FAST SHORT-TIME ORTHOGONAL WAVELET PACKET TRANSFORM ALGORITHMS

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ABSTRACT

Orthogonal wavelet packet transforms, presented recently as powerful tools for nonstationary signal processing, suffered from the lack of efficient and time-domain transparent algorithms for their implementation. In this paper, we present an overlapped block lattice structure that allows fast short-time wavelet packet transform algorithms to be built in an equivalent manner to other short-time transforms, such as the short-time Fourier transform (STFT) based on the fast Fourier transform (FFT) algorithms. The outputs obtained with the overlapped block lattice are equivalent to those achieved by the well known tree-structured filter bank approach, though the implementation algorithms are different.

1. INTRODUCTION

The theory of multirate systems and lattice filter banks has been presented in [1]. This theory is well suited for treating uniform filter banks and tree-structured wavelet transforms.

The orthogonal wavelet packet transform [2] is a generalization of the orthogonal wavelet transform [3] that allows nonuniform partitions of the time-frequency plane to be performed. It is commonly implemented using tree-structured filter banks, since the cascading of two-band paraunitary filter banks in a tree manner has been shown to be a sufficient condition to construct orthogonal wavelet packet transforms possessing the perfect-reconstruction property [4].

Some orthogonal non-overlapped transforms, such as the Hadamard-Haar transforms and the Fourier transform, are known to possess equivalent parallel and tree-structured implementations [5] [6]. In general, the tree to parallel conversion procedure is simplified in the case of such non-overlapped orthogonal block transforms, since orthogonal matrix factorization approaches provide solutions that can directly be translated into fast transform algorithms. On the other hand, lapped transforms such as the lapped orthogonal transform (LOT), the modulated lapped transform (MLT) and the extended lapped transform (ELT), possess fast algorithms for uniform decompositions of the frequency domain, but without direct translations into binary tree structures.

The overlapped transform case to be discussed here possesses a structural solution to this conversion that has been described in [7].

Up to now, no parallel algorithms have been proposed for the implementation of general short-time wavelet packet transforms with overlapped block orthogonal lattices. This is the approach presented here.

2. OVERLAPPED ORTHOGONAL TRANSFORMS

The main idea is first to implement the two-channel filter bank structure as a matrix product of a delay chain, having the length L of the prototype lowpass filter $H(z)$, and an orthogonal overlapped block transform $\mathbf{T}$ having no delay operators. This approach has been described in [7]. The structure is then extended to any N-band filter bank — N being a power of 2. At synthesis, the lapped block transform becomes the transpose $\mathbf{T}^T$. A criss-cross or butterfly operation is employed in the algorithms. It is the orthogonal operator described in Fig. 1. If we consider the case $N = 8$ and $L = 4$, this procedure produces the analysis and synthesis algorithms presented in Figs. 2 and 3, respectively. In these figures, $\beta = \prod_{i=0}^{1} \left(1 + \gamma_i^2\right)^{-1/2}$ is a lattice normalization factor and the γ_i are the lattice parameters. The overlapped transform is computed in $\log_2 N$ stages. Such an orthogonal transform performs the time-frequency decomposition described by the grid in Fig. 4.

It is important to consider the output coefficients at each stage of the transform which are related to the N-sample input block being transformed. In Figs. 2 and 3, these coefficients are highlighted with black dots. When a transform is calculated on a new input sample block, the coefficients that are not marked with dots in the structure can be shown to be identical to those previously computed for preceeding input blocks. This means that not all the butterfly operations have to be computed when shifting from one N-point transform block to the next one. A computational

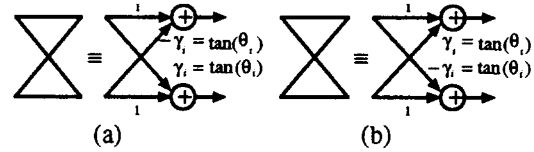

Figure 1: *Criss-cross operation: (a) at analysis, (b) at synthesis; θ_i is a rotation angle.*

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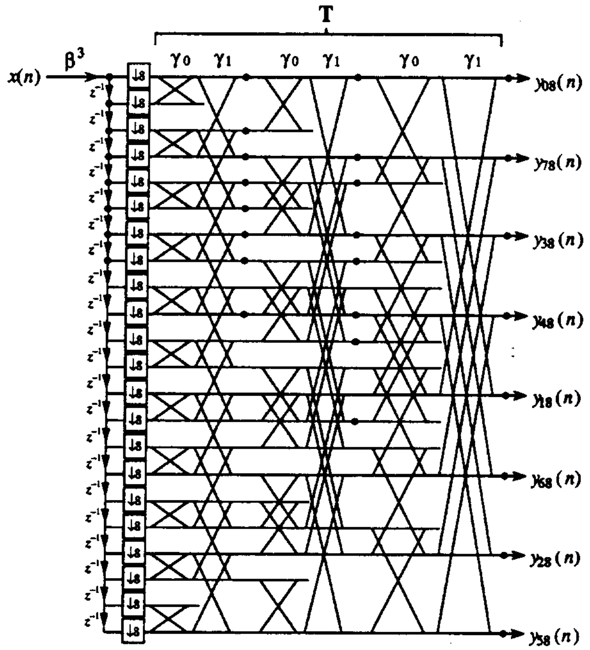

Figure 2: *Eight-channel orthogonal transform: analysis section.*

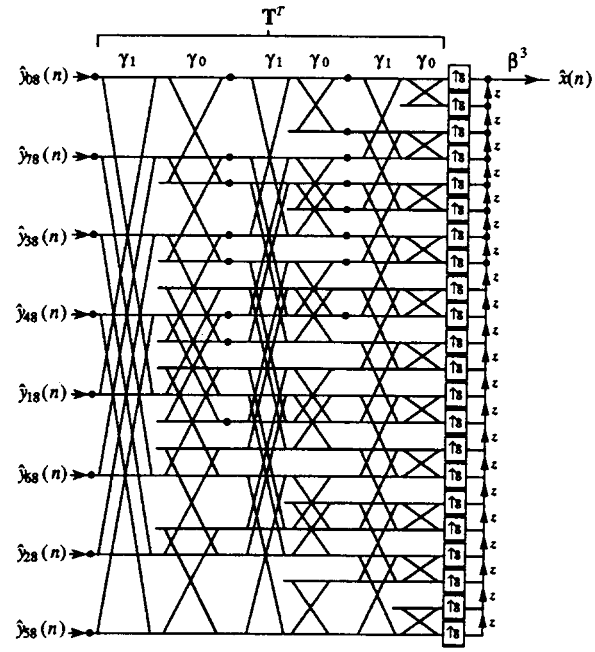

Figure 3: *Eight-channel orthogonal transform: synthesis section.*

gain is obtained by pruning the overlapping computations among the lattice structures applied to consecutive blocks.

The coefficients appearing at the transform output are in the Hadamard or natural order. The output coefficient $y_{i8}(n)$ corresponds to the band i, in increasing order in the frequency domain, after transformation of samples 1 to 8. At the intermediate stages, the order of the coefficients marked with black dots is also related to Hadamard ordering.

The structures presented do not need to be maximally subsampled. Any subsampling factor varying from 1 to N, by powers of 2, can be employed in the case of perfect reconstruction. If the subsampling factor is smaller than N, the algorithm implements a short-time overlapped orthogonal transform. If only the analysis operation is necessary, the subsampling factor can take any integer value between 1 and N.

The derivation of lattice tree structures that can implement different filter bank solutions, by varying the orthogonal lattice operators and the delay factors, has been covered in [7]. A choice of rotations, symmetries and permutations can be made to fit into the diverse filter types proposed in the literature, preserving the global structure of the lattice. This approach can be employed with the parallel structures described here, so that it retains the flexibility inherent to the lattices.

The computational complexity C of the overlapped structure can be shown to be identical to the complexity of tree structures. We can denote the number of multiplications by C_μ and additions by C_σ. After removing all the redundant operations in Figs. 2 and 3, we have $C_\mu = L/2 \cdot N \cdot \log_2 N + N$ and $C_\sigma = L/2 \cdot N \cdot \log_2 N$. L is length of the prototype low-pass filter or twice the number of lattice parameters — 4 in

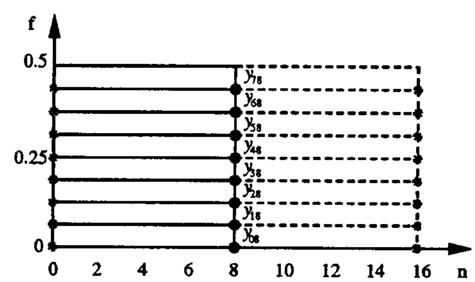

Figure 4: *Eight-channel time-frequency partition and output samples.*

our example. N is the number of output channels. The additional term N for C_μ comes from the normalization factor β.

3. OVERLAPPED ORTHOGONAL WAVELET PACKET TRANSFORMS

Up to now we have only presented a transform solution which allows the calculation of uniform time-frequency decompositions. The design of nonuniform partitions of the time-frequency plane can be easily achieved with the help of tree-structured filter banks. For this, it is sufficient to choose a subtree of a uniform N-channel tree structure. The different coefficients of the transform are then obtained at the outputs of the subtree at different time intervals. If the overlapped structure is used, the derivation of nonuniform time-frequency decompositions is achieved in an analogous manner. To compute an orthogonal wavelet packet transform it is sufficient to choose a subtransform of a complete

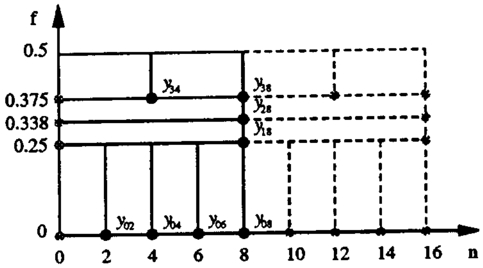

Figure 5: *Four-channel time-frequency partition and output samples. The downsampling factor is 8.*

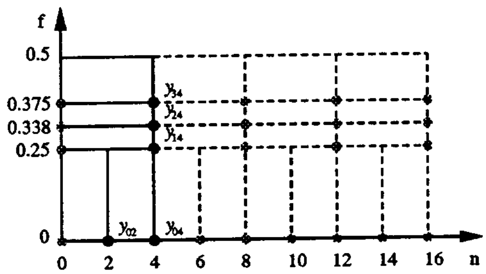

Figure 6: *Four-channel time-frequency partition and output samples. The downsampling factor is 4.*

N-channel transform. This is equivalent to choosing subtrees in complete tree structures.

As a design example, we can consider a four-band orthogonal wavelet packet decomposition realizing the time-frequency partition presented in Fig. 5. The corresponding lattice block algorithm appears in Fig. 7. The sample block shift or downsampling factor is 8. Here, only the non-redundant operations have been represented. This algorithm is derived from the eight-channel orthogonal transform of Fig. 2 by extracting a subtransform. However, the time-spectral coefficients of the wavelet packet decomposition can be directly obtained from the analysis algorithm in Fig. 2. The coefficients of any subtransform are always available at the different stages of its corresponding N-channel complete transform. The four bands are indexed from 0 to 3 and this corresponds to the index i of the output coefficient y_{ij}. The second index j denotes the temporal location of the segment being analyzed.

As a second example, an illustration of a short-time decomposition of the input signal is shown by the time-frequency grid in Fig. 6. The wavelet packet decomposition is still the same, but the block shift is now 4 samples. The corresponding lattice block algorithm of this example is represented in Fig. 8.

4. DESIGN ISSUES

After choosing a structure to perform a wished time-frequency decomposition, the lattice parameters γ_i must be selected to implement filters with well chosen characteristics. In the previous examples, we can choose the parameter set $\{\gamma_0, \gamma_1\}$ as in Table 1. Such choices of γ_i correspond to the implementation of the prototype low-pass filters $H(z)$

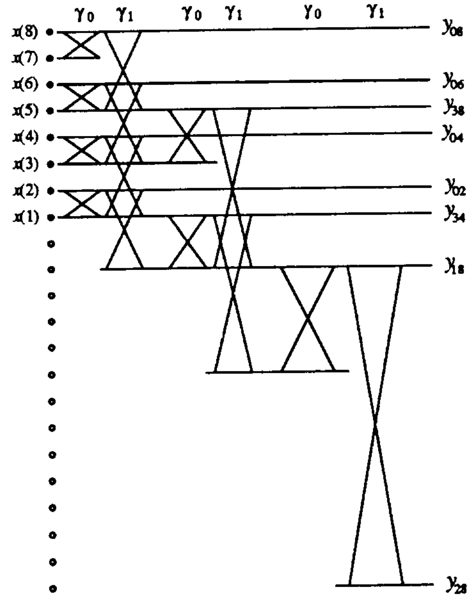

Figure 7: *Four-channel orthogonal wavelet packet transform with a downsampling factor of 8.*

appearing in Table 2, for an orthogonal two-band decomposition. The Remez designed low-pass filter has a transition bandwidth of 0.2.

As presented in the tables, prototype filters can also be designed via optimization procedures in order to obtain a variety of orthogonal prototype filters, with balanced regularity, frequency selectivity, number of orthogonal operators, and phase. Such a unified filter design procedure has been proposed in [8]. It can be employed to compute different types of filters such as Daubechies filters, binomial filters, MLT filters, ELT filters, those developped for orthogonal wavelet transforms, and other paraunitary filter sets.

The resulting magnitude characteristics of the four-channel wavelet packet transform, after making the suggested choices, appear in Fig. 9.

5. CONCLUSIONS

The new structurally efficient approach to wavelet packet transforms presented in this paper, readily provides a general structure for fast orthogonal overlapped block transform algorithms. This new time-domain transparent formulation —overlapped block lattice structure solution— developped here for time invariant transforms also provides a simple procedure for designing time-varying wavelet packet

Transform type	γ_0	γ_1
ELT (I)	-1.2562	0.3494
Daubechies (II)	-1.7321	0.2679
Remez designed (III)	-1.8687	0.4143

Table 1: *Lattice parameters.*

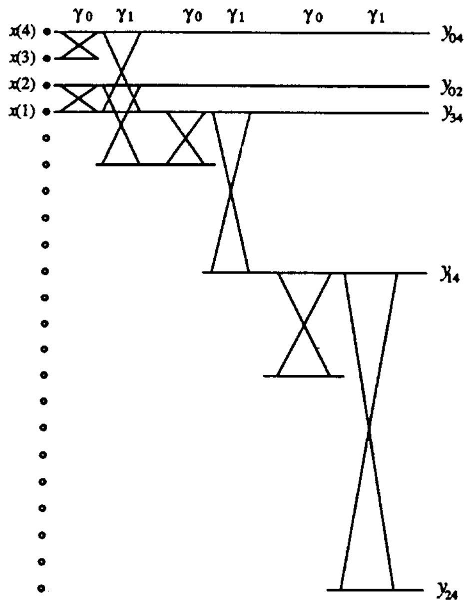

Figure 8: *Four-channel orthogonal wavelet packet transform with a downsampling factor of 4.*

Transf. type	low-pass filter $H(z)$
I	$0.5880 + 0.7386z^{-1} + 0.2581z^{-2} - 0.2055z^{-3}$
II	$0.4830 + 0.8365z^{-1} + 0.2241z^{-2} - 0.1294z^{-3}$
III	$0.4359 + 0.8146z^{-1} + 0.3374z^{-2} - 0.1806z^{-3}$

Table 2: *Prototype low-pass filters.*

transforms [9].

The main result of this work is a solution to the open problem of constructing perfect-reconstruction filter banks using polyphase block lattice structures as an alternative to cascaded lattice tree structures. This leads to fast short-time orthogonal overlapped block transform algorithms, with multiresolution time-spectral properties. Generalization of this new time-domain transparent formulation allows for an uncountable number of short-time wavelet packet transforms to be generated by choosing a type of criss-cross operator, a lattice block structure, a subsampling factor (lattice block shift), and the rotation parameters of the lattices.

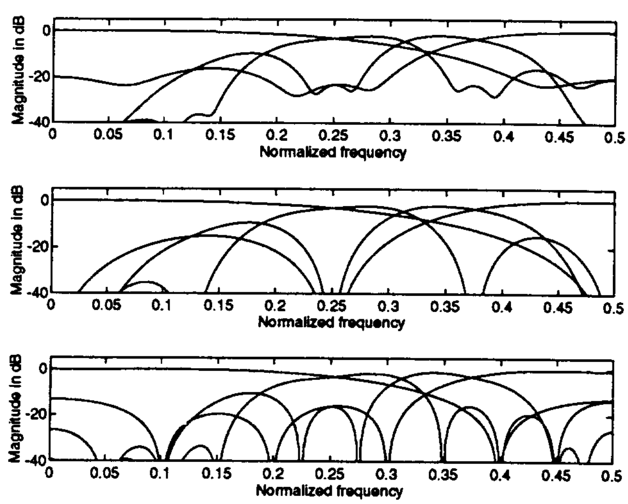

Figure 9: *Four-channel orthogonal wavelet packet transform magnitude responses. From top to bottom: ELT, Daubechies and Remez designed.*

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