A CONTINUOS-TIME ADAPTIVE FILTER STRUCTURE

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ABSTRACT

Adaptive filters have been traditionally developed in a digital environment which involves large number of computations to get the coefficients that make the desired approximation. Most of the time, this calculations required a great capacity machines and that is not practical for some applications like channel equalization in cellular systems. This paper proposes a continuos-time adaptive filter which is based on representing the impulse response of adaptive filter as a linear combination of a set of orthogonal exponentials. An important practical advantage of it is that if a satisfactory representation can be obtained by exponentials and simple filter structures can be synthesized.

1. INTRODUCTION

Adaptive filters have been used in many practical systems, as echo and noise cancelers, equalizers and predictors, etc. Such signal processing applications have relied on adaptive digital filters. That is because, with the advance of digital technology, the digital signal processors (DSP) became more powerful allowing implementation of more efficient adaptive digital algorithms. However, in most real time applications the signal bandwidth must be kept within the audio range, or systems with several DSPs operating in parallel or master slave configuration must be used, because of processing speed limitations.

Another important problem is the convergence rate for tracking variations on the statistics of input and reference signals. It is well known that algorithms with relatively high computational complexity (RLS) have very fast initial convergence, while the algorithms with low computational complexity (LMS) have a slower convergence rates. Because the convergence rate and computational complexity are very important issues in most adaptive filter applications, great efforts have been carried out for reduce computational complexity of adaptive filters, and improved their convergence and tracking ability. However even if the characteristics of the adaptive filters algorithms are greatly improved, it is very difficult to achieved very high processing speed (sampling rates on the order of several mega hertz) due to technological limitations. These problems suggest the necessity of development of reliable continuous time adaptive filter structures for applications in which the bandwidth of input and reference signals is very wide (several hundred of kilohertz) [1], or for application which require very fast initial convergence and tracking, such a channel equalization of cellular telephones, because continuous time adaptive filters would be able to provide extremely fast convergence rates and smaller size than digital adaptive filters.

In following sections it will show an analog adaptive filter structure that improve time convergence of conventional relizations usinga continuos time LMS algorithm to reduce the error between the reference system and the adaptive filter.

2. PROPOSED STRUCTURE.

Orthogonal polynomials such as Laguerre, Legendre, Hermite and Chebyshev, have been widely used in the derivation of several system identification algorithms, most of them involving matrix operations. However, since proposed adaptive filter structure is intented to be used when large order adaptive filters are requiered and is to be implemented in a continuos-time way, the based of functions and the approach used to generate it, most lead into a simple structure and easy implementations. A suitable set is the Laguerre orthogonal functions, due to the fact that, by using a simple change of variables, they can be rewritten as a set of orthogonal exponentials. An important practical advantage is that if a satisfactory representation can be obtained by exponentials, simple filter structures can be synthesized.

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The modified Laguerre functions, orthogonal on the interval $[0,\infty)$, are the result of the orthogonalization of the function e^{-at} in the interval $[0,\infty)$. [1].

By using the method of orthogonalization in frecuency domain proposed by Lee [1], it is found that the modified Laguerre functions can be generated as the impulse response of a network with transfer function given by

$$P_m(s) = \frac{1}{s+a} \prod_{i=1}^{m-1} H_i(s) \tag{1}$$

where

$$H_i(s) = \frac{s-a}{s+a} \tag{2}$$

and a is a positive constant selected such that the multiple pole be larger than the higher frecuency on the transfer function of the system to be identified.

It can be probed that since modified Laguerre functions form a complete set of orthogonal functions on the interval $[0,\infty)$ the impulse response of any causal systems can be represented, approximately, by the first N terms of a convergent Laguerre series expansion given by

$$h(t) = \sum_{m=1}^{N} A_m\, p_m(t) \tag{3}$$

where A_m are the coefficients expansion chosen such that the mean square value of approximation error is kept to a minimum.

To derive the algorithm for on-line estimation of the expansion coefficients, consider a causal linear system with impulse response h(t) and input signal x(t). If y(t) denotes the output signal of h(t) and d(t) is the desired response, then the output error will be given by

$$e(t) = d(t) - y(t) \tag{4}$$

$$e(t) = d(t) - \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau \tag{5}$$

Thus, sustituting (3) into (5) it follows that

$$e(t) = d(t) - \int_{-\infty}^{t} x(\tau) \sum_{m=1}^{N} A_m\, p_m(t-\tau)\, d\tau \tag{6}$$

$$e(t) = d(t) - \sum_{m=1}^{N} A_m \int_{-\infty}^{t} x(\tau)\, p_m(t-\tau)\, d\tau \tag{7}$$

$$e(t) = d(t) - \sum_{m=1}^{N} A_m\, u(t) \tag{8}$$

where

$$u(t) = \int_{-\infty}^{t} x(\tau)\, p_m(t-\tau)\, d\tau \tag{9}$$

denotes de output signal of a system with impulse response $p_m(t)$ and input signal x(t).

Equation (8) represents output error of a finite impulse response (FIR) system. Then almost any FIR type adaptive algorithm may be used. Because the signals u(t) are continuos, a suitable choice could be the continuos time LMS algorithm given by

$$A_m = \mu \int_{0}^{t} e(\tau)\, u(\tau)\, d\tau \tag{10}$$

where μ is a factor that controls stability and convergence rate. Equations (1), (8) and (10) lead to the proposed adaptive filter structure shown in figure # 1.

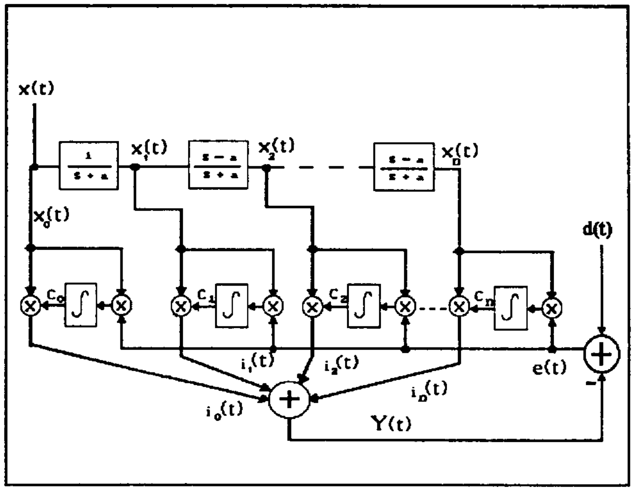

Figure # 1

2.1. Convergence of the continuos time LMS algorithm.

Consider the continuos time LMS algorithm given by

$$A_k(t) = -\mu \int_0^t \overline{\nabla}_k(\tau)\, d\tau \qquad (11)$$

where

$$\overline{\nabla}_k(t) = -2e(\tau)\, u_k(t) \qquad (12)$$

and

$$e(t) = d(t) - \sum_{m=1}^{N} A_m(t)\, u_m(t) \qquad (13)$$

for k = 1, 2, ..., N.

Using matrix notations, equations (11) to (13) can be rewritten as

$$\mathbf{A}(t) = -\mu \int_0^t \overline{\nabla}(\tau)\, d\tau \qquad (14)$$

where

$$\overline{\nabla}(t) = -2d(t)\, \mathbf{U}(t) + 2\, \mathbf{U}(t)\, \mathbf{U}^T(t)\, \mathbf{A}(t) \qquad (15)$$

Taking the expectation of (14), (15) asuming that the weights are uncorrelated with the input signal it follows that

$$E[\mathbf{A}(t)] = -\mu \int_0^t \nabla(\tau)\, d\tau \qquad (16)$$

where

$$\nabla(t) = -2E[d(t)\mathbf{U}(t)] + 2E[\mathbf{U}(t)\mathbf{U}^T(t)]\, E[\mathbf{A}(t)] \qquad (17)$$

Taking the derivated of (16) respect to t we obtain

$$\frac{d}{dt} E[\mathbf{A}(t)] = -\mu \nabla(t) \qquad (18)$$

$$\frac{d}{dt} E[\mathbf{A}(t)] = 2\mu \mathbf{P} - 2\mu \mathbf{R}\, E[\mathbf{A}(t)] \qquad (19)$$

$$\frac{d}{dt} E[\mathbf{A}(t)] = -2\mu \mathbf{R}\, E[\mathbf{A}(t)] + 2\mu \mathbf{P} \qquad (20)$$

It can be shown that the solution of (20) is

$$E[\mathbf{A}(t)] = \mathbf{R}^{-1}\mathbf{P} + e^{-2\mu\mathbf{R}t}\,(\mathbf{W}(o) - \mathbf{R}^{-1}\mathbf{P}) \qquad (21)$$

Sustracting $\mathbf{R}^{-1}\mathbf{P}$ in both sides of (21) and taking the limit

$$E[\mathbf{A}(t)] - \mathbf{R}^{-1}\mathbf{P} = e^{-2\mu\mathbf{R}t}\,(\mathbf{W}(o) - \mathbf{R}^{-1}\mathbf{P}) \qquad (22)$$

when t goes to infinite it follows that

$$\lim_{t\to\infty} (E[\mathbf{A}(t)] - \mathbf{R}^{-1}\mathbf{P}) = \lim_{t\to\infty} (e^{-2\mu\mathbf{R}t}\,(\mathbf{W}(o) - \mathbf{R}^{-1}\mathbf{P})) \qquad (23)$$

$$\lim_{t\to\infty} (E[\mathbf{A}(t)] - \mathbf{R}^{-1}\mathbf{P}) = 0 \qquad (24)$$

Thus as $t \to \infty$, $E[\mathbf{A}(t)]$ approaches the optimal Wiener solution. From (23), it follows that the convergence factor μ can be set arbitrarily large but greater than zero.

2.2 Analog structure.

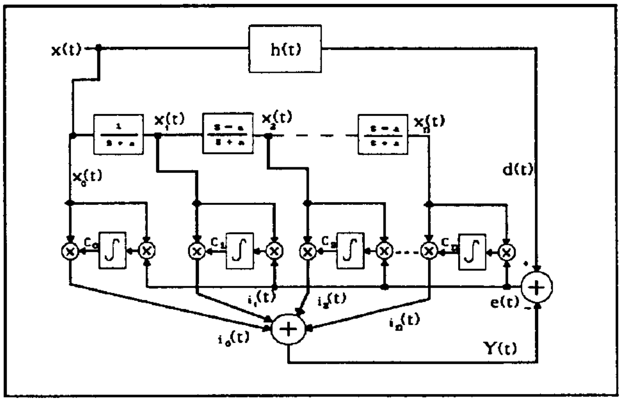

Figure # 2

Figure # 2 shows the analog LMS algorithm structure with the delay line formed in concordance with Laguerre functions, working as a identification configuration.

Signal x(t) is feeded into delay line and into the h(t) block which is the block that wants to be identified. The desired signal d(t) is given by the convolution between x(t) and h(t). This signal is feeded into a difference circuit in order to produce the error signal like equation (13). For the figure # 2, signal Y(t) is given by

$$Y(t) = \sum_{n=0}^{n} i_{out_n} \qquad (25)$$

where the output currents are given by:

$$i_{out_n} = c_n \, x_n(t) \qquad (26)$$

The coefficients c_n are calculated by the circuit formed by the two multiplying circuits and the integrator, which conform the analog LMS algorithm. The signal error e(t) is passed to the first multiplying and is multiplied with the corresponding signal $x_n(t)$, then, this product is integrated yielding the coefficient c_n. The factor 2μ is related with the RC parameters of the integrator circuit. The coefficient c_n is then multiplied with the signal $x_n(t)$ yielding the output current that is passed to the add circuits to produce the output signal Y(t) of the transversal filter.

3. RESULTS

The results that are showed in this paper was obtained by computational simulations (PSPICE simulations) using some VLSI structures to implement the proposed adaptive filter. Figure # 3 show a transfer functon approximated by the propose configuration in which it can observed that the working range is between 10mHz to 1 MHz. This simulation was generated with 3 coefficients. Figure # 4 shows the corresponding error function.

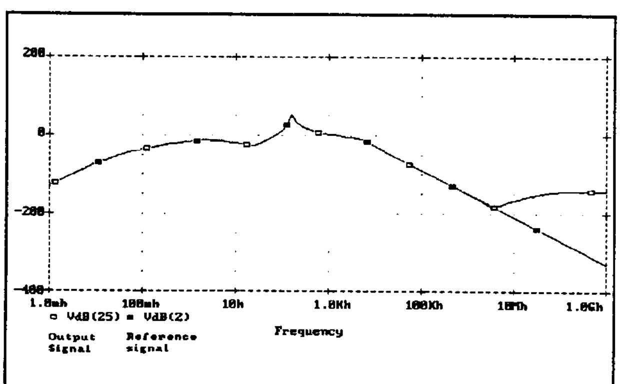

Figure # 3

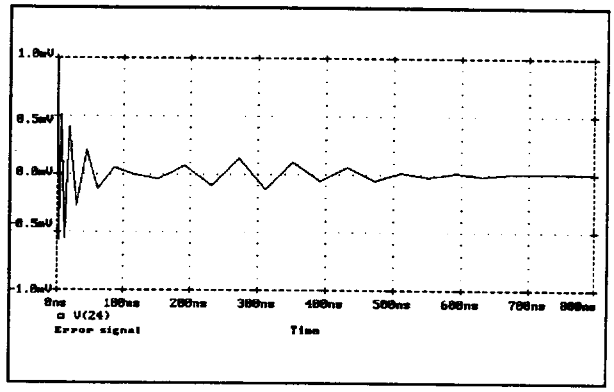

Figure # 4

4. CONCLUSIONS.

This paper shows the implementation of an analog adaptive filter. Proposed structure was obtained by using a set of orthogonal functions. This realization leads into a filter structure which consists of a low pass function and N-1 all pass functions. The adaptation is perform by using the analog LMS algorithm. Spice simulations shows the feasibility of the proposed model and as the figure # 3 shows that the output signal Y(t) identify the system. Figure # 4 shows that the convergence time is very fast and this suggest applications in which the statistics of the signal to be identified change in time with high speeds. Future work suggest to implement on VLSI structures in order to improve the operating frequency of the circuits to reach higher frequencies.

5. REFERENCES.

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