BLIND EQUALIZATION USING SECOND-ORDER STATISTICS

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ABSTRACT

When a message signal is transmitted through a linear dispersive system, the system output may contain severe intersymbol interference (ISI). The removal of the ISI without the aid of training signals is referred to as blind equalization.

We present a new algorithm that achieves blind equalization of possibly nonminimum phase channels, based only on the second-order statistics of the source symbols. Source symbols may have an arbitrary distribution; specifically, they do not have to be independently identically distributed (i.i.d.). This is an extension to previous work done by Tong, Xu and Kailath [1].

Simulations show that the new algorithm compares favorably to the algorithm given in [1].

1. Introduction

In digital communication over linear dispersive systems, intersymbol interference is considered to be the primary source of signal distortion, not additive noise [2]. Conventional equalizers make use of a training period, during which the desired signal is known at the receiving end. There are however cases, where sending a training signal does not seem feasible or where channel characteristics may abruptly change, thereby requiring renewed equalizer adjustment. Therefore, it would be desirable if the equalizer could adapt itself to the channel *blindly*, without a desired response.

Consider digital radio systems as an example. The time-varying multipath propagation inherent in wireless communication systems, can produce severe channel fading which may lead to system outage. If this occurs during the training process, the adaptive equalizer will not converge since the desired response is missing. In this situation a blind equalizer would be required because it does not rely on knowledge about the exact training signal or on a predefined channel model.

A multiplicity of additional applications are discussed by various authors in [3]. This reference gives also an excellent summary of several important methods of blind equalization.

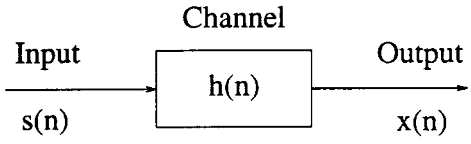

Figure 1: Model of a Communication System

1.1. Motivation for this work

The problem of blind equalization has now been studied for nearly twenty years. A variety of algorithms have been developed. Most of them, however, are built on the conception that higher-order statistics are needed for identifying nonminimum phase transfer functions. Many of these algorithms have problematic convergence properties due to local minima in the nonconvex cost functions, upon which typically a gradient descent algorithm operates. It has been shown recently that higher-order statistics may not be required for the identification of nonminimum phase systems if the input to the system is non-stationary [4]. Tong *et al.* were the first to use this property in a true blind equalization algorithm without training signal. However, all the available literature, including [1], considers only system inputs which are independently and identically distributed (i.i.d.). This excludes important source models such as *Markov* sources.

In this paper we describe a new blind equalization algorithm based on second-order statistics for systems driven by a non-stationary and not memoryless communication source, i.e. symbols are correlated and not i.i.d. .

In Section 2 we set up the mathematical formulation of the blind equalization problem. Section 3 gives an outline of the proposed new algorithm. Results of simulations are presented in Section 4 and conclusions are drawn in Section 5.

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2. Background

Let the channel in Figure 1 be a linear time-invariant system. Then the system output $x(n)$ is given by the linear convolution of the source sequence $s(n)$ with the sampled channel impulse response $h(n)$

$$x(n) = h(n) \star s(n) = \sum_{k=-\infty}^{\infty} h(k)s(n-k). \quad (1)$$

The discrete impulse response $h(n)$ includes pulse-shaping filter, dispersive channel and receiver filters. Our objective is to transform (1) into vector-matrix form. If $h(n)$ is assumed to have finite support, evidently only a finite number of source symbols $s(i)$ will have an effect on each output symbol $x(j)$. Thus, if we define a vector of output symbols, it can be computed as the product of some matrix $\mathbf{H}$ (a channel convolution matrix) with a corresponding finite length vector $\mathbf{s}(n)$. If the received signal is sampled at a rate higher than the baud rate, it can be shown (for example [5]) that a model

$$\mathbf{x}(n) = \mathbf{H}\mathbf{s}(n) \quad n = 0, 1, 2, \ldots \quad (2)$$

can be used to describe the system, where $\mathbf{H}$ is constant but vectors $\mathbf{x}(n)$ and $\mathbf{s}(n)$ are windowed segments of the output and input sequences respectively. The dimensions of matrix $\mathbf{H}$ depend on the length L of the impulse response, the rate of oversampling a, and the length of the observation window. There are $m = ra$ rows and $d = L + r - 1$ columns, where r is an integer chosen to ensure $m > d$. Calculating the autocorrelation of $\mathbf{x}$ we obtain

$$\mathbf{R}_x(k) = \mathbf{H}\mathbf{R}_s(k)\mathbf{H}^T \quad k = 0, 1, 2, \ldots \ . \quad (3)$$

$()^T$ denotes matrix-transpose and k indicates a time-lag.

The $\{\mathbf{R}_x(k) : k = 0, 1, \ldots\}$ are the autocorrelation matrices of the received sequence and can be estimated using standard methods. Each $\mathbf{R}_s(k)$ is an autocorrelation matrix of the source sequence and is assumed to be known (in all blind equalization techniques knowledge of source statistics is assumed).

The problem of blind equalization now corresponds to identifying $\mathbf{H}$ from the system of equations (3), and then inverting the channel. The latter step is performed by standard least-squares or maximum-likelihood estimation. Tong *et al.* [1] have solved the problem of identifying $\mathbf{H}$ under the constraint that $s(k)$ is an i.i.d. sequence, which implies a very specific structure for $\mathbf{R}_s(k)$. The new algorithm of Section 3 works for arbitrary source statistics, but utilizes more of the equations (3).

3. The New Algorithm

Consider the first equation in the system of equations of (3). At lag $k = 0$ both autocorrelation matrices are symmetric and positive semi-definite. Replace $\mathbf{R}_x(0)$ by its eigenvalue decomposition, $\mathbf{R}_x(0) = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$, and $\mathbf{R}_s(0)$ by its Cholesky factorization, $\mathbf{R}_s(0) = \mathbf{L}\mathbf{L}^T$. Using these factorizations and performing a matrix square root operation on both sides we can solve for the channel convolution matrix

$$\mathbf{H} = \mathbf{U}_d\mathbf{\Lambda}_d^{1/2}\mathbf{Q}\mathbf{L}^{-1}. \quad (4)$$

$()^{1/2}$ denotes the matrix square root, and $()^{-1}$ the matrix inverse. In equation (4) $\mathbf{U}_d$ contains the first d columns of $\mathbf{U}$ and $\mathbf{\Lambda}_d$ is a diagonal matrix of the d most significant eigenvalues; the only unknown factor remaining is the $d \times d$ unitary matrix $\mathbf{Q}$, which is an ambiguity arising from the matrix square root operation.

The ambiguity in $\mathbf{Q}$ can be resolved from the next $d - 1$ equations in (3). Substituting (4) for $\mathbf{H}$ and simplifying gives

$$\mathbf{R}_k = \mathbf{Q}\mathbf{A}_k\mathbf{Q}^H \quad k = 1, \ldots, d-1 \ , \quad (5)$$

where

$$\begin{aligned} \mathbf{R}_k &= \mathbf{\Lambda}_d^{-1/2}\mathbf{U}_d^T\mathbf{R}_x(k)\mathbf{U}_d\mathbf{\Lambda}_d^{-T/2}, & (6) \\ \mathbf{A}_k &= \mathbf{L}^{-1}\mathbf{R}_s(k)\mathbf{L}^{-T} \quad k = 1, \ldots, d-1 \ . & (7) \end{aligned}$$

$()^H$ denotes the conjugate transpose of the matrix.

The matrices $\{\mathbf{R}_k\}$ and $\{\mathbf{A}_k\}$ are available at the receiver. Let $\mathbf{x}_k, \mathbf{y}_k$ be the singular vectors corresponding to the smallest singular values (σ_k) of $\mathbf{R}_k$ and $\mathbf{A}_k$ respectively. Multiplying (5) by $\mathbf{x}_k$ yields

$$\mathbf{R}_k\mathbf{x}_k = \mathbf{Q}\mathbf{A}_k\mathbf{Q}^H\mathbf{x}_k = \sigma_k\mathbf{u}_k \quad k = 1, \ldots, d-1 \ . \quad (8)$$

$\mathbf{u}_k$ is the corresponding left singular vector, and not of particular interest here. Since $\mathbf{Q}$ is unitary it can be shown that

$$\mathbf{Q}^H\mathbf{x}_k = \pm\mathbf{y}_k \quad k = 1, \ldots, d-1 \ . \quad (9)$$

The set of equations in (9) together with the constraint that $\mathbf{Q}$ has to be unitary are sufficient to identify a set of 2^d possible matrices. For practical reasons, the sign ambiguity in (9) (which leads to this *set* of matrices) is best resolved by an exhaustive search. The identification of $\mathbf{Q}$ is improved if the singular vectors $\mathbf{x}_k, \mathbf{y}_k$ are found as those corresponding to the smallest singular values of $[\mathbf{R}_k^T, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}]^T$ and $[\mathbf{A}_k^T, \mathbf{y}_1, \ldots, \mathbf{y}_{k-1}]^T$, respectively. In this way the set of vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ are mutually orthogonal.

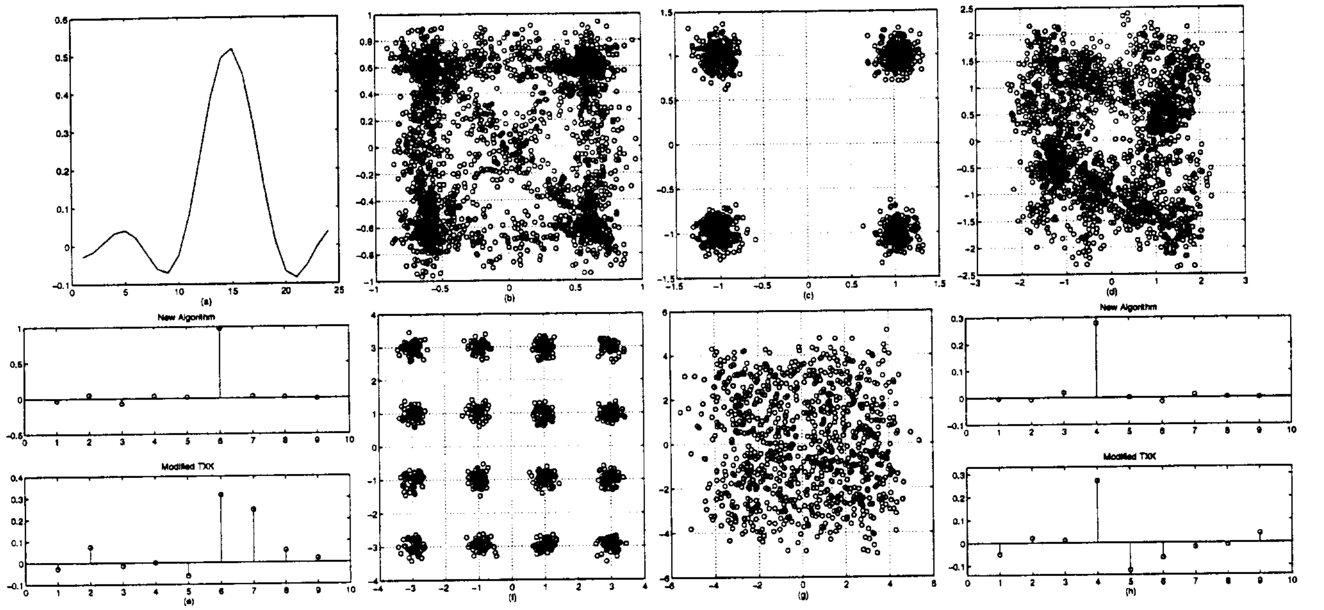

Figure 2: (a) Channel Impulse Response, (b) Unequalized Received Sequence, (c) Test 1: New Algorithm, (d) Test 1: TXK Algorithm, (e) Test 1: Combined Channel-Equalizer Response, (f) Test 2: New Algorithm, (g) Test 2: TXK Algorithm, (h) Test 2: Combined Channel-Equalizer Response.

Now that the channel is identified in terms of its channel convolution matrix, the sequence estimation can be performed. The least-squares approach, for example, gives

$$\hat{\mathbf{s}}_{LS}(n) = \mathbf{H}^{\dagger}\mathbf{x}(n) \quad (10)$$

$$= \mathbf{L}\mathbf{Q}^{H}\mathbf{\Lambda}_d^{-1/2}\mathbf{U}_d^{T}\mathbf{x}(n) \quad n = 0, 1, \ldots \quad (11)$$

$\mathbf{H}^{\dagger}$ is the pseudo-inverse of the channel convolution matrix.

4. Simulations

Preliminary tests were performed to compare the new algorithm to the algorithm of Tong, Xu and Kailath [1]. We refer to their original algorithm as TXK. For the non-i.i.d. case we modify the TXK algorithm by including the inverse of an imaginary whitening filter in the channel convolution matrix $\mathbf{H}$; this algorithm is referred to as *modified* TXK. There are two test cases.

Markov Source

1000 data points are generated by a first-order markov process. The points are drawn from a 4-QAM constellation. To obtain the received sequence, the data is convolved with the sampled impulse response of Figure 2a, and low-power white noise is added to the result at a signal-to-noise ratio of ca. 25 dB. The channel impulse response (adapted from [1]) is generated from three delayed raised cosine pulses and has nonminimum phase. As can be seen from Figure 2b, the unequalized received sequence suffers from severe ISI and does not allow reliable decisions. The received sequence is then equalized by each algorithm. The outputs, Figures 2c and 2d, are obtained after estimating the channel convolution matrix using 400 points of the received sequence, and calculating the least-squares estimates with (11). It can be seen that the new algorithm by far outperforms modified TXK. The improved performance is also evident in Figure 2e. Ideally, the combined response of the channel-equalizer cascade should show only one non-zero element. The new algorithm comes closer to this ideal than modified TXK. The output signal-to-interference-and-noise ratio, defined by

$$SINR = \frac{|c_{max}|^2}{\sum_n |c_n|^2 - |c_{max}|^2} \quad (12)$$

is 50 dB for the new algorithm and only 12.7 dB for TXK. In equation (12) the c_i are the coefficients of the combined response of channel and equalizer

IID Source

In the second test, data are drawn uniformly and independently from a 16-QAM constellation. Using the same channel as before and 400 elements of the received sequence in the estimation, the equalized outputs for both algorithms are shown in Figures 2f and 2g. From these plots and again from the combined channel-equalizer response in Figure 2h, one can see that the new algorithm is more accurate (numerically stable) even on i.i.d. source sequences. The output SINRs are 43.6 dB and 16.4 dB for the new algorithm and the TXK algorithm, respectively.

5. Conclusions

We have proposed a novel method of Blind Equalization. The method, based on previous work by Tong *et al.* [1], identifies nonminimum phase channels driven by (possibly) non-i.i.d. source sequences, using second-order statistics only. This method has many advantages over existing higher-order statistics based approaches. There are no convergence problems due to local minima and fewer samples are required to estimate second-order statistics. The new algorithm identifies channels asymptotically exactly without putting any restrictions on the source statistics. At the cost of slightly higher computational complexity, the new algorithm is more numerically stable than that proposed in [1].

6. References

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