ON-LINE IDENTIFICATION OF ECHO-PATH IMPULSE RESPONSES BY HAAR-WAVELET-BASED ADAPTIVE FILTER

Miloš Doroslovački
Compunetix, Inc.
2000 Eldo Road
Monroeville, Pennsylvania 15146
U.S.A.

H. (Howard) Fan
Department of ECE
University of Cincinnati
Cincinnati, Ohio 45221-0030
U.S.A.

ABSTRACT

Using a small number of coefficients in Haar-wavelet-based models we can efficiently identify echo paths which have certain typical impulse response shapes. The obtained energy of modeling error is low (less than 2%). A simple wavelet-based LMS adaptive filter can be used for on-line estimation of coefficients. A low number of time-consuming computations is obtained per input sample due to the usage of Haar wavelets. This number is less than the ones obtained by FIR or DFT domain based modeling.

1. INTRODUCTION

Long-distance telephone communications of good quality require absence of echos. Examples of typical impulse responses of the echo path of four-to-two wire hybrid in a long-distance telephone system are given in Figures 1 and 2. We can notice that the impulse responses have a long tail. Also, they change fast at the beginning and slow at the end. Physical characteristics of an echo path are time varying and the impulse responses measured at different times can differ from each other. This means that an echo canceller must track the changes. If we can have a good representation of the impulse responses by a small number of coefficients multiplying some known functions and if the computation of the coefficients is not time consuming, we can expect fast adaptation and tracking in on-line identification of echo paths. The FIR model of the impulse responses [1] will require many significant coefficients for a small modeling error, especially with fast sampling. A good IIR model will also require many poles and zeros. The slow decay of the impulse responses is not appropriate for modeling by functions with an exponential decay, for one will need poles or multiple poles close to the unit circle. This shifts the problem to the convergence of adaptive IIR algorithms. We can experience a lack of convergence or a very slow one. Another approach that is usually resorted to is to try modeling in the DFT-domain [2]. Much computational savings are achieved at the expense of somewhat slower convergence due to less frequent updating. In this paper we propose modeling by wavelets which is supposed to give economical bases for signals composed of short and long components of similar shapes. It will be shown that our approach has the advantages of few adaptive coefficients, thus low computational complexity, with a convergence speed that is at least as good as that of the FIR LMS algorithm.

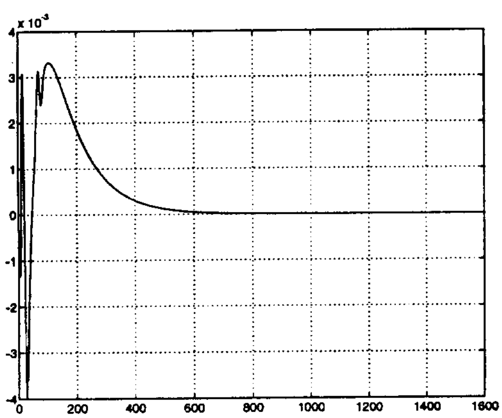

Figure 1: Impulse response of an echo path

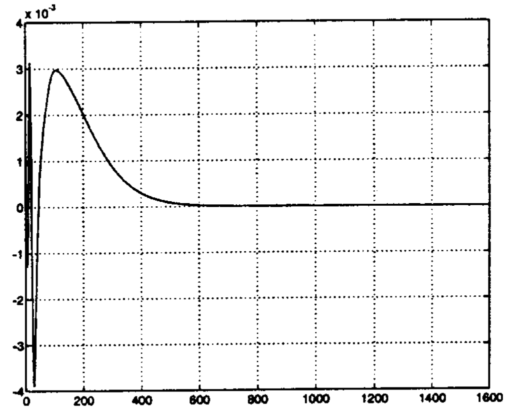

Figure 2: Impulse response of the same echo path as in Fig. 1 but measured at a different time

2. HAAR-WAVELET-BASED APPROACH

For on-line identification of echo-path impulse responses a wavelet-based adaptive filter is used. A theoretical setting for the filter is given in [3]-[5]. A ma-

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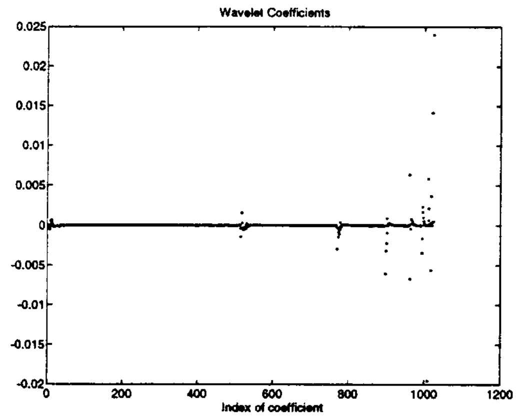

Figure 3: Haar-wavelet coefficients of the impulse response in Fig. 1

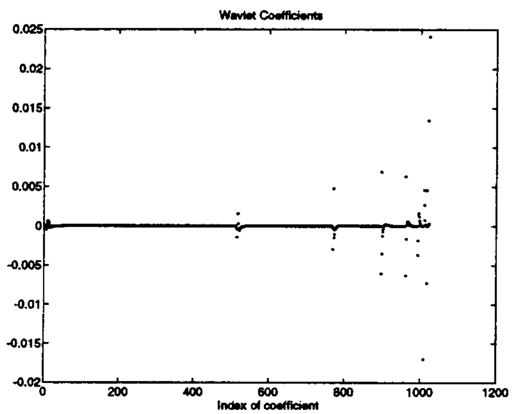

Figure 4: Haar-wavelet coefficients of the impulse response in Fig. 2

jor issue in such modeling is the choice of wavelets. They can have different shapes (e.g., symmetric, anti-symmetric, asymmetric, oscillatory), lengths and inter-relationships (e.g., orthogonal, semi-orthogonal, non-orthogonal). In this application we have tried different kinds of wavelets such as Daubechies' orthogonal [6] and some non-orthogonal (biorthogonal) ones [7]. For the class of impulse responses we are considering and with the objective that the energy of modeling error being less than 2%, the smallest number of coefficients for representation is obtained using Haar wavelets.

The Haar wavelets are discrete-time orthonormal sequences $\psi_{mn}(t)$, defined by

$$\psi_{mn}(t) = \psi_{m0}(t - 2^m n), \tag{1}$$

and

$$\psi_{m0}(t) = \begin{cases} 2^{-\frac{m}{2}}, & \text{for } 0 \le t \le 2^{m-1} - 1; \\ -2^{-\frac{m}{2}}, & \text{for } 2^{m-1} \le t \le 2^m - 1; \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The indices m and n correspond to the scale and translation respectively. Here m is a natural number and n is an integer. Let $h(t)$ be the impulse response of some discrete-time linear time-invariant system, then we can write

$$h(t) = \sum_{(m,n)\in\mathcal{D}} a_{mn}\psi_{mn}(t), \tag{3}$$

where $\psi_{mn}(t)$ belongs to a set $\mathcal{D}$ of discrete-time wavelets, e.g., the Haar wavelets given by (1) and (2). The LMS algorithm is used for the adaptation of filter coefficients $\hat{a}_{mn}$, i.e.,

$$\hat{a}_{mn}(t+1) = \hat{a}_{mn}(t) + \mu r_{mn}(t)e(t), \tag{4}$$

where μ is the adaptation gain, $e(t)$ the error between the desired signal and adaptive filter output

$$\hat{y}(t) = \sum_{(m,n)\in\hat{\mathcal{D}}} r_{mn}(t)\hat{a}_{mn}(t), \tag{5}$$

and $r_{mn}(t)$ the convolution of the input signal $u(t)$ and wavelet $\psi_{mn}(t)$

$$r_{mn}(t) = \sum_{l} \psi_{mn}(t-l)u(l). \tag{6}$$

The index set $\hat{\mathcal{D}}$ satisfies $\hat{\mathcal{D}} \subseteq \mathcal{D}$, i.e., for practical purpose "reduced order" modeling is considered. The desired signal is the output of an echo-path "unknown system" and it can be expressed using (3) as

$$y(t) = \sum_{(m,n)\in\mathcal{D}} r_{mn}(t)a_{mn}(t). \tag{7}$$

The inputs of the unknown system and adaptive filter are the same (the system identification configuration). They are white noise having Gaussian or bipolar (± 1) distribution. Selection of the set of Haar wavelets which will be used in the adaptive filter, i.e. the specification of $\hat{\mathcal{D}}$, is made based on a set of measured impulse responses. The wavelet coefficients a_{mn} of the impulse responses in Figures 1 and 2 are shown in Figures 3 and 4. Note that we have mapped the two-index pair (m, n) to one index (let us denote it by i) in order to draw the coefficient values. More precisely, vectors of wavelet coefficients for different scales are mutually concatenated. Assume that the scale parameter m goes from 1 to M, then we can write

$$i = 2^{M+1} - 2^{M+2-m} + n, \tag{8}$$

where

$$0 \le n \le 2^{M+1-m} - 1. \tag{9}$$

The last two indices $i = 2^{M+1} - 2$ and $i = 2^{M+1} - 1$ are reserved for coefficients of the scaling functions $\phi_{M0}(t)$ and $\phi_{M1}(t)$. The scaling functions are not explicitly given in (3) since they can be represented through wavelets at scales $m > M$. Note that $\phi_{M0}(t)$ is a rectangle having the height $2^{-\frac{M}{2}}$ and defined for $t = 0, 1, \cdots, 2^{M-1}$. The scaling function $\phi_{M1}(t)$ is

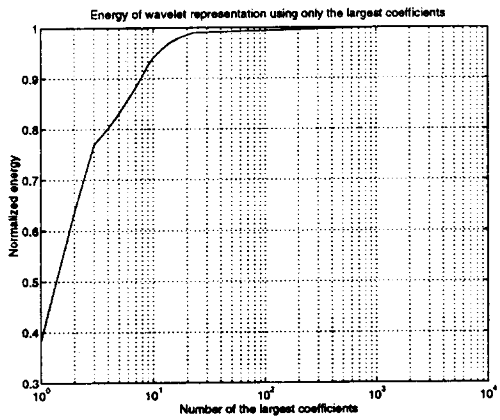

Figure 5: Number of the largest coefficients and corresponding energy

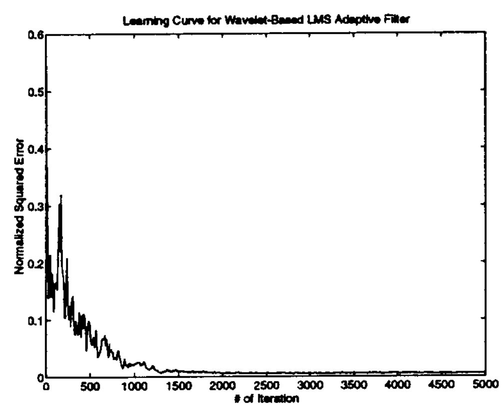

Figure 6: Learning curve for Haar-wavelet-based LMS adaptive filter in identifying echo path of Fig. 1

$\phi_{M0}(t)$ translated by 2^M. The total number of the wavelet and scaling-function coefficients is 2^{M+1}. It can be seen from Figures 3 and 4 that these are sparse representations, i.e. only a small number of coefficients is significantly different from zero. It is also seen that the coefficient sets for the two models heavily overlap. We have chosen for representation the largest coefficients representing 98% of the impulse response energy. The number of the largest coefficients and the corresponding energy of representation for the impulse response in Figure 1 is given in Figure 5. It is seen that only about 30 or so coefficients are needed for 98% energy representation. The energy plot for the model in Figure 2 is similar to Figure 5, requiring even fewer coefficients. The final $\hat{\mathcal{D}}$ is obtained by taking the union of the coefficients for different known (measured) impulse responses.

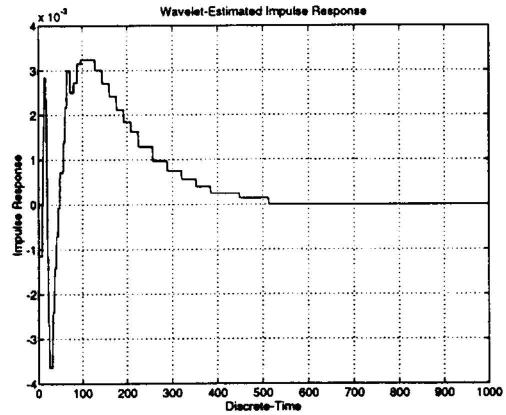

Figure 7: Haar-wavelet-estimated impulse response of echo path in Fig. 1

3. RESULTS

It appears that the total number of representation sequences $\psi_{mn}(t)$ and $\phi_{Mn}(t)$ is 35 and that we have to use nine different scales, i.e. $1 \leq m \leq 9$. The adaptation gain of the LMS algorithm is chosen such that the convergence occurs and the misadjustment is small enough that we can estimate the modeling mean square error from constant part of the learning curve, e.g. the one in Figure 6. For the same modeling error an FIR adaptive filter would need about 350 coefficients. That is, the Haar-wavelet-based adaptive filter needs much fewer coefficients and consequently it saves computation and can also converge faster. A comparison with the DFT based approaches is not straightforward, due to their block processing nature. Nevertheless, from Table 2 of [2] some DFT based algorithms take about one tenth of the computations of the LMS algorithm, similar to our results. However, our algorithm is being updated at every sample, not once per block as the DFT based algorithms, and hence we do not sacrifice the convergence speed. In fact, wavelet-based LMS adaptive filters generally have even faster convergence than the FIR LMS algorithm. See [4] for details.

The Haar-wavelet-estimated impulse response of the echo-path in Figure 1 is shown in Figure 7. The staircase nature of the approximation can be easily seen. This introduces a low-energy high-frequency content which is not present in the original impulse response. Our approach results in a small energy of modeling error but an open question is whether this error is perceptually acceptable. Low errors of some other types (frequency selective, L_1-norm, etc.) may be more appropriate. An easy way to suppress effects of the small high-frequency content is to apply low-pass filtering to the output of the adaptive filter. The filtered output can be used to cancel the echo. The low-pass filter should have characteristics satisfying perceptual requirements. The low-pass filter that already exists at the D/A converter of the echo canceller may well serve the purpose. Simulations show that changes in the input signal dis-

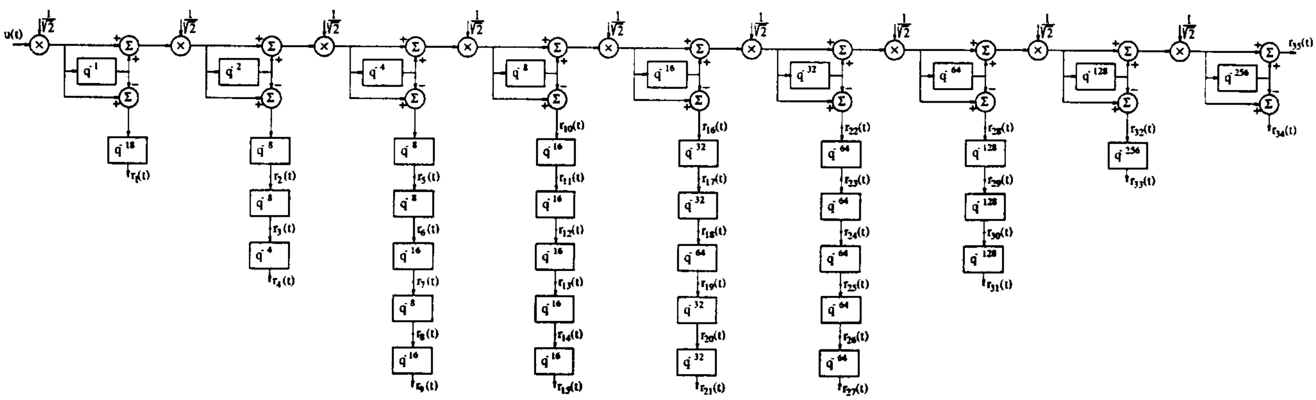

Figure 8: Haar-wavelet convolver

	×	±
Convolver	M	2M
Adap. filter output	C	C-1 1
LMS update	C	C
Total	M+2C	2M+2C
	3M+4C	

Table 1: Number of multiplications and additions/ subtractions per iteration

tribution type (Gaussian or bipolar) does note affect significantly the results of identification.

Since we are using the Haar wavelets, the implementation of the convolution (6) needs only a minimal number of multiplications, which can be attractive from a computational point of view. In Figure 8 we can see nine stages of a Haar-wavelet convolver where each stage consists of shift registers, two additions/subtractions, and one multiplication. The multiplication provides the energy normalization for wavelets used in the convolutions. Each stage corresponds to one scale m and the shift registers within one stage correspond to the translation parameter n. The outputs of the convolver are denoted using one index which takes values from 1 to 35. The convolution between the input signal $u(t)$ and the scaling function $\phi_{9,0}(t)$ is given by the output $r_{35}(t)$. The number of multiplications and additions/subtractions is given in Table 1, where M is the number of scales used and C the number of coefficients. Since typically M and C are small (in our example $M = 9$ and $C = 35$), the total number of computations is much less than that in the FIR modeling and less than that in the DFT domain modeling [2].

The (relatively) fast convergence of the wavelet-based adaptive filters makes them suitable for tracking time-varying echo paths. As long as $\hat{\mathcal{D}}$ is chosen appropriately, the LMS algorithm presented in this paper can be used for tracking slowly time-varying echo paths with μ a trade-off factor between the tracking speed and the residual mean square error. For fast time-varying echo paths, the approach of [8] can be used in a similar fashion following the development of this paper.

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