A TIME-VARYING AR MODELING OF HEART WALL VIBRATION

Hiroshi Kanai, Noriyoshi Chubachi, and Yoshiro Koiwa

Kanai and Chubachi's address: Dept. of Electrical Engineering, Faculty of Eng., Tohoku University, Sendai 980-77, Japan
e-mail: hkanai@chubachi.ecei.tohoku.ac.jp
Koiwa's address: First Department of Internal Medicine, School of Medicine, Tohoku University, Sendai 980-77, Japan

ABSTRACT

In this paper, we present a new method to estimate spectrum transition of a nonstationary signals in low signal-to-noise ratio (SNR) cases. If the spectrum transition pattern is complex and/or there are large differences in the transition patterns among the individual nonstationary signals, it is difficult to estimate the transition pattern stably by the previously proposed time-varying AR modeling because the results are considerably dependent on the choice of the basic functions to be used. We propose a new approach of modeling to estimate the spectrum transition of the nonstationary signals by using a linear algorithm without assuming any basic functions. Instead of basic functions we use the spectrum transition constraint. By applying this method to the analysis of vibration signals on the interventricular septum of the heart, noninvasively measured by the method developed in our laboratory using ultrasonic, spectrum transition pattern is clearly obtained during one beat period. The proposed method will serve a tool for the noninvasive acoustic diagnosis of heart diseases in near future.

1. INTRODUCTION

Much work has been done on the parametric spectrum estimation using autoregressive (AR) model. A strong restriction of these methods lies in the necessary assumption that the signals may be considered to be stationary over the observation interval. Time-varying parametric approaches of modeling have been proposed to overcome this limitation and to take the effects of nonstationary signals into account explicitly. To estimate the parameters using a linear algorithm, the unknown time-varying parameters are approximated by linearly weighted combinations of a small number of known functions. The choice of the basic functions is an important part of such modeling process. A convenient way is to replace the time-varying coefficients with their second-order expansion [1], or an arbitrary order expansion [2],[3]. Legendre [4],[5], Fourier [6], prolate spheroidal [7], and B-spline [8] are usually chosen for the basic functions. Since the number of unknown parameters is large, efficient equivalent representations for the modeling have been also proposed such as lattice filters [2],[7],[9].

However, if the spectrum transition pattern is complex and/or there are large differences in the transition patterns among the individual nonstationary signals, it is difficult to estimate the transition pattern stably by choosing a set of basic functions *a priori*.

We have proposed a method for analyzing the spectrum transition of the multiframe signals of the fourth heart sounds detected during the stress test [10]. In the method, however, the analyzable signals are limited to multiple short length signals and the spectrum transition pattern between these signals are obtained. In this paper, by modifying the method we propose a new approach of modeling to estimate the spectrum transition of a nonstationary signal by using a linear algorithm without any basic function. We also propose a singular-value-decomposition (SVD)-based method to obtain more accurate nonstationary AR coefficients according to the above-mentioned linear optimization in low SNR cases.

In order to noninvasively diagnose the acoustic characteristics of the heart muscle, it is necessary to measure the small vibration signals on the heart wall from the chest surface and analyze the resultant nonstationary signal during one beat period.

For the former problem, we have developed a new method to noninvasively measure a small vibration signal on the heart wall using ultrasound [11]. For the latter problem, we apply the developed time-varying modeling to the nonstationary small vibration signals on the interventricular septum in order to diagnose the acous-

This work was supported in part by the Grant-in-Aid for Scientific Research (No. 06304010, 05555108, 06213205, 06555113, 06750430) from the Ministry of Education, Science and Culture of Japan.

0-7803-2431-5/95 $4.00 © 1995 IEEE

tic characteristics of the heart muscle. These characteristics and the transition patterns may be applied to acoustic diagnosis of heart diseases.

2. PRINCIPLE

Let us divide an original nonstationary signal $y(n)$ into succeeding F short signals $\{x(n;j)\}, n = 0,1,...,N-1, j = 0,1,...,F-1$, each is called by a *frame*, where F is the number of frames. Let us assume that each frame signal $x(n;j)$ be an AR signal of order M, represented by the backward recursion such that

$$x(n;j) = -\sum_{i=1}^{M} a_i(j) \cdot x(n+i;j) + e(n;j) \quad (1)$$

where $e(n;j)$ is a stationary white noise process of the jth frame with zero-mean and variance σ_e^2, and $\{a_i(j), i = 0,1,...,M\}$ are linear predictive coefficients of the jth frame signal $(a_0(j) = 1)$. We assume that each signal $x(n;j)$ is stationary over the jth frame and the AR coefficients $\{a_i(j)\}$ are slowly time-varying between the succeeding frames. When the length of each frame signal is short and the SNR is low, to achieve a meaningful spectrum transient pattern of nonstationary signals when the length of each frame signal is short and the SNR is low, we must design a new cost function J in such a way that the resulting solution is sufficiently smooth under the assumption that the AR parameters over the succeeding frames do not vary rapidly. In addition to this assumption, we assume that the differences $\{\Delta a_i(j) \stackrel{\text{def}}{=} a_i(j) - a_i(j-1)\}$ between ith AR parameters of the succeeding frames are random drawing from a stationary Normal distribution having zero mean and variance σ_i^2. Thus, the *framed* AR parameters $\{a_i(j), i = 1,2,...,M, j = 0,1,...,F-1\}$ are assumed to be from a process with independent increments. This model is, therefore, specified by a set of parameters $\{a_i(0), i = 1,...,M\}$, $\{\Delta a_i(j), i = 1,...,M, j = 1,2,...,F-1\}$, and $\{\sigma_i^2, i = 1,...,M\}$. To determine the spectrum transient pattern without any basic functions as is done in existing time-varying spectrum estimation algorithms, we introduce the following constrained least-square modeling in the formulation of the cost function J as minimizing the normalized residual power subject to the condition that the transition power of the ith set of AR parameters equals the constant σ_i^2. Using the method of Lagrange multipliers,

$$J = \frac{E_{n,j}[|\, e(n;j)\,|^2]}{E_{n,j}[|\, x(n;j)\,|^2]} - \sum_{i=1}^{M} \lambda_i \{E_j[|\,\Delta a_i(j)\,|^2] - \sigma_i^2\}, \quad (2)$$

where λ_i is a Lagrange multiplier. Let us define $P_0 = \frac{1}{F(N-M)} \sum_{j=0}^{F-1} \sum_{n=0}^{N-M-1} |x(n;j)|^2$, $\lambda_i' = \lambda_i \cdot \frac{P_0 F(N-M)}{F-1}$ and $\sigma_i'^2 = \sigma_i^2(F-1)$, a $(N-M)$-dimensional vector $\mathbf{x}_j = [x(0;j), x(1;j), \cdots, x(N-1-M;j)]^T$, a M-dimensional vector $\mathbf{a}_j = [a_1(j), ..., a_M(j)]^T$, where the superscript T denotes the matrix transpose, $(N-M)$-by-M matrix X_j:

$$X_j = \begin{bmatrix} x(1;j) & \cdots & x(M;j) \\ x(2;j) & \cdots & x(M+1;j) \\ \vdots & \ddots & \vdots \\ x(N-M;j) & \cdots & x(N-1;j) \end{bmatrix},$$

a $F(N-M)$-by-FM matrix G:

$$G = \begin{bmatrix} X_0 & 0_{N-M,M} & \cdots & 0_{N-M,M} \\ X_1 & X_1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0_{N-M,M} \\ X_{F-1} & X_{F-1} & \cdots & X_{F-1} \end{bmatrix},$$

and a FM-by-FM matrix Λ':

$$\Lambda' = \begin{bmatrix} 0_{M,M} & \cdots & \cdots & 0_{M,M} \\ \vdots & I_{M,M}\Gamma' & \cdots & \vdots \\ \vdots & \cdots & \ddots & 0_{M,M} \\ 0_{M,M} & \cdots & \cdots & I_{M,M}\Gamma' \end{bmatrix}$$

where $0_{N_1,N_2}$ and I_{N_1,N_2} denote a N_1-by-N_2 zero matrix and a N_1-by-N_2 unit matrix, respectively, and Γ' is a M-by-M diagonal matrix $\Gamma' = \text{diag}(\lambda_1', \lambda_2', \ldots, \lambda_M')$. Let $\overrightarrow{\lambda'}$ and $\overrightarrow{\sigma'}$ be M-dimensional vectors $(\lambda_1', \lambda_2', \cdots, \lambda_M')^T$ and $(\sigma_1', \sigma_2', \cdots, \sigma_M')^T$, respectively. By using these matrices, vectors, and the relation

$$\mathbf{a}_j = \mathbf{a}_0 + \sum_{k=1}^{j} \Delta\mathbf{a}_k, (j = 1,2,...,F-1), \quad (3)$$

where $\Delta\mathbf{a}_j$ is a M-dimensional vector of $[\Delta a_1(j), ..., \Delta a_M(j)]^T$. The cost function J', which is obtained by multiplying J in Eq. (2) by $P_0 F(N-M)$, is rearranged as follows:

$$J' = (\mathbf{X} + G\mathbf{A})^T(\mathbf{X} + G\mathbf{A}) - (\mathbf{A}^T\Lambda'\mathbf{A} - \overrightarrow{\lambda'}^T \overrightarrow{\sigma'}) \quad (4)$$

where $\mathbf{A} = (\mathbf{a}_0^T, \Delta\mathbf{a}_1{}^T, \cdots, \Delta\mathbf{a}_{F-1}{}^T)^T$ and $\mathbf{X} = (\mathbf{x}_0{}^T, \mathbf{x}_1{}^T, \cdots, \mathbf{x}_{F-1}{}^T)^T$. Taking the derivatives of J' with respect $\mathbf{a}_0$, $\Delta\mathbf{a}_j$, and $\{\lambda_i'\}$, the following simultaneous equations are obtained.

$$(G^TG - \Lambda')\mathbf{A} = -G^T\mathbf{X}, \quad (5)$$

$$\sum_{j=1}^{F-1} |\Delta a_i(j)|^2 - \sigma_i'^2 = 0. \qquad (i = 1,2,\cdots M) \quad (6)$$

Therefore, it is concluded that using the transition constraint of Eq. (2), the solutions $\widehat{\mathbf{A}}$ of the AR coefficients of F frames are simultaneously obtained from

$$\widehat{\mathbf{A}} = -(G^T G - \Lambda')^{+} \cdot G^T \mathbf{X}, \qquad (7)$$

where $^+$ denotes the generalized inverse operation. By substituting Eqs. (5) and (7) into J' of Eq. (4), the achieved minimum cost J'_{min} is given by

$$J'_{min} = \mathbf{X}^T\mathbf{X} + \mathbf{X}^T G\widehat{\mathbf{A}} + \overrightarrow{\lambda'}^T \overrightarrow{\sigma'}. \qquad (8)$$

Thus, the spectrum transient pattern is determined from the resultant estimates of the AR coefficients of each frame. In this paper, we assume that the variance of every order i of the AR parameters $\{a_i(j)\}$ has the same value, that is, $\sigma_1'^2 = \sigma_2'^2 = \cdots = \sigma_M'^2 \stackrel{\text{def}}{=} \rho'^2/M$, and then $\lambda_1' = \lambda_2' = \cdots = \lambda_M' \stackrel{\text{def}}{=} \lambda'$. We describe how to automatically choose the value of $\rho'^2 = \sigma_1'^2 + \cdots + \sigma_M'^2$ in the next section.

After determining the average value ρ'^2/M of the variance σ_i^2 of AR parameters, the Lagrange multipliers λ_i' in Λ' of Eq. (7) must be adjusted so that the constraint of the second term of Eq. (4) or Eq. (6) is satisfied. By finding a zero of the function $\zeta = \sum_{j=1}^{F-1} ||\overrightarrow{\Delta \mathbf{a}_j}||^2 - \rho'^2$ which changes sign based on the bisection method, the optimization of Eq. (5) under the constraint in Eq. (6) can be done in a simple iterative fashion. When the SNR is low, in order to improve the accuracy of the estimates, we apply the SVD-based approach [12] to the above modeling.

3. EXPERIMENTAL RESULTS

We applied this method to the analysis of small vibration signals $y(n)$ on the interventricular septum in the heart wall of a normal man of 54 years old in Fig. 1(c) for the noninvasive acoustical diagnosis of myocardial dysfunction.

One beat signal $y(n)$ in the first beat period in Fig. 1(c) is divided into succeeding 27 short frame signals $x(n;j)$, each of which has 30 points in length, by multiplying the Hamming window with a length of 30 points. That is, each frame signal is about 150 ms in length since the signal is A/D converted at a sampling period of 5 ms. Adjacent short signals overlap each other by their three-quarter-length. Since each duration time of the first heart sound (I) and the second heart sound (II) in Fig. 1(b) is about 150ms in length, let us assume that each frame signal $x(n;j)$ is stationary over each frame.

Since the SNR is low and the duration time of each frame signal is very short, there are large fluctuations and many phantom peaks appear in spectra of Fig. 2(a) estimated by independently applying the discrete Fourier transform (DFT) to each frame signal (N=30, F=27).

On the other hand, by applying the proposed method (M = 8) to the same multiframe signals, the resultant spectrum transition patterns are shown in Fig. 2(b) for the same signal as Fig. 2(a). In these experiments we found that the optimum value of λ is chosen as follows: As the value of λ is increased, though the transition restriction becomes stronger, the third term $\overrightarrow{\lambda'}^T \overrightarrow{\sigma'} = \lambda' \sum_{j=1}^{F-1} ||\Delta \mathbf{a}_j||^2$ of J'_{min} in Eq. (4) is increased. We chose the value of λ as the value of $\overrightarrow{\lambda'}^T \overrightarrow{\sigma'}$ takes the local maximum.

In the resultant spectra in Fig. 2(b), the frequency transition from the systole period, which lies between the first heart sound (I) and the second heart sound (II), to the diastole period is clearly obtained.

4. CONCLUDING REMARKS

We present a new method to estimate spectrum transition of a nonstationary signal in low SNR cases using

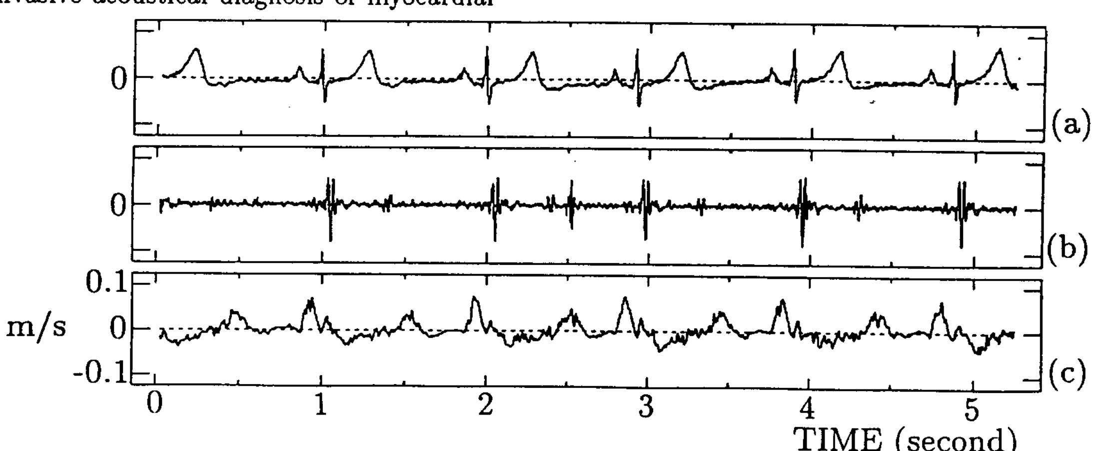

Fig. 1 (a) an electrocardiogram, (b) heart sounds, and (c) small vibration signal $y(n)$ with high frequency ↗ components on the interventricular septum of the heart measured by the newly developed method in our lab.[11].

a linear algorithm without any basic function. By applying the proposed method to the heart wall vibrations, we found there are clear spectrum transition patterns.

The electrocardiogram or the heart sounds contain only low frequency components and each of them does not continue within one beat period. However, small vibration signals accurately measured by our method contain the information enough to diagnose all four stages in one cardiac cycle. Thus, a new scientific field of noninvasive acoustic diagnosis of the heart dysfunction will be developed soon by the measurement of the heart wall vibrations and their analysis as proposed in this paper.

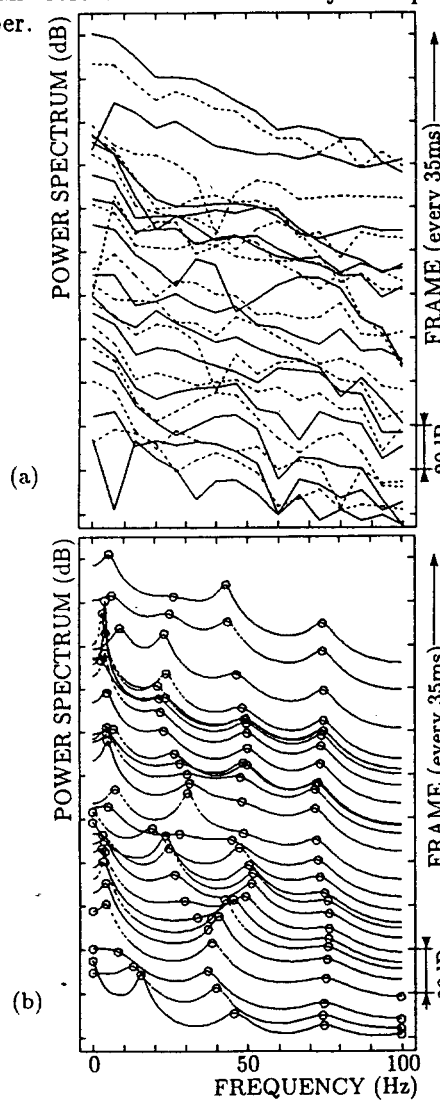

Fig. 2. (a) The spectra of signals $x(n;j)$ obtained by the DFT with the Hamming window of 30 point in length. The even and odd frames are shown in solid lines and dotted lines, respectively. (b) The spectrum transition estimated by the proposed method in this paper for the same normal person. Each estimated pole frequency is indicated by "o".

References

[1] T. Subba Rao,"The Fitting of Non-Stationary Time-Series Models with Time-Dependent Parameters," *J. of the Royal Statist. Soc.* Series B, Vol. 32, No. 2, pp.312-322, 1970.

[2] L. A. Liporace, "Linear Estimation of Non-Stationary Signals," *J. Acoust. Soc. Amer.*, Vol. 58, No. 6, pp.1288-1295, 1975.

[3] K. C. Sharman and B. Friedlander, "Time-Varying Autoregressive Modeling of a Class of Nonstationary Signals," in *Proc. IEEE ICASSP*, 1984, pp. 22.2.1-22.2.4.

[4] F. Kozin, "Estimation and Modeling of Nonstationary Time-series," in *Proc. Symp. Appl. Comput. Meth. Eng.*, Vol. 1, Los Angeles, CA, 1977, pp. 603-612.

[5] G. Alehgrin and M. Barlaud, and J. Menez, "Unbiased Parameter Estimation of Nonstationary Signals and Noise," *IEEE Trans.* vol. ASSP-34, no. 5, pp. 1319-1322, 1986.

[6] M. Hall, A. V. Oppenheim, A. Willsky, "Time-Varying Parametric Modeling of Speech," in *Proc. IEEE Decision and Control Conf.*, New Orleans, LA, pp. 1085-1091, 1977.

[7] Y. Grenier, "Time-Dependent ARMA Modeling of Nonstationary Signals," *IEEE Trans.* vol. ASSP-31, no. 4, pp. 899-911, 1983.

[8] M. J. Flaherty, "Application of Polynomial Splines to a Time-Varying Autoregressive Model of speech," in *Proc. IEEE ICASSP*, 1988, pp. 2220-2223.

[9] E. Karlsson and M. H. Hayes, "ARMA Modeling of Time Varying Systems with Lattice Filters," in *Proc. IEEE ICASSP*, 1986, pp. 2335-2338.

[10] H. Kanai, N. Chubachi, K. Kido, Y. Koiwa, T. Takagi, J. Kikuchi, and T. Takishima: "A New Approach to Time Dependent AR Modeling of Signals and Its Application to Analysis of the Fourth Heart Sound," *IEEE Transactions on Signal Processing.* Vol. 40, No. 5, pp. 1198-1205, 1992.

[11] H. Kanai, K. Hirose, H. Sato, and N. Chubachi: "A New Method for Measuring Small Local Vibrations in the Heart Using Ultrasound," *IEEE Transactions on Biomedical Engineering.* Vol. 40, No. 12, 1993.

[12] R. Kumaresan and D. W. Tufts, "Estimating the Parameters of Exponentially Damped sinusoids and pole-zero modeling in noise," *IEEE Trans.* vol. ASSP-30, no. 6, pp. 833-840, 1982.