BLIND LINEAR RECURSIVE EQUALIZER WITH DECORRELATION ALGORITHM

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ABSTRACT

High-speed communications suffer from ISI introduced by the channel. In order to combat ISI one commonly uses equalizers that need a training sequence to adjust the equalizer tap weights. When sending a training sequence is not appropriate, blind equalization has to be used. In this paper, a new blind linear equalizer is proposed.

The equalizer has a recursive structure. To avoid stability problems, a soft limiter is used in the feedback loop. The weights are controlled by means of the decorrelation algorithm.

For channels without precursors, the equalizer converges to the desired weights in around 500 iterations, depending on the amount of distortion. The novel blind equalization technique is globally convergent for minimum and non-minimum phase channels.

1. INTRODUCTION

The problem of decision feedback equalizers with linear feedforward sections was intensively researched by many authors [1], [2], [3]. The blind linear equalizer was also considered using LMS algorithms to find the optimum of cost functions which differ from one research to another [4], [5], [6]. However, since these functions were non-convex, the algorithms suffer from ill-convergence.

In recent work [7], [8], a blind decision feedback equalizer based on decorrelation of the data samples at the input of the slicer was proposed and studied. It was shown that the algorithm converges to the optimum weights irrespective of the initial conditions or error rate and hence, unlike other blind equalizer algorithms, it is globally convergent.

In this paper, a linear equalizer that uses the decorrelation algorithm in controlling the weights is studied. In dealing with the customary communication ISI channel (moving average) it was shown that if a FIR (transversal) filter is used, this algorithm might face some difficulties that need to be further researched to obtain successful equalization. However, when a linear recursive structure is used, the algorithm led to convergence of the weights to steady state value commensurate with the channel impulse response. Similar to the decision feedback equalizer it results in total illumination of ISI with no additive noise. Clearly this structure may have a stability problem if the channel is non-minimum phase (contains zeros outside the unit circle). However, simulations with a modified structure, which we termed "linear recursive equalizer with soft limiter," resulted in successful convergence to the correct weight values and hence cancellation of ISI even for non-minimum phase channels.

The importance of the linear equalizer in comparison to the decision feedback equalizer is in the ability of the former to cancel precursors in the channel response. Hence, a combination of both controlled by the decorrelation algorithm might give an important solution to the general blind equalization problem without the drawbacks of ill-convergence. The study of cancelling precursors with the blind recursive equalizer is ongoing.

2. BLIND LINEAR RECURSIVE EQUALIZER

Fig. 1 depicts the system model with equalizer structure in detail. The channel is modelled as an FIR filter with length N, having the transfer function

$$H(z) = 1 + h_1 z^{-1} + h_2 z^{-2} + \ldots + h_N z^{-N}$$

Discarding the soft limiter, the equalizer output is given by

$$A_k = Y_k - \sum_{i=1}^{N} w_i A_{k-i} = I_k + \sum_{i=1}^{N} h_i I_{k-i} - \sum_{i=1}^{N} w_i A_{k-i} \qquad (1)$$

where, for mathematical convenience, the number of equalizer taps is assumed to be equal to the number of channel taps. A limited discrepancy in the number of taps will only slightly degrade equalizer performance. In (1), I_k are the data symbols, assuming an i.d.d. random variable, $I_k \in \{-1,1\}$. Rewriting the equalizer output in matrix form, we get

$$A_k = I_k + \mathbf{I}'_{k-1}\,\mathbf{h} - \mathbf{A}'_{k-1}\,\mathbf{w} \qquad (2)$$

where prime stands for transpose and

$$I_{k-1} = [I_{k-1}, I_{k-2}, \ldots, I_{k-N}]^T, \ \mathbf{A}_{k-1} = [A_{k-1}, A_{k-2}, \ldots, A_{k-N}]^T,$$

$$\mathbf{h} = [h_1, h_2, \ldots, h_N]^T, \ \mathbf{w} = [w_1, w_2, \ldots, w_N]^T.$$

The equalizer weights are updated according to the decorrelation algorithm [7], yielding

$$w_i^{(k+1)} = w_i^{(k)} + \mu \overline{A_k A_{k-i}}, \quad i = 1, \ldots, N$$

To derive the steady state of the equalizer, we multiply (2) by $\mathbf{A}_{k-1}$ and obtain, after averaging:

$$\overline{A_k \mathbf{A}_{k-1}} = \overline{I_k \mathbf{A}_{k-1}} + \overline{\mathbf{A}_{k-1}\mathbf{I}'_{k-1}}\,\mathbf{h} - \overline{\mathbf{A}_{k-1}\mathbf{A}'_{k-1}}\,\mathbf{w} \qquad (3)$$

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The first term in (3) vanishes since the channel output does not depend on the data symbols in the future:

$$\overline{I_{k-m}A_{k-n}} = 0 \quad n>m \tag{4}$$

Let the matrix $\mathbf{B} = \overline{\mathbf{A}_{k-1}\mathbf{I}'_{k-1}}$. From (4), it is clear that $\mathbf{B}$ is an upperside triangle matrix. Evaluation of the elements on the main diagonal of $\mathbf{B}$ using (4) and the fact that the transmitted data symbols are uncorrelated, yields

$$\begin{aligned}\overline{A_{k-m}I_{k-m}} &= \overline{I_{k-m}I_{k-m}} + \overline{I_{k-m}I_{k-m-1}}h_1 + \ldots + \overline{I_{k-m}I_{k-m-N}}h_N \\ &\quad - \overline{I_{k-m}A_{k-m-1}}w_1 - \ldots - \overline{I_{k-m}A_{k-m-N}}w_N \\ &= \overline{I_{k-m}I_{k-m}} = \sigma_I^2 = 1 \quad ,k \geq m\end{aligned}$$

Hence we get

$$\mathbf{B} = \begin{bmatrix} 1 & \overline{A_{k-1}I_{k-2}} & \cdots & \overline{A_{k-1}I_{k-N}} \\ 0 & 1 & \cdots & \overline{A_{k-2}I_{k-N}} \\ \vdots & & & \vdots \\ 0 & \cdots & 0 & 1 \end{bmatrix} \tag{5}$$

The elements of $\mathbf{B}$ above the main diagonal can be written as ($n>m, k \geq n$)

$$\begin{aligned}\overline{A_{k-m}I_{k-n}} = \overline{I_{k-n}\{I_{k-m}} &+ \overline{I_{k-m-1}}h_1 + \ldots + \overline{I_{k-m-N}}h_N - \\ &\overline{A_{k-m-1}w_1 - \ldots - A_{k-m-N}w_N\}}\end{aligned}$$

Letting $n = m+l$ the following expression is obtained:

$$\begin{aligned}\overline{A_{k-m}I_{k-m-l}} &= \overline{I_{k-m-l}I_{k-m-l}}h_l - \overline{I_{k-m-l}A_{k-m-1}}w_1 - \\ &\quad \ldots - \overline{I_{k-m-l}A_{k-m-l}}w_l \\ &= h_l - w_l - \sum_{i=1}^{l-1}\overline{I_{k-m-l}A_{k-m-i}}w_i \quad ,l \geq 1\end{aligned}$$

This expression is valid for every m provided $n>m$ and $k>m+l$, thus we can take m equal zero, resulting in

$$\overline{A_k I_{k-l}} = h_l - w_l - \sum_{i=1}^{l-1}\overline{I_{k-l}A_{k-i}}w_i$$

Combining (3) and (5) yields for the correlation vector:

$$\overline{A_k\mathbf{A}_{k-1}} = \mathbf{Bh} - \overline{\mathbf{A}_k\mathbf{A}'_k}\mathbf{w} \tag{6}$$

3. STEADY STATE

In this paragraph, attention will be paid to the steady state behaviour of the equalizer. This means, the behaviour of the equalizer weights is analysed, assuming that they have converged to the values that result in decorrelation of the equalizer outputs, i.e., $E\{A_k A_{k-i}\} = 0, i = 1 \ldots N$. Here, $k-N$ must be larger than t_{st}, the time at which the steady state is reached. Furthermore, if $E\{A_k\} = 0$, the second moment of the stochastic variable A_k is found as:

$$E\{A_k A_k\} = \sigma_A^2 \tag{7}$$

where σ_A^2 is the variance of A_k. The fact that $E\{A_k\} = 0$ can be shown using

$$\overline{A_k} = \overline{I_k} + \overline{I_{k-1}}h_1 + \ldots + \overline{I_{k-N}}h_N - \overline{\mathbf{A}'_{k-1}}\mathbf{w} = -\overline{A_k}(w_1 + \ldots + w_N)$$

Assuming $w_1 + w_2 + \ldots + w_N \neq -1$,then $\overline{A_k}$ must be zero.

In steady state, when $k-N > t_{st}$, it follows from (7) and from the fact that $E\{A_k A_{k-i}\} = 0, i = 1 \ldots N$, that $\overline{\mathbf{A}_{k-1}\mathbf{A}'_{k-1}}$ in (6) is a diagonal matrix with σ_A^2 on the main diagonal. That is, with decorrelation we have

$$\mathbf{0} = \mathbf{Bh} - \sigma_A^2\mathbf{w} \tag{8}$$

To get an expression for the variance, we combine (2) and (7):

$$\begin{aligned}\sigma_A^2 &= \overline{(I_k + \mathbf{I}'_{k-1}\mathbf{h} - \mathbf{A}'_{k-1}\mathbf{w})(I_k + \mathbf{I}'_{k-1}\mathbf{h} - \mathbf{A}'_{k-1}\mathbf{w})} \\ &= \overline{I_k I_k} + 2\cdot\overline{I_k\mathbf{I}'_{k-1}}\mathbf{h} - 2\cdot\overline{I_k\mathbf{A}'_{k-1}}\mathbf{w} - 2\cdot\overline{\mathbf{I}'_{k-1}\mathbf{h}\mathbf{A}'_{k-1}\mathbf{w}} \\ &\quad + \overline{\mathbf{I}'_{k-1}\mathbf{h}\mathbf{I}'_{k-1}\mathbf{h}} + \overline{\mathbf{A}'_{k-1}\mathbf{w}\mathbf{A}'_{k-1}\mathbf{w}}\end{aligned} \tag{9}$$

In the appendix we show that (8) is equivalent to

$$\sigma_A^2 = [1 - \mathbf{w}'\mathbf{w}]^{-1}[1 - 2\mathbf{w}'\mathbf{Bh} + \mathbf{h}'\mathbf{h}] \tag{10}$$

Now one can easily show that the ISI is zero if and only if $\mathbf{w} = \mathbf{h}$. It could also be shown that if $\sigma_A^2 = 1$ and $\mathbf{w} = \mathbf{h}$ then the output of the equalizer is decorrelated. However it is difficult to show, due to the non-linear property of (8), that in the general case decorrelation leads to $\mathbf{w} = \mathbf{h}$ and hence zero ISI. However, a few examples presented in next section lead us to conclude that the answer to the question is positive.

4. EXAMPLES

To elucidate the problem, the equalizer weights will be calculated for $N = 2$. The equations obtained will be combined with (10) for the variance. This will result in possible solutions for the equalizer weights in steady state.

For $N = 2$, and assuming the equalizer is in steady state, i.e., the equalizer outputs are decorrelated, we have:

$$\begin{bmatrix} \overline{A_k A_{k-1}} \\ \overline{A_k A_{k-2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

It then follows from the general expression for the correlation vector (6) that

$$w_1 = \frac{h_1(1+h_2)}{\sigma_A^2 + h_2}, \quad w_2 = \frac{h_2}{\sigma_A^2} \tag{11}$$

Evaluating the expression for the variance yields

$$\begin{aligned}\sigma_A^2 &= (1 - \mathbf{w}'\mathbf{w})^{-1}\left(1 - 2\{\mathbf{h}'\mathbf{w} + h_2 w_1\overline{I_{k-2}A_{k-1}}\} + \mathbf{h}'\mathbf{h}\right) \\ &= \frac{(1 - 2h_1w_1 - 2h_2w_2 - 2h_1h_2w_1 + 2h_2w_1^2 + h_1^2 + h_2^2)}{1 - w_1^2 - w_2^2}\end{aligned} \tag{12}$$

To get more insight in the solutions of σ_A^2, w_1 and w_2 for different channels, Eqs. (11) and (12) are solved for three numerical examples:

1. $H_1(z) = 1 + 0.5z^{-1} + 0.3z^{-2}$ minimum phase, $\mathbf{h}^T\mathbf{h} < 1$;
2. $H_2(z) = 1 + 0.9z^{-1} + 0.7z^{-2}$ minimum phase, $\mathbf{h}^T\mathbf{h} > 1$;
3. $H_3(z) = 1 + 0.9z^{-1} - 0.7z^{-2}$ non-minimum phase.

The corresponding solutions, generated by "Mathematica," can be found in Tables 1, 2 and 3. Complex solutions are not depicted.

σ_A^2	w_1	w_2
0.09	1.67	3.33
1	0.5	0.3

Table 1. Solutions for ex. 1: $h_1 = 0.5, h_2 = 0.3$

σ_A^2	w_1	w_2
0.49	1.29	1.43
1	0.9	0.7

Table 2. Solutions for ex. 2: $h_1 = 0.9, h_2 = 0.7$

σ_A^2	w_1	w_2
0.25	-0.6	-2.8
0.49	-1.29	-1.43
1	0.9	-0.7
1.96	0.21	-0.35

Table 3. Solutions for ex. 3: $h_1 = 0.9, h_2 = -0.7$

In examples 1 and 2, in which the channel is minimum phase, we observe that the equalizer weights corresponding to unit variance are the correct ones, i.e., for unit variance the ISI is cancelled exactly. Since the channel is minimum phase, the poles of the recursive equalizer with correct weights are inside the unit circle, hence the equalizer is stable.

With respect to example 3, we observe that the first solutions in Table 3 result in an equalizer with poles outside the unit circle, i.e., the equalizer is unstable. Only one set of weights, the set corresponding to the largest variance, forces the poles of the equalizer to be inside the unit circle, yielding a stable equalizer. However, these weights do not yield zero ISI since $\mathbf{w} \neq \mathbf{h}$. In fact, simulations show that the equalizer converges to the 'stable' weights, i.e., no ISI cancellation.

To avoid instability of the equalizer, a soft limiter is implemented in the feedback loop, as depicted in Fig. 1. The soft limiter performs the following operation:

$$Out = \begin{cases} In & \text{for } |In| \le 1 \\ \mp 1 & \text{for } In < -1, In > 1 \end{cases} \quad (13)$$

where In and Out are the limiter input and limiter output, respectively. Clearly, the soft limiter has a slope of 45°.

Due to the limiter, the equalizer can operate in two different modes. The first mode of operation is when $|In| \le 1$, in which the limited equalizer simplifies to the regular recursive equalizer. In the second mode, i.e., when $|In| > 1$, the limiter slices the input, or, in other words, it acts as a decision device, and hence it behaves as a decision feedback equalizer [7] in this case.

Simulations with this scheme show that the recursive equalizer with a soft limiter converges to the correct weights, irrespective of whether the channel is minimum or non-minimum phase.

5. SIMULATIONS

In this paragraph, learning curves and admissibility plots [9] of the equalizer will be shown. The figures show the tap weights obtained by taking the Monte Carlo average of 200 experiments. The step size μ equals 0.01; the channel is assumed to be noiseless.

Fig. 2 depicts the learning curves for the blind linear recursive equalizer (BLE) without a limiter, compared with the decorrelation decision feedback equalizer (DFE) for a channel according to example 1. We observe that the convergence speed is approximately equal for both equalizers. In Fig. 3 the learning curves for the BLE with and without a limiter are presented. There is a slight increase in convergence speed when adding the limiter.

Fig. 4 shows the learning curves for the BLE with a soft limiter and the DFE for a channel according to example 3. One can observe that the BLE converges rather slow in this case.

In Fig. 5 and 6 the admissibility plots for the first and third example, respectively, are presented. From these plots one can conclude that the blind linear recursive equalizer is globally convergent.

6. CONCLUSIONS

In this paper we examined the behaviour of a blind equalizer, having a recursive structure and being controlled by the decorrelation algorithm. It was shown that stability problems can be avoided by implementing a soft limiter in the equalizer's feedback loop.

Steady state solutions were obtained mathematically. Due to some analysis difficulties, sufficient proof is still being researched. Nevertheless, many simulations show positive results. The convergence speed of the recursive equalizer is slightly lower than that of the decorrelation DFE. However, when the channel is heavily distorted, the decrease in convergence speed becomes significant.

The recursive equalizer with soft limiter converges to the correct weights, regardless of the initial settings.

REFERENCES

[1] Proakis, J.G., "Adaptive Equalization for TDMA Digital Mobile Radio," IEEE Trans. on Veh. Techn., Vol. 40, No. 2, May 1991

[2] Qureshi, S.U.H., "Adaptive Equalization," Proc. of the IEEE, Vol. 73, No. 9, pp. 1349-1387, Sept. 1985

[3] Clark, A.P., Equalizers for Digital Modems, Pentech Press 1985.

[4] Sato, Y, "A Method of Self-Recovering Equalization for Multilevel Amplitude-Modulation Systems," IEEE Trans. on Commun., Vol. com-23, pp. 679-682, June 1975.
[5] Godard, D.N., "Self-Recovering Equalization and Carrier Tracking in Two-Dimensional Data Communication Systems," IEEE Trans. on Commun., Vol. com-28, No. 11, pp.1867-1875, Nov. 1980.
[6] Bellini, S., "Bussgang Techniques for Blind Equalization," Proc. Globecom '86, pp. 46.1.1-46.1.7, 1986.
[7] Kamel, R.E. and Bar-Ness, Y., "Decorrelation Algorithm for Blind Decision Feedback Equalization," presented at Comm. Theory Mini Conference/Globecom '93, Nov. 1993.
[8] Kamel, R.E. and Bar-Ness, Y., "Error Performance of Blind Decision Feedback Equalizer using Decorrelation," to be presented at Milcom 1994, New Jersey.
[9] Johnson, C.R., "Admissibility in Blind Adaptive Channel Equalization," IEEE Control Systems Magazine, pp. 3-15, Jan. 1991

APPENDIX

We will evaluate the terms in this equation to derive a simpler form. The first term in this expression clearly equals one. Since we have uncorrelated data symbols, the second term vanishes. The equalizer outputs $\mathbf{A}_{k-1}$ do not depend on future data symbols, (4), so the third terms vanishes too. Skipping the fourth term for this very moment, the fifth term can be reduced:

$$\overline{\mathbf{I}'_{k-1}\mathbf{h}\mathbf{I}'_{k-1}\mathbf{h}} = \overline{\left[h_1 I_{k-1}+\ldots+h_N I_{k-N}\right]\left[h_1 I_{k-1}+\ldots+h_N I_{k-N}\right]} = \mathbf{h}'\mathbf{h}$$

In the same way, the last term of (9) simplifies to

$$\overline{\mathbf{A}'_{k-1}\mathbf{w}\mathbf{A}'_{k-1}\mathbf{w}} = \sigma_A^2\mathbf{w}'w$$

Evaluation of the fourth term in (9) results in:

$$\overline{\mathbf{I}'_{k-1}\mathbf{h}\mathbf{A}'_{k-1}\mathbf{w}} = \overline{\mathbf{I}'_{k-1}\mathbf{h}\mathbf{w}'\mathbf{A}_{k-1}} = \mathrm{trace}\left(\overline{\mathbf{A}_{k-1}\mathbf{I}'_{k-1}\mathbf{h}\mathbf{w}'}\right)$$

$$= \mathrm{trace}\left(\mathbf{B}\mathbf{h}\mathbf{w}'\right) = \mathbf{w}'\mathbf{B}\mathbf{h}$$

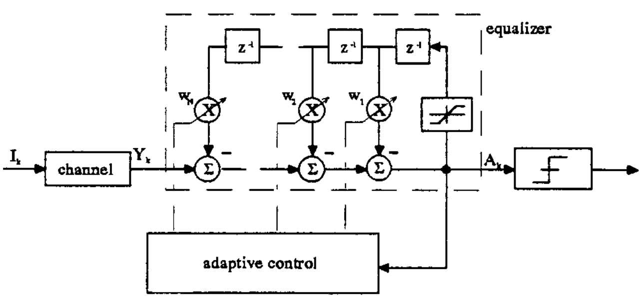

Fig. 1 Recursive equalizer with soft limiter

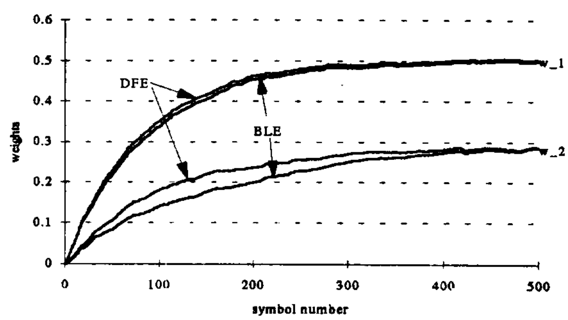

Fig. 2 Learning curve for the BLE and DFE, $H_1(z)$ channel

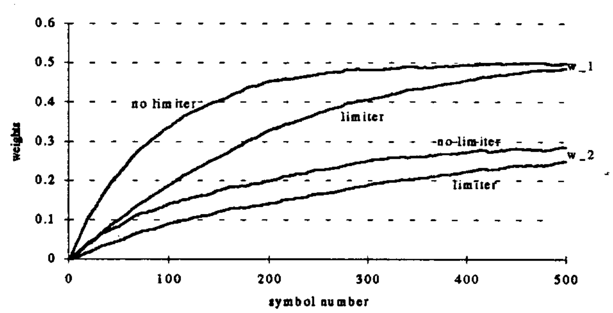

Fig. 3 Learning curve of BLE with and without limiter, $H_1(z)$ channel

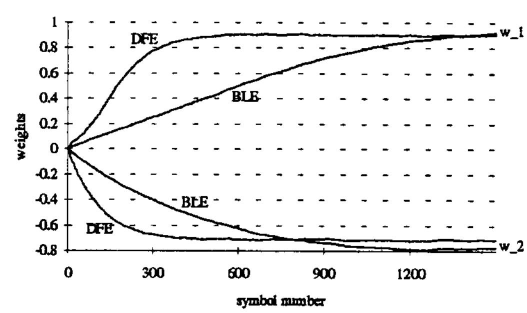

Fig. 4 Learning curve of BLE with limiter and DFE, $H_3(z)$ channel

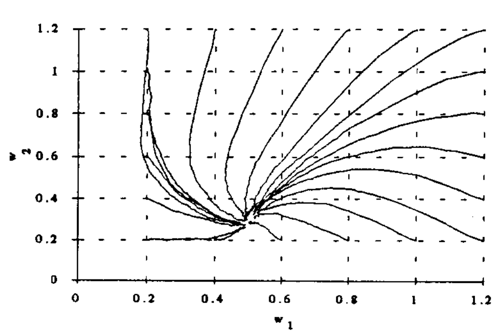
Fig. 5 Admissibility curve for BLE without limiter, $H_1(z)$ channel

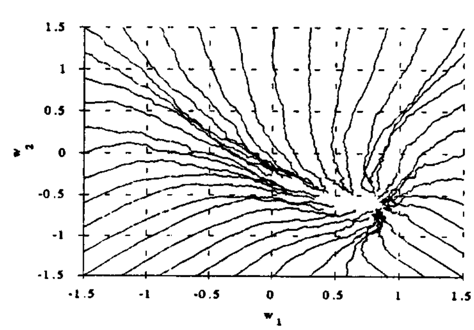
Fig. 6 Admissibility curve for BLE with limiter, $H_3(z)$ channel