ON THE CONVERGENCE ENHANCEMENT OF THE WAVELET TRANSFORM BASED LMS

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ABSTRACT

The wavelet transform least mean squares algorithm (WTLMS) has been recently proposed as an alternative to the simple and transform based (DCT) LMS algorithms, because of its faster convergence. In this paper, we first show the influence of the regularity of the wavelet low-pass filter on the convergence behavior of the normalized WTLMS algorithm (NWTLMS). Then, we show that the subband decomposition of the input signal along a regular subband tree, which splits the signal frequency band uniformly, gives better results, i.e., a faster convergence rate than the dyadic subband tree, which splits the signal frequency band dyadically. Finally, we show that perfect reconstruction quadrature mirror filters (PR-QMFs), which are less regular, can lead to as good results while the multiplier-free PR-QMFs offer, furthermore, a very reduced computational complexity, and hence can be used as an alternative to the wavelet filters for accelerating the convergence rate of the NWTLMS algorithm.

1. Introduction

The conventional time-domain LMS adaptive filter algorithm has the advantage of being very simple, easy to implement and has a very low computational complexity. However, when the input signal is highly colored, the LMS convergence is slowed down [1]. One way to alleviate this problem is to prewhiten the input signal using a certain number of transforms, which can be efficiently computed with a small extra computational load, such as the fast Fourier transform (FFT) and the discrete cosine transform (DCT) [2]. The use of the discrete wavelet transform (DWT) has been shown to lead to a faster convergence [3], [4]. In this paper, more results on the convergence of the WTLMS algorithm are presented: first, we investigate the influence of the regularity of the wavelet filters on the convergence behavior of the normalized WTLMS. To this end, we use the Daubechies wavelet filters of 4, 8, 10, 12, 14 and 16 coefficients since we know that for these filters the regularity increases with length [6]. We shall show that, when such filters are used, the convergence rate is improved as the regularity increases. Then, we investigate the influence of the subband decomposition structure on the convergence behavior of the NWTLMS. We will show that the regular subband tree (fig. 2) leads to better results than the dyadic tree (fig. 1). Finally, the use of multiplier-free PR-QMFs, which are less regular than Daubechies wavelet filters but have comparable frequency responses, can give as good results while offering, at the same time, a very reduced computational complexity.

2. The Discrete Wavelet Transform and the Concept of Regularity

If $x(t)$ is any square integrable function, then it can be decomposed onto a set of square integrable basis functions, constructed by dilating and translating a single wavelet $\varphi(\omega)$, as follows [5]:

$$x(t) = \sum_{j,k} \sqrt{2^j}\, x_{j,k}\, \varphi(2^j t - k)$$

where

$$\varphi(\omega) = H_1(e^{j\frac{\omega}{2}})\phi(\tfrac{\omega}{2})$$

and

$$\phi(\omega) = \prod_{k=1}^{\infty} H_0(e^{j\frac{\omega}{2^k}})$$

$\phi(\omega)$ is called the scaling function. $H_0(z)$ and $H_1(z)$ should satisfy the 2-band FIR PR-QMF bank conditions [5]. Moreover, they must each have at least one zero at $z=-1$, and $z=1$, respectively. In practice, the discrete wavelet transform is computed using a dyadic binary subband tree (fig. 1). In order to ensure that the infinite product of the scaling function converges to a smooth function rather than breaking into fractals, this latter should have some regularity, i.e., a certain number of continuous derivatives [6]. In [7], several algorithms are developed to accurately compute the regularity in the Hölder and Sobolev spaces. In the remainder of this paper, we shall use only Hölder regularity.

3. The Wavelet/PR-QMFs Transform based Normalized LMS

The dependency of the LMS convergence on the signal conditioning can be reduced considerably by applying the wavelet transform defined in terms of (NxN) matrix $\mathbf{T_w}$, to

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the input vector x(n), to form a second vector z(n) such that

$$z(n) = \mathbf{T}_w x(n)$$

$\mathbf{T}_w$ is constructed out of the wavelet low-pass and high-pass filters. The convolution boundary effects are handled by periodizing the data vector x(n). If d(n) is the desired signal, the autocorrelation matrix and the intercorrelation vector of the new vector z(n) are given, respectively, by

$$\begin{aligned}\mathbf{R}_{zz} &= E[z(n)z^T(n)] \\ &= E[\mathbf{T}_w x(n)x^T(n)\mathbf{T}_w^T] \\ &= \mathbf{T}_w R_{xx} \mathbf{T}_w^T\end{aligned}$$

and

$$\begin{aligned}\mathbf{P}_{zd} &= E[d(n)z(n)] \\ &= \mathbf{T}_w P_{xd}\end{aligned}$$

where R_{xx} and P_{xd} are, respectively, the conventional autocorrelation matrix and the intercorrelation vector of the input data vector x(n). The error to be minimized is defined as:

$$e(n) = d(n) - y(n)$$

The optimum Wiener solution which minimizes the mean square error (MSE) is given by

$$\mathbf{g}_{opt} = \mathbf{R}_{zz}^{-1}\mathbf{P}_{zd}$$

Since $\mathbf{T}_w$ is orthogonal, then we can easily get back the conventional impulse respone $\mathbf{h}_{opt}$ of the filter, knowing that

$$\mathbf{g}_{opt} = \mathbf{T}_w \mathbf{h}_{opt}$$

By analogy with the normalized time-domain LMS, we can define the NWTLMS as

$$g(n+1) = g(n) + \hat{\mathbf{R}}_{zz}^{-1}(n)z(n+1)e(n)$$

where

$$\mathbf{R}_{zz}^{-1}(n) = \mathrm{diag}[\hat{Z}_0^{-1}(n)\ \hat{Z}_1^{-1}(n) \ldots \hat{Z}_{N-1}^{-1}(n)]$$

and N is the adaptive filter order. The diagonal elements are estimated in the following way:

$$Z_i(n) = \beta Z_i(n-1) + (1-\beta)(z_i^2(n)),\ 0 < \beta < 1$$

Now, if we replace $\mathbf{T}_W$ by a more general orthogonal transform matrix $\mathbf{T}$, which may not be normalized, then

$$\mathbf{g}_{opt} = \frac{1}{M}\mathbf{T}\mathbf{h}_{opt}$$

and keeping the rest of equations unchanged, we get a more generalized subband transform LMS (SBTLMS). In order to ensure a good convergence of the algorithm, we should make a good choice of the parameters α and β as well as the initial conditions $Z_i(0)$. In practice, the following choice is often made [2] :

$$\alpha = \frac{1}{N}$$

and

$$\beta = 1 - \alpha$$

The initial conditions are chosen as

$$Z_i(0) = M\hat{\sigma}_x^2(0),\ i = 0, 1, \ldots, N-1.$$

where $\hat{\sigma}_x^2(0)$ is the initial variance of the data vector x(n). For the NWTLMS, M is, of course, equal to 1, whereas, for the multiplier-free PR-QMF matrix based NLMS, M is greater than or equal to 1.

In fact, any paraunitary PR-QMF bank can be used to form an orthogonal matrix as well. The only difference from wavelets is that the multiplier-free PR-QMFs do not lead to normalized transform matrices. They are designed such that they have only the allowed coefficient values

$$h(n) = \pm 2^{k_n} \pm 1,\ n = 0, 1, 2, \ldots 2N-1.$$

where k_n is an integer. They are, generally, called suboptimal filters [5].

4. Simulation Results

System identification is often considered as a good context for testing the performance of adaptive algorithms. This context is used to evaluate the influence of the regularity of the wavelet low-pass filter and the subband decomposition structure on the convergence behavior of the NWTLMS algorithm. The performance measure used is given by

$$\rho(n) = E[(\mathbf{h}(k) - \mathbf{h}_{opt})^T(\mathbf{h}(k) - \mathbf{h}_{opt})]$$

where $\mathbf{h}_{opt}$ and $\mathbf{h}$ are the unknown system and the adaptive filter impulse responses (16 coefficients), respectively. This expectation has been estimated by the ensemble average over 30 simulation trials. The channel and the unknown system gains have been chosen to have unit gain and the input signal x(n) to have a unit variance. Therefore the output estimation error converges to the power (variance) of the additive noise.

The simulations of the NWTLMS for different degrees of regularity of the wavelet filters have been carried out for an additive noise of about -70 dB and a channel which correlates the input signal samples such that the EVR, i.e., the eigenvalue ratio of the input signal autocorrelation matrix is about 50. For each case, the estimated error is plotted versus time. In figure 3, we show the results obtained when the decomposition structure level is 1 (fig.1), i.e., the signal is decomposed into two subband signals only. We notice the dramatic improvements in the convergence of the NWTLMS. The wavelet transform succeeds in whitenning the colored input signal to a great extent. This covergence increases as the regularity of Daubechies filters is increased from 0.55, which corresponds to a filter length of 4, to 2.18, which corresponds to a filter length of 12. Figure 4 shows the results of the influence of the subband decomposition structure (dyadic (fig.1) and regular (fig. 2), decomposition level=2) on the convergence behavior of the NWTLMS. It is clear that the transform matrix based on the dyadic subband tree (DST-NWTLMS) converges much slower than the regular subband tree based NWTLMS (RST-NWTLMS) for the same subband decomposition structure level. Figures 5 and 6 show the results obtained when using the following multiplier-free PR-QMFs: filter 1=[2 6 3 -1], filter 2=[4 16 16 0 -4 1] and filter 3=[-8 8 64 64 8 -8 1 1]. These filters, presented by Akansu in [5], have frequency responses comparable with Daubechies wavelet filters with the same duration, but are less regular. In any case, the normalized multiplier-free PR-QMF based LMS (NMFPRQMFTLMS) performs as well as or better than the DST-NWTLMS.

5. Complexity Considerations

In order for the following discussion to have a meaning, we consider, independently from any processor architecture, that the multiplication complexity is higher than the addition complexity which is, itself, higher than the binary shift complexity. Moreover, the input signal is considered to be real.

From the previous simulation results, we have noticed that the most interesting results have been found for wavelet filter lengths less than or equal to 12. For short filters, the use of FFT based methods to reduce the computational load of the convolution is inefficient. However, there are some other methods which can handle short filters and reduce the computational load by 30% at most [8]. But this may be still unacceptable for real-time applications. Therefore, suboptimal multiplier-free PR-QMFs, which have the advantage of being multiplierless, are more attractive. Let us consider, for the sake of illustration, the 4 coefficients multiplier-free PR-QMF [2 6 3 -1]. It is clear that the convolution of the signal with this filter can be performed only with a set of binary shifts and additions.

6. Conclusion

In this paper, we have shown that the NWTLMS leads to dramatic improvements in the convergence rate of the algorithm when the input signal is colored, even if the subband decomposition level is only 1. We have shown that the regularity of the wavelet filter does influence the convergence behavior of the NWTLMS. But this influence is noticeable only for filter lengths not exceeding 12.

The regular subband tree structure based NWTLMS has been shown to give a faster convergence than the one based on the dyadic subband tree structure.

The multiplier-free PR-QMFs have been shown not only to give as good results as the wavelet filters with maximum regularity, but to offer, furthermore, a computational complexity which is made up of only binary shifts, additions and subtractions. This makes The normalized multiplier-free PR-QMF transform based LMS a good candidate for real-time applications.

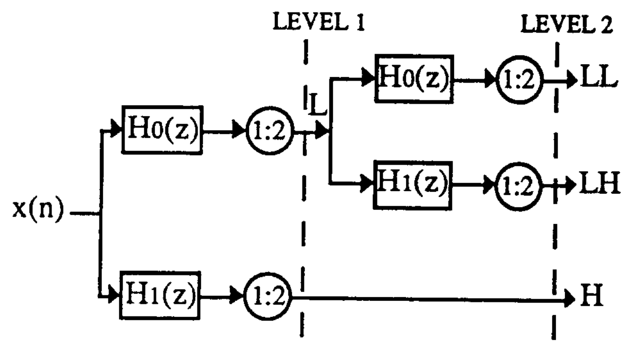

Fig.1. Dyadic subband tree structure

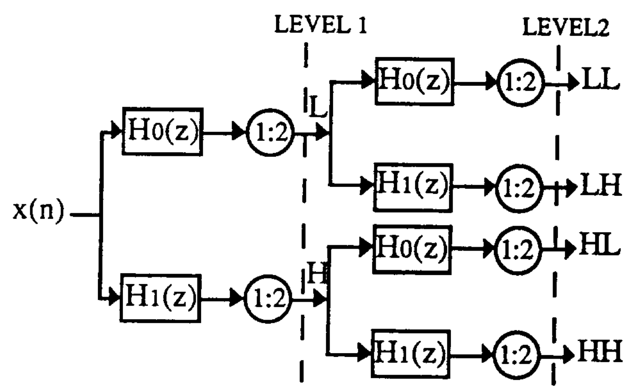

Fig.2. Regular subband tree structure

7. Acknowledgement

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8. References

[1] B. Widrow and S. D. Stearns, Adaptive Signal Processing. Englewood Cliffs, Prentice-Hall, NJ, 1985.

[2] J. C. Lee and C. K. Un, "Performance of Transform-Domain LMS Adaptive Digital Filters," *IEEE Trans.*

Acoust., Speech, Signal Processing, vol. ASSP-34, pp. 499-510, June 1986.
[3] S. Hosur and A.H. Tewfik, "Wavelet Transform Domain LMS Algorithm," Proc. ICASSP-93, pp. 508-510, April 1993, Minneapolis, Minnesota, USA.
[4] N. Erdol and F. Basbug, "Performance of Wavelet Transform Based Adaptive Filters," Proc. ICASSP-93, pp. 500-503, April 1993, Minneapolis, Minnesota, USA.
[5] A. N. Akansu and R. A. Haddad, Multiresolution Signal Decomposition, New york: Academic Press, 1992.
[6] I. Daubechies, "Orthonormal Bases of Compactly Supported Wavelets," Comm. in Pure and Applied Math., vol. 41, pp. 909-996, 1988.
[7] O. Rioul, "A simple, optimal regularity estimate for wavelets," in Proc. European Signal Processing Conf. (EUSIPCO) (Brussels, Belgium), vol. II, Aug. 1992, pp. 937-940.
[8] Z. J. Mou and P. Duhamel, "Short-length FIR filters and their use in nonrecurssive filtering," *IEEE Trans.Signal Proc.*, vol 39, pp. 1322-1332, June 1991.

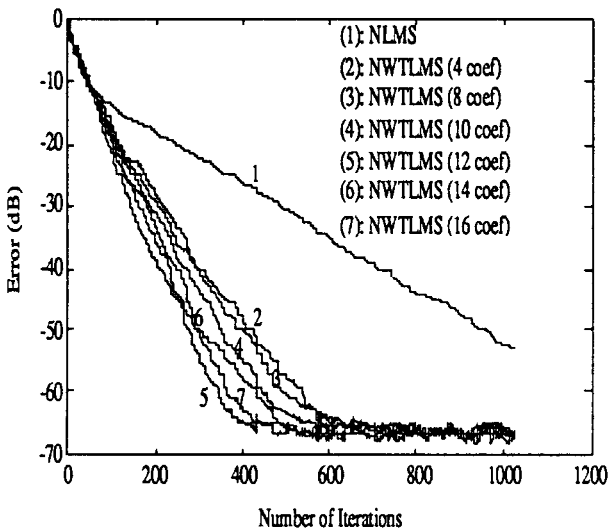

Fig.3. NLMS and NWTLMS results (colored input signal, subband decomposition level=1)

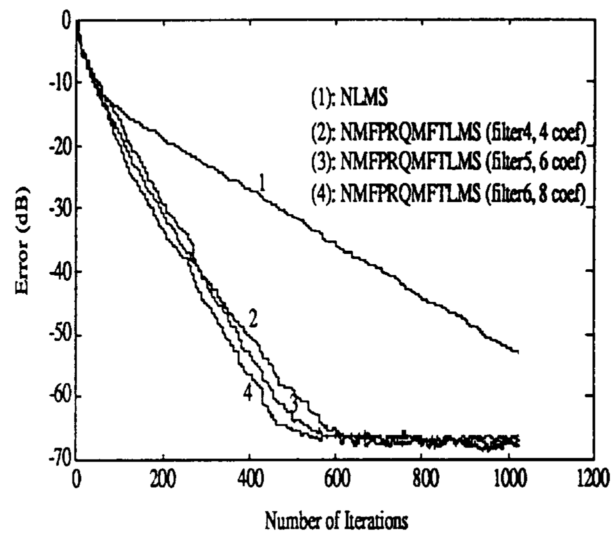

Fig.5. Multiplier-free PR-QMF transform based LMS results (colored input signal, subband decomposition level=1)

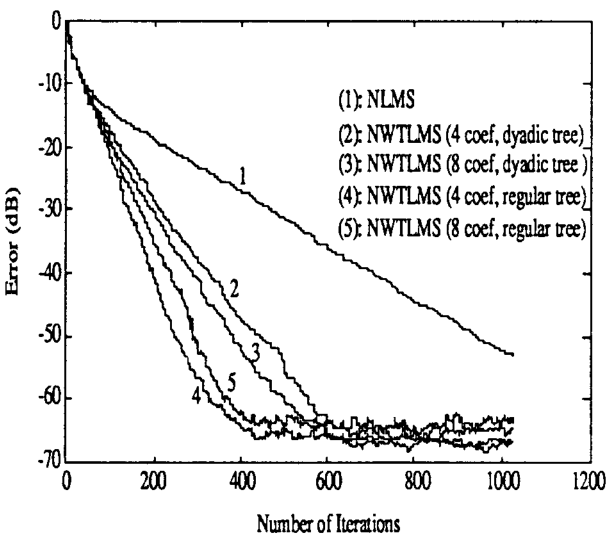

Fig.4. Dyadic and Regular subband tree based NWTLMS results (colored input signal, subband decomposition level =2)

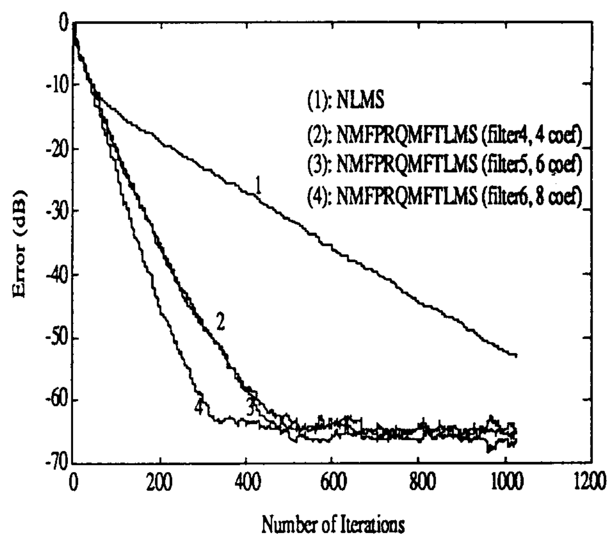

Fig.6. Multiplier-free PR-QMF transform based LMS results (colored input signal, subband decomposition level=2)