computational cost and improve the performance, a specific filter coefficient coding scheme has been studied and implemented. The proposed method can cope with a free mixture of frequency and time domain filter specifications, is able to find solutions also for reasonably long filters and provides sub optimal solutions with good performance.

Moreover, due to its implicit parallel nature, this GA approach can explore many possible solutions at each generation and can be easily and efficiently implemented on parallel machines.

2. THE DESIGN METHOD

Let us consider only the case of linear-phase symmetric real FIR filters with odd length, since the extension to other linear-phase filters is straightforward. The frequency response $H(f)$ of such filters is given by:

$$H(f) = h(0) + \sum_{n=1}^{(N-1)/2} 2h(n)\cos(2\pi fn); \qquad (1)$$

where f is the normalized frequency ($0 \le f \le 0.5$), N is the (odd) length and $h(n)$, $n=-(N-1),..,0,..,(N-1)$, are the filter coefficients. Due to the symmetry, it holds true that

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$h(n)=h(-n)$, which allows to reduce the free parameters of the filter to $(N+1)/2$, $h(n)=0,\ldots,(N-1)/2$.

Such coefficients are then constrained to be a simple sum of power-of-two terms, i.e. they must belongs to the domain $\boldsymbol{D}$:

$$\boldsymbol{D}=\left\{\begin{array}{l}\alpha\ :\ \alpha=\sum_{k=1}^{\lambda}c_k 2^{-g_k},\\ c_k\in\{-1,0,1\},\ g_k\in\{0,1,\ldots,B\}\end{array}\right\};\qquad(2)$$

which is ordered as follows:

$$\boldsymbol{D}=\{\alpha_1,\alpha_2,\ldots,\alpha_L\}\ ;\qquad \alpha_1<\alpha_2<\ldots<\alpha_L.$$

Where λ is the number of powers of two which form the coefficients (usually $\lambda=1$ or $\lambda=2$), B is the maximum number of shifts allowed by the domain, and L is the number of elements of the domain $\boldsymbol{D}$. The minmax criterion minimizes the maximum error between the filter frequency response $H(f)$ and the ideal response $T(f)$ on a dense grid of frequency points f_m, $m=1,\ldots,F$, equally spaced between 0 and 0.5, i.e. minimizes:

$$F_0(G,h(n))=\max_{m=1,\ldots,F}\left|W(f_m)\left[\frac{H(f_m)}{G}-T(f_m)\right]\right|;\quad(3)$$

where $W(f)$ is a suitable weighting function, F is a-priori selected, G^{-1} is the passband gain, and $n=0,\ldots,(N-1)/2$. Since it is known that the coefficient G of the quantized filter assumes a particular relevance [4,5,7], the G value is considered an additional free parameter of the problem. In fact due to the non uniform distribution of the elements of the domain $\boldsymbol{D}$, a gain variation can improve the frequency response more than 6 dB, by a reduction of the coefficients quantization error [4,5].

In the proposed method a GA approach is employed for minimizing F_0 with respect to the $(N+3)/2$ free parameters G and $h(n)=0,\ldots,(N-1)/2$ where G is quantized and the $h(n)$ belong to the domain $\boldsymbol{D}$ with λ and B a-priori given. The simplest way for coding these free parameters into the genetic chromosomes of the optimization algorithm, is to use directly the binary or Gray representation of their values [10]. When this coding scheme is used, the optimization process requires a computational time which is related to the parameters' quantization level, i.e. the number of bits used to represent them. If this number of bits increases, the search space becomes very large and the GA design method begins to exhibit poor performance (see for example [10]). In order to reduce the search space and consequently the computational time, instead of coding the quantized $h(n)$ and G values, we code the deviations, inside specified interval, from some leading values. As leading values for the coefficients $h(n)$, the corresponding unquantized optimal coefficients $h_0(n)\in\boldsymbol{R}$ are chosen, computed by the Parks-McClellan algorithm [1]. As leading value of G, the value $G_0\in\boldsymbol{R}$ nearest to 1 is selected, which minimizes the following function [5] for $G\in[0.5,1]$:

$$\max_{n=-(N-1),\ldots,(N-1)}\left|h_0(n)-\frac{Q_D[G\ h_0(n)]}{G}\right|;\qquad(4)$$

where $Q_D[.]$ represents the rounding to the nearest power-of-two belonging to the domain $\boldsymbol{D}$.

Let $h_{0D}(n)=Q_D[h_0(n)]$ where $h_{0D}(n)\in\boldsymbol{D}$ represent the quantized optimal filter coefficients which will be used as leading values.

Let $i_{0D}(n)$ be the index of $h_{0D}(n)$ in the domain $\boldsymbol{D}$:

$$i_{0D}(n)=P_N[h_{0D}(n)];\qquad i_{0D}(n)\in\boldsymbol{N};\qquad(5)$$

where $P_N[.]$ returns the index of the position of the argument inside the ordered domain $\boldsymbol{D}$.

As previously stated, for each index value $i_{0D}(n)$ computed by (5), corresponding to a leading value $h_{0D}(n)$, is associated an interval $I_{0D}(i_{0D}(n))$ of indexes of allowed values for the n-th coefficient. To describe such intervals, a pair of integer values (a_n, z_n), $a_n\in N$, $z_n\in N$, is used for each $I_{0D}(i_{0D}(n))$. The term a_n represents the number of the allowed values around $i_{0D}(n)$ and $z_n<a_n$ represents the relative position of $i_{0D}(n)$ in this interval (bias factor). The z_n value is selected in order to roughly center the real-valued interval corresponding to the index interval $I_{0D}(i_{0D}(n))$ around the leading value $h_D(n)$. Fig.1 shows an example of this coding scheme in a simple case where each a_n, corresponding to the n-th filter coefficient, is coded by 4 bits (16 possible values around the leading value).

For the G factor, the interval $[G_0-0.15,\ G_0+0.15]$ is coded using a binary code of 8 bits (256 levels around the real leading value). The chromosome then contains the binary codes of the indexes a_n $n=0,\ldots,(N-1)/2$ plus the binary code of G.

Using such integer values as genetic strings, a population of quantized filters is created. The evolution then takes place with given probabilities of mutation and crossing-over. The fitness of each individual is computed by expr. (3) and then normalized so that the lowest fitness in the population is set to 0 and the highest to 10000. To avoid the genetic drift in small populations, we have chosen to use a multiple crossing-over [8] on the genetic strings of two individuals. An elitist mechanism, similar to the De Jong's model [8], has been also implemented, increasing the fitness of the best individuals in the population by a fixed amount roughly proportional to the fitness itself.

After a given number of generations, the performance is measured by the maximum weighted error e_{dB} expressed in decibel [4,5]:

$$e_{dB} = \left[\frac{\delta}{\left(H_{\max}(f) + H_{\min}(f)\right)/2}\right]_{dB} ; \qquad (6)$$

where δ is the peak weighted ripple, $H_{\max}(f)$ and $H_{\min}(f)$ are respectively the maximum and minimum value of the frequency response in the passband.

3. EXPERIMENTAL RESULTS

The proposed genetic technique (GA), the simulated annealing algorithm (SA) in [5], the computation of the real-valued optimum minmax filter obtained by the Parks-McClellan algorithm [1] and the direct rounding to the nearest power-of-two ("rounded" label), have been implemented on a computer system. Many linear-phase FIR filter design experiments have been carried out with the implemented methods varying the filter lengths and the initial conditions for the GA and SA algorithms. The best results are here reported.

Table 1 and Table 2 report the peak weighted error in decibel e_{dB} versus the number of taps of the filter. The filter lengths and the other parameters (λ=2, B=9, F=512) have been selected in order to compare the results also with those obtained by the mixed integer linear programming (MILP) technique (see [3,5]) and by the proportional relation-preserve (PRP) design method (see [4,5]). The probabilities of mutation and crossing-over for the proposed GA method were set to 0.006 and 0.9 respectively, with populations from 30 to 100 individuals.

In Table 1 (filter length from 15 to 35), it can be noted that the GA can obtain the same results of the SA technique, while its performance is slightly better than the PRP method, but slightly worse than the MILP approach. In Table 2 (filter length from 51 to 59), it is shown that the GA approach can attain better results than the PRP and SA techniques since longer filters are involved. No data are available for the MILP, due to its high computational cost.

4. CONCLUSIONS

The proposed design method, together with the SA approach, are very general, since they can cope with any free mixture of frequency and time domain filter specifications. As the SA, it is able to find solutions also for reasonably long filters providing sub optimal solutions with good performance. However, the SA algorithm usually requires several runs to provide the best results, while the GA method is able to find them in a single run, since it explores many different solutions in parallel.

It is known that on a single CPU the SA method can be much faster than a classical GA approach, However, by the use of the proposed efficient coding for the chromosomes, our GA design method in the test cases resulted only 1.5 to 2 times slower than the SA, but it was able to easily overcome the simulated annealing algorithm when implemented on a parallel machine.

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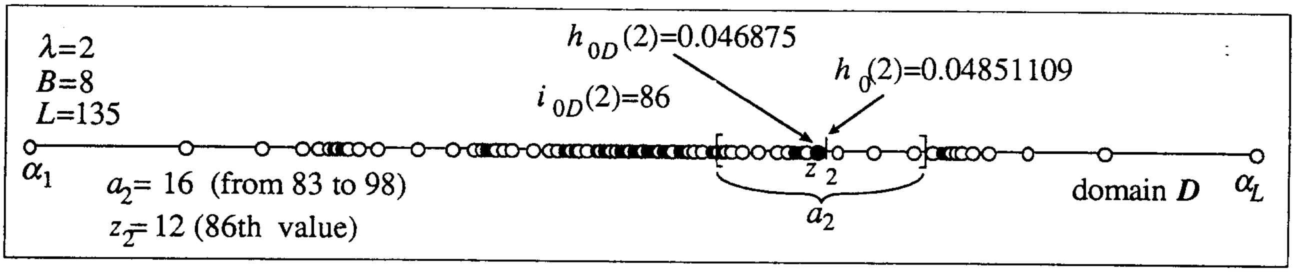

Fig. 1. The pair (a_n, z_n) for filter coefficient coding over the power-of-two domain D.

Table 1. Performance comparison for filter length from 15 to 35 taps.

FIR length	rounded	MILP	GA	PRP	SA
15	-23	-29	-29	-29	-29
17	-24	-32	-32	-32	-32
19	-25	-37	-37	-37	-37
21	-24.5	-37.5	-37.3	-37.3	-37.3
23	-25	-39	-39	-38	-39
25	-25	-41	-40.5	-38	-40.5
27	-26	-42	-41.3	-40.5	-41.3
29	-26.5	-44.5	-43.1	-41	-43.1
31	-26	-44.5	-43.1	-40	-43.1
33	-26.5	n. a.	-44.8	-39	-44.5
35	-26	n. a.	-45	-41	-44.6

Table 2. Performance comparison for filter length from 51 to 59 taps.

FIR length	rounded	McClellan	GA	PRP	SA
51	-24.6	-34.0	-33.2	-31.7	-33.0
53	-25.1	-34.7	-33.5	-30.0	-33.0
55	-24.8	-35.5	-33.8	-32.0	-33.2
57	-25.7	-36.9	-35.1	-34.0	-34.4
59	-26.4	-37.1	-35.2	-33.5	-34.5

Table 3. Comparison of the gain factor G for the SA and GA.

FIR length	SA-gain	GA-gain
51	0.906	0.905
53	0.910	0.904
55	0.908	0.907
57	0.906	0.918
59	0.910	0.911