A SUBBAND ADAPTIVE FILTER WITH THE OPTIMUM ANALYSIS FILTER BANK

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ABSTRACT

Conventional subband ADF's(adaptive digital filters) using filter banks have shown degradation in performance because of the non-ideal nature of filters. For this problem, we propose a new type of subband ADF incorporating two types of analysis filter bank. In this paper, we show that we can design the optimum filter bank which minimizes the LMSE(least mean squared error). In other words, we can design a subband ADF with less MSE than that of conventional subband ADF's.

1. INTRODUCTION

In recent years, there has been interests in adaptive signal processing applications that require filters with very long impulse responses; for example, several thousands of FIR(finite impulse response) filter coefficients are needed for adaptive noise cancellers, acoustic echo cancellers and so forth[1]-[5]. To implement such kinds of FIR ADF's(adaptive digital filter) with a very long impulse response, the use of subband adaptive processing incorporating filter banks is suitable. This technique decomposes a very long impulse response of the adaptive filter into a short length; so it can achieve parallel processing of adaptive filters. Hence some adaptive algorithms based on the subband technique have been developed[1],[5],[6].

However, these conventional subband algorithms are not sufficient to implement subband adaptive filters. This is mainly because they cannot perfectly avoid the degradation effect of aliasing due to decimation. Gilloire et al. have improved the effect of aliasing by allowing cross terms[5]. This technique is, however, an approximated method. Somayazulu et al. have proposed a new adaptive structure with auxiliary subbands [1]. This structure, however, cannot carry out maximal decimation. To avoid aliasing problems, we have proposed a new class of maximally decimated adaptive filter using FSF(frequency sampling filter) bank[7]-[10].

In this paper we propose another type of subband ADF which also enables us maximal decimation. The proposed subband ADF has the identical structure as conventional ADF's; however, two different types of analysis filter banks are used. First, we define a cost function by using a correlation function before decimation and after decimation. Our definition of the cost function is much simpler than that based on the equivalent model[11] because there is no need for a polyphase decomposition with regard to an unknown system. We also derive the optimum ADF coefficients and LMSE(least mean squared error). The LMSE is a function in terms of the coefficients of the analysis filter bank. Hence, we can design an analysis filter bank which makes the LMSE minimum. Lastly, some design examples are given in order to show the efficiency of the proposed ADF.

2. PROPOSED SUBBAND ADF

2.1. Architecture

Figure 1 shows a proposed subband ADF. The proposed subband ADF has two analysis banks. The analysis filter bank 2, which splits reference signals into several bins, is a pair of PR(perfect Reconstruction) banks with the synthesis filter bank. It should be noted that we have freedom of design in the analysis filter bank 1 with regard to input signal. This is a merit of the proposed subband ADF. Figure 2 shows the equivalent

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structure of the kth bin.

2.2. Cost Function

In this section, we define a cost function in terms of the kth bin. The $x_k(m)$ and $d_k(m)$ are an input and reference signal with regard to kth bin's ADF. The $h_k(m)$ is an ADF coefficient of the kth bin. The $f_k(m)$ is a band limited filter coefficient of the kth bin. L_{h_k} denotes the length of the $h_k(m)$. The output signal $y_k(m)$ is given by

$$y_k(m) = \mathbf{h}_k^T \mathbf{x}_k(m) \tag{1}$$

where

$$\mathbf{h}_k^T = [\ h_k(0)\ h_k(1) \cdots h_k(L_{h_k}-1)\ ]$$

$$\mathbf{x}_k^T(m) = [\ x_k(m)\ x_k(m-1) \cdots x_k(m-L_{h_k}+1)\ ]$$

We define a cost function in terms of the kth bin as follows:

$$\begin{aligned} MSE_k &= E[e_k^2(m)] \\ &= E\left[\left\{d_k(m) - \mathbf{h}_k^T \mathbf{x}_k(m)\right\}^2\right] \\ &= \sigma_{d_k}^2 - 2\mathbf{P}_{d_k x_k}^T \mathbf{h}_k + \mathbf{h}_k^T \mathbf{R}_{x_k x_k} \mathbf{h}_k \end{aligned} \tag{2}$$

where

$$\sigma_{d_k}^2 = E[d_k^2(m)]$$

$$\mathbf{P}_{d_k x_k}^T = E[d_k(m)\mathbf{x}_k^T(m)]$$

$$\mathbf{R}_{x_k x_k} = E[\mathbf{x}_k(m)\mathbf{x}_k^T(m)]$$

Now, we can rewrite $x_k(m)$ in (1) by using the kth bin's band limited filter coefficients. D is a decimation number.

$$x_k(m) = \mathbf{f}_k^T \mathbf{x}(mD) \tag{3}$$

where

$$\mathbf{f}_k^T = [\ f_k(0)\ f_k(1) \cdots f_k(L_{f_k}-1)\ ]$$

$$\mathbf{x}^T(n) = [\ x(n)\ x(n-1) \cdots x(n-L_{f_k}+1)\ ]$$

The correlation coefficient $r_{x_k x_k}(\tau)$ is given by

$$\begin{aligned} r_{x_k x_k}(\tau) &= E[x_k(m) x_k(m+\tau)] \\ &= \mathbf{f}_k^T E[\mathbf{x}(mD)\mathbf{x}((m+\tau)D)]\mathbf{f}_k \end{aligned} \tag{4}$$

Hence, the correlation matrix $\mathbf{R}_{x_k x_k}$ is given by

$$\mathbf{R}_{x_k x_k} = \mathbf{F}_k^T \mathbf{R}_{xx} \mathbf{F}_k \tag{5}$$

where $L = L_{f_k} + (L_{h_k}-1)D$, $\mathbf{0} = [\ 0\ 0 \cdots 0\ ]^T$ $(D \times 1)$

$$\mathbf{R}_{xx} = \begin{bmatrix} r_{xx}(0) & r_{xx}(1) & \cdots & r_{xx}(L-1) \\ r_{xx}(1) & r_{xx}(0) & \cdots & r_{xx}(L-2) \\ \vdots & \vdots & \ddots & \vdots \\ r_{xx}(L-1) & r_{xx}(L-2) & \cdots & r_{xx}(0) \end{bmatrix}$$

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{f}_k & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{f}_k & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{f}_k \end{bmatrix}$$

Similarly, we obtain

$$\mathbf{P}_{d_k x_k} = \mathbf{f}_k^T \mathbf{P}_{d'_k x} \tag{6}$$

where

$$\begin{aligned} \mathbf{p}_{d'_k x}^T(\tau) &= [\ p_{d'_k x}(\tau)\ p_{d'_k x}(\tau+1) \\ &\qquad \cdots p_{d'_k x}(\tau + L_{f_k} - 1)\ ] \end{aligned}$$

$$\begin{aligned} \mathbf{P}_{d'_k x} &= [\ \mathbf{p}_{d'_k x}(0)\ \mathbf{p}_{d'_k x}(D) \\ &\qquad \cdots \mathbf{p}_{d'_k x}((L_{h_k}-1)D)\ ] \end{aligned}$$

Substituting (5) and (6) into (2), we get

$$MSE_k = \sigma_{d_k}^2 - 2\mathbf{f}_k^T \mathbf{P}_{d'_k x} \mathbf{h}_k + \mathbf{h}_k^T \mathbf{F}_k^T \mathbf{R}_{xx} \mathbf{F}_k \mathbf{h}_k \tag{7}$$

2.3. Coefficients of the Optimum ADF

Here we derive the optimum filter coefficients of subband ADF's. The derivative of (7) with respect to $\mathbf{h}_k$ is

$$\frac{\partial MSE_k}{\partial \mathbf{h}_k} = -2\mathbf{P}_{d'_k x}^T \mathbf{f}_k + 2\mathbf{F}_k^T \mathbf{R}_{xx} \mathbf{F}_k \mathbf{h}_k \tag{8}$$

In order to minimize MSE_k, let (8) be zero. Then we get the equation with respect to $\mathbf{h}_k$

$$\mathbf{h}_{k,opt} = \left\{\mathbf{F}_k^T \mathbf{R}_{xx} \mathbf{F}_k\right\}^{-1} \mathbf{P}_{d'_k x}^T \mathbf{f}_k \tag{9}$$

Hence, we obtain the optimum filter coefficients of the kth bin's ADF. Eq.(9) shows that the optimum filter coefficient vector of the ADF is the function of the analysis filter coefficient vector $\mathbf{f}_k$.

2.4. Least Mean Squared Error

In this section, we derive a LMSE. Substituting (9) into (7), we get

$$LMSE_k = \sigma_{d_k}^2 - \mathbf{f}_k^T \mathbf{P}_{d'_k x} \left\{\mathbf{F}_k^T \mathbf{R}_{xx} \mathbf{F}_k\right\}^{-1} \mathbf{P}_{d'_k x}^T \mathbf{f}_k \tag{10}$$

Hence the LMSE is a function of $\mathbf{f}_k$. It is very important in view of suggesting the possibility of designing the optimum filter $\mathbf{f}_k$. If we make use of (9) and (10), we can design the optimum $\mathbf{f}_k$ which makes LMSE minimum. The concrete design algorithm is as follows:

1. Get $\sigma^2_{d_k}$, $\mathbf{R}_{xx}$ and $\mathbf{P}_{d'_k x}$
2. Substitute above parameters into (10) and find the analysis filter coefficients which minimize $LMSE_k$ by using a nonlinear optimization method.

3. DESIGN EXAMPLE AND SIMULATION RESULT

We show a design example under following conditions.

Unknown system	:	64 tap FIR LPF
Input Signal $x(n)$	:	1st order AR
Adaptive Algorithm	:	NLMS
Filter Bank 1 (Optimized)	:	8,16,32 tap
Filter Bank 2 (QMF)	:	8,16,32 tap
$N = 2$, $D = 2$		

Figure 3 shows the amplitude response of the unknown system. Figure 4 shows the amplitude response with regard to the QMF and the optimum filter bank in terms of 16tap case. Figure 5 shows the learning curves with regard to the conventional and the proposed method. It is obvious that the proposed scheme has less MSE than conventional methods.

4. CONCLUSION

We have proposed a new type of subband adaptive filter, which can reduce the LMSE compared with that of the conventional subband ADF, by using two different types of analysis filter banks.

We have shown that the optimum filter coefficients of the subband ADF and LMSE are the function with respect to analysis filter coefficient vector $\mathbf{f}_k$. We have also shown the design method of the optimum analysis filter bank 1 with regard to the input signals by minimizing the LMSE. Lastly, we have shown a design example and simulation result. Simulation result shows that the proposed method attains less MSE than that of conventional subband ADF's.

5. REFERENCES

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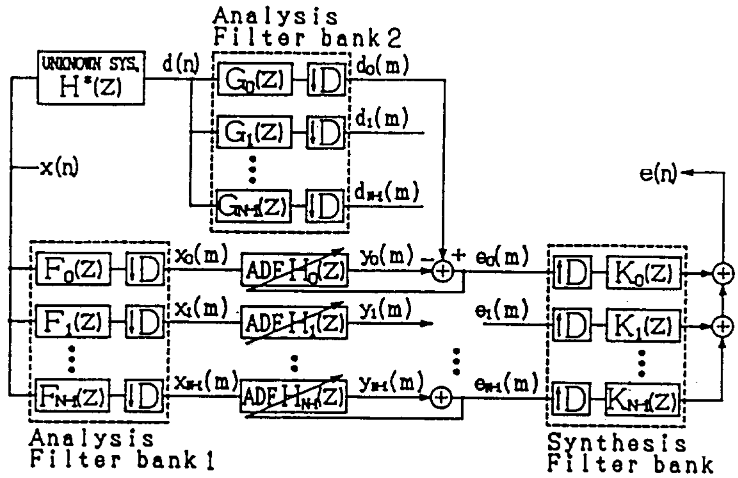

Figure 1: Subband Adaptive Filter

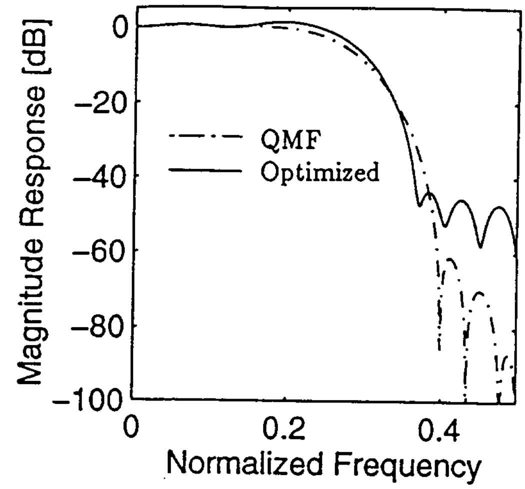

Figure 4(a): Filter Bank (16tap,Low)

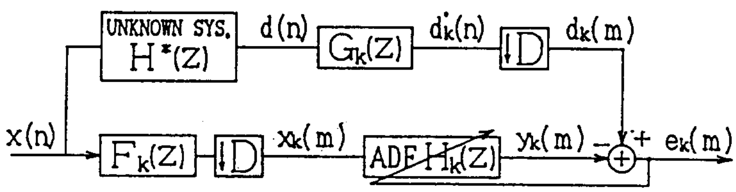

Figure 2: kth bin

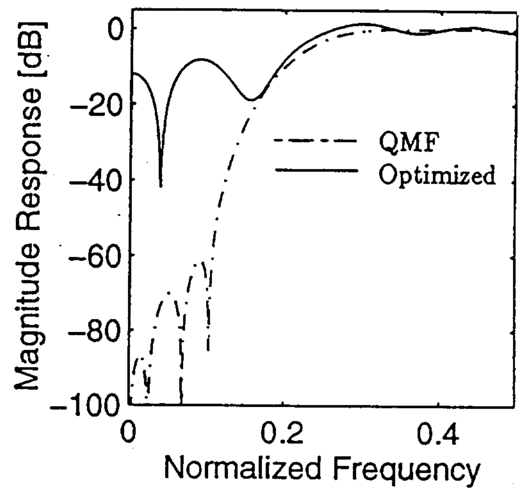

Figure 4(b): Filter Bank (16tap,High)

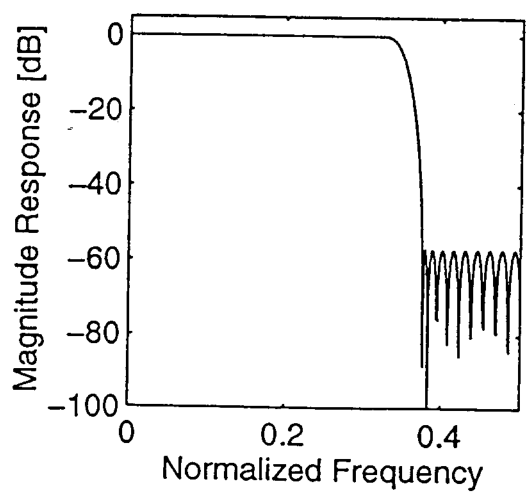

Figure 3: Unknown System

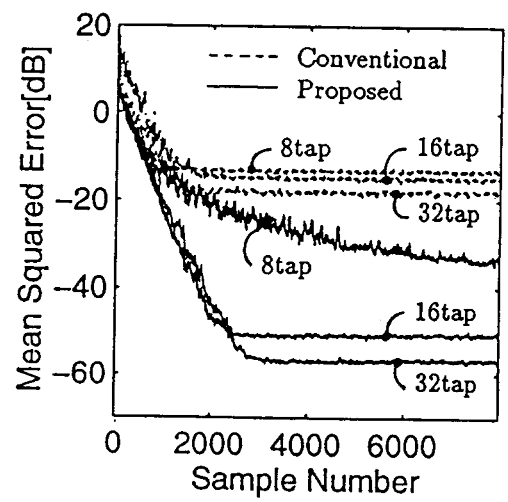

Figure 5: Learning Curves