OUTPUT-ERROR LMS BILINEAR FILTERS WITH STABILITY MONITORING

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ABSTRACT

This paper introduces output-error LMS bilinear filters with stability monitoring. Bilinear filters are recursive nonlinear systems that belong to the class of polynomial systems. Because of the feedback structure, such models are able to represent many nonlinear systems efficiently. However, the usefulness of adaptive bilinear filters is greatly restricted unless they are guaranteed to perform in a stable manner. A stability monitoring scheme is proposed to overcome the stability problem. The paper concludes with simulation results that demonstrate the usefulness of our technique.

1. INTRODUCTION

While linear filters and system models have been very useful in a large variety of applications and are usually simple from conceptual and implementational points of view, there are several applications in which they will not perform well at all.

A very common system model that has been employed with relatively good success in nonlinear filtering applications is the truncated Volterra system model. Several researchers have developed adaptive filters based on truncated Volterra series expansions [3], [6], [7], [12], [13]. The main problem associated with such filters is the extremely large number of coefficients (and the correspondingly large computational complexity) that is usually required to adequately model the nonlinear system under consideration. An alternate approach is to use nonlinear system models with feedback. Just as linear IIR filters can model many linear systems with greater parsimony than FIR filters, there are a large number of nonlinear systems that can be approximated by nonlinear feedback models using a relatively small number of parameters. Consequently, one can expect that the corresponding adaptive filters can be implemented with good computational efficiency. This paper is concerned with adaptive nonlinear filtering algorithms that employ bilinear system model satisfying the following difference equation

$$y(n) = \sum_{i=0}^{r} a_i^o x(n-i) + \sum_{j=1}^{s} b_j^o y(n-j) + \sum_{i=0}^{r}\sum_{j=1}^{s} c_{i,j}^o x(n-i)y(n-j). \quad (1)$$

There is one major problem with bilinear system models in that most bilinear systems are inherently unstable. By this, we mean that it is possible to find bounded input signals that can drive the output signal to become unbounded for almost any given bilinear system. Consequently, it is very important to either develop adaptive bilinear filters that are guaranteed to operate in a stable manner all the time, or equip the adaptive filters with some sort of stability monitoring device that checks the filters for any indication of unstable behavior. When the test indicates potential instability, the system must take proper actions on the filter coefficients to prevent the adaptive filter from becoming unstable.

Even though the stability problem is of great importance for adaptive recursive nonlinear filters, there are only few results available in the literature for combating the problem. Fnaeich and Ljung [4] appended a Kalman filter to the basic adaptive filter to ensure the stability of the overall system. This approach is comptationally very costly. The Kalman filter that stabilizes the system is often significantly more complex than the basic adaptive filter itself. The authors of this paper recently showed that many exact realizations of extended least squares adaptive bilinear filters are inherently stable in the sense that the time average of the estimation error is bounded under relatively mild conditions [9]. They have also presented LMS adaptive bilinear filters [11]. However, they did not consider the stability issue in [11]. Recently, Bose and Chen [2] developed a conjugate gradient adaptive bilinear filter. They incorporated a stability monitoring mechanism that is somewhat similar to our method for their adaptive filter.

The purpose of this paper is to introduce a new scheme to overcome the stability problems associated with most adaptive bilinear filters. In particular, we develop output-error LMS bilinear filters with stability monitoring. The rest of the paper is organized as follows. Section 2 derives output-error adaptive bilinear filters. Section 3 introduces

This work was supported in part by the National Science Foundation under Grant MIP 8922146 and the Project No. 10VB700 conducted by the ITRI under sponsorship of the Minister of Economic Affairs, R.O.C.

0-7803-2431-5/95 $4.00 © 1995 IEEE

the new stability monitoring scheme for the adaptive bilinear filters. Section 4 contains simulation results that demonstrate the usefulness of our technique. The concluding remarks are made in Section 5.

2. LMS BILINEAR FILTERS

The basic problem of adaptive bilinear filtering may be formulated as follows. Given a desired response signal $d(n)$ and an input signal $x(n)$, we want to estimate $d(n)$ adaptively as

$$\hat{d}(n) = \sum_{i=0}^{r} a_i(n-1)x(n-i) + \sum_{j=1}^{s} b_j(n-1)\hat{d}(n-j)$$

$$+\sum_{i=0}^{r}\sum_{j=1}^{s} c_{i,j}(n-1)x(n-i)\hat{d}(n-j). \qquad (2)$$

Note that the output of the adaptive filter is used in a recursive manner in estimating the desired response signal. This approach belongs to the class of output-error adaptive filters. Because of the recursive structure of the estimation process, the development of the adaptive bilinear filters is somewhat more complicated than that of adaptive Volterra filters.

For compactness of presentation, let us define the input vector Z_n and the coefficient vector W_n as

$$Z_n = [x(n), ..., x(n-r), \hat{d}(n-1), ..., \hat{d}(n-s),$$

$$x(n)\hat{d}(n-1), x(n)\hat{d}(n-2), ..., x(n-r)\hat{d}(n-s)]^T, \quad (3)$$

and

$$W_n = [a_0(n), ..., a_r(n), b_1(n), ..., b_s(n),$$
$$c_{0,1}(n), c_{0,2}(n), ..., c_{r,s}(n)]^T, \qquad (4)$$

respectively. The objective is to find a stochastic gradient descent solution for the coefficients of the adaptive filter which attempts to minimize the cost function

$$J(n) = E(d(n) - \hat{d}(n))^2 = E(d(n) - W_{n-1}^T Z_n)^2 \qquad (5)$$

in a recursive manner.

The stochastic gradient descent solution is given by

$$W_n = W_{n-1} - (\mu/2)\frac{\partial(d(n) - W_{n-1}^T Z_n)^2}{\partial W_{n-1}}, \qquad (6)$$

where μ is a small positive constant that controls the rate at which the adaptive filter converges. Note that $\hat{d}(n)$ and therefore Z_n are functions of W_{n-1}. Thus, the gradient in (6) may be obtained as

$$-\frac{\partial(d(n) - W_{n-1}^T Z_n)}{\partial W_{n-1}} = Z_n + \sum_{j=1}^{s} b_j(n-1)\frac{\partial\hat{d}(n-j)}{\partial W_{n-1}}$$

$$+\sum_{i=0}^{r}\sum_{j=1}^{s} c_{i,j}(n-1)x(n-i)\frac{\partial\hat{d}(n-j)}{\partial W_{n-1}}. \qquad (7)$$

Equation (7) indicates the necessity of re-evaluation of the derivatives of the past values of $\hat{d}$ with respect to W_{n-1}.

An assumption commonly employed in the adaptive IIR filtering literature is that μ is sufficiently small such that $W_{n-1} \approx W_{n-2} \approx ... \approx W_{n-N}$ [5]. Using this approximation and the fact that $d(n)$ is not a function of W_{n-1}, we may write (7) as

$$\frac{\partial\hat{d}(n)}{\partial W_{n-1}} = Z_n + \sum_{j=1}^{s} b_j(n-1)\frac{\partial\hat{d}(n-j)}{\partial W_{n-1-j}}$$

$$+\sum_{i=0}^{r}\sum_{j=s}^{N-1} c_{i,j}(n-1)x(n-i)\frac{\partial\hat{d}(n-j)}{\partial W_{n-1-j}}. \qquad (8)$$

Let us define a new vector Ψ_n as

$$\Psi_n = \frac{\partial\hat{d}(n)}{\partial W_{n-1}}. \qquad (9)$$

Equation (8) can then be compactly written as

$$\Psi_n = Z_n + \sum_{j=1}^{s} b_j(n-1) + \sum_{i=0}^{r}\sum_{j=1}^{s} c_{i,j}(n-1)x(n-i)\Psi_{n-j}. \qquad (10)$$

We are now in a position to summarize the output-error LMS bilinear filter as follows:

$$\alpha(n) = d(n) - W_{n-1}^T Z_n, \qquad (11)$$

$$\Psi_n = Z_n + \sum_{j=1}^{s}\{b_j(n-1) + \sum_{i=0}^{r} c_{i,j}(n-1)x(n-i)\}\Psi_{n-j}, \qquad (12)$$

and

$$W_n = W_{n-1} + \mu\alpha(n)\Psi_n. \qquad (13)$$

The above algorithm has a computational complexity of $O(rs^2)$ multiplications per iteration. The computational burden can be significantly reduced by recognizing that $rs+s-1$ elements of Z_n in (10) are delayed versions of the other $r+s+2$ elements of Z_n. This motivates the approximation of replacing the corresponding $rs+s-1$ elements of Ψ with appropriate delayed versions of the other $r+s+2$ elements of Ψ. That is, instead of (12), we could use

$$\hat{\Psi}_n = \hat{Z}_n + \sum_{j=1}^{s}\{b_j(n-1) + \sum_{i=0}^{r} c_{i,j}(n-1)x(n-i)\}\hat{\Psi}_{n-j}, \qquad (14)$$

where $\hat{\Psi}$ and $\hat{Z}$ denote the vectors that contain the aforementioned $r+s+2$ entries of Ψ and Z, respectively. We then construct an approximate version of Ψ, which is needed for (13), from the current and past values of $\hat{\Psi}$. With this approximation, the filter requires only $O(rs)$ multiplications per iteration.

The algorithms described above are not guaranteed to be stable in the sense that the output $\hat{d}(n)$ may grow without bound during adaptation. Therefore, the filters must be monitored at all times for any indication of unstable behavior. In the next section, we propose a stability monitoring scheme to accomplish this objective.

3. STABILITY MONITORING

The basic philosophy behind the derivation of the adaptive bilinear system with stability moniotoring may be described as follows. After each coefficient update, the system will check the coefficients to see if they satisfy some sufficient conditions which guarantee the stability of the bilinear system generated by the adaptive filter. If the coefficients satisfy the conditions, the update is complete and the adaptive filter will wait for the next sample to arrive. Otherwise, it will project the coefficients to a space that satisfies the sufficient conditions.

The stability condition that we will employ is based on a sufficient condition recently derived by the authors [8] for time-invariant bilinear systems. We check if the coefficients satisfy the following:

$$\begin{cases} |q_i(n)| < 1, \textit{for } i = 1,\ 2, \ldots,\ s,\ \textit{and} \\ M_x \sum_{i=0}^{r} \sum_{j=1}^{s} |c_{i,j}(n)| < \prod_{j=1}^{s} (1 - |q_j(n)|), \end{cases} \tag{15}$$

where $q_1(n)$, $q_2(n)$,..., and $q_s(n)$ denote the zeros of the polynomial $q^s(1 - \sum_{j=1}^{s} b_j(n) q^{-j})$, and M_x denotes the maximum absolute value of $x(n)$ in some interval. Since the above condition involves the calculation of the roots of a polynomial, we first examine if the coefficients satisfy another easily calculated sufficient condition [1]:

$$\sum_{j=1}^{s} |b_j(n) + \sum_{i=0}^{r} c_{i,j}(n) x(n-i)| < 1 - \delta, \tag{16}$$

where δ is a small positive constant (typically 0.001 or 0.01). The conditions of (15) are checked only if the coefficients fail the simpler test.

If neither of (15) and (16) holds, we reduce the amount of adjustment for the coefficients in the hope that the resulting update will pass the tests and guarantee the boundedness of $\hat{d}(n)$. For obvious reason, we need to limit the number of iterations for this adjustment. Based on this idea, an algorithm of output-error LMS bilinear filter with stability monitoring is summarized below.

(T0) Wait for new sample. Set $\Delta = 0$

Error Calculation

(T1) $\alpha(n) = d(n) - W_{n-1}^T Z_n$

Gradient Calculation

(T2) Calculate Ψ_n as described in Section 2 using (14).

(T3) $\varepsilon(n) = \mu \alpha(n) \Psi_n$

Coefficient Update

(T4) $W_n = W_{n-1} + \varepsilon(n)$

Stability Monitoring

(T5) If (7) holds, go to (T0)

(T6) If (6) holds, go to (T0)

(T7) If Δ equals a pre-selected threshold, set $W_n = W_{n-1}$ and go to (T0). In our experiments we set the threshold to 3.

(T8) $\Delta = \Delta + 1$

(T9) Set $\varepsilon(n) = \varepsilon(n)/2$ and go to (T4)

Note that both of the conditions employed are only sufficient. Also, the coefficient set that satisfy (16) is not a proper subset of the coefficient set that satisfy (15) and vice versa. The use of both the tests results in a more robust stability monitoring system.

4. SIMULATION RESULTS

We now present simulation results that illustrate the usefulness of the foregoing algorithm. The results presented are ensemble averages over 20 independent runs. The problem considered in the experiments was that of identifying an unknown, time-invariant bilinear system with $r = s = 2$. The coefficients of the unknown system were given by

$$W^o = [1, 1, 1, 0.2, 0.48, 0.3, 0.1, 0.2, 0.2, 0.1, 0.3]^T. \tag{8}$$

The input signal $x(n)$ was a white, zero-mean and pseudorandom Gaussian noise with variance 0.2. Another white, zero-mean, pseudorandom Gaussian noise that was uncorrelated with the input signal was added to the output of the bilinear system. The resulting signal-to-noise ratio was 20 dB. The adaptive filter was implemented with the same structure and the same number of coefficients as that of the unknown system. The step-size μ was set to 0.0025.

We have conducted several experiments involving output error adaptive bilinear filters with and without the stability monitoring mechanism. The results indicate that the adaptive filter without the stability monitoring device suffers from significant stability problem. With the help of stability monitoring scheme, the output-error LMS bilinear filter performed reasonably well. The evolution of some filter coefficients with and without the stability monitoring scheme is shown in Figures 1 (a) and (b), respectively.

5. CONCLUDING REMARKS

We presented an output-error LMS adaptive bilinear filter equipped with a stability monitoring device in this paper. Results of some preliminary experiments presented showed that the system equipped with the stability monitoring device did not exhibit the unstable behavior shown by adaptive bilinear filters that lacked such devices. It appears that unsupervised operation of output-error LMS adaptive bilinear filters may now be possible in practical applications.

We are at present working on improving the system in two different ways. The authors of this paper recently presented a sufficient stability condition for time-varying bilinear systems [10]. Early results of adaptive filters that employ the new condition appear to be very promising.

Cascade or parallel realizations of certain portions of the adaptive filter will facilitate easier stability monitoring. This topic also is currently being investigated by the authors.

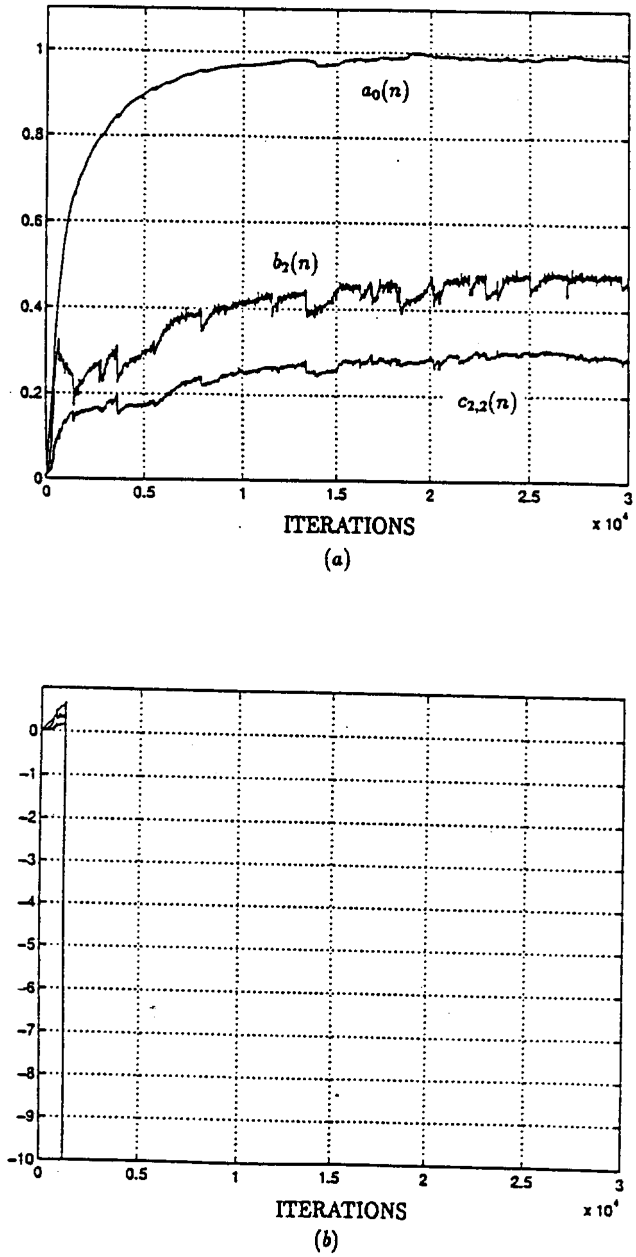

Figure 1: Mean trajectories of filter coefficients

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