STABILITY ANALYSIS OF ROBUST ADAPTIVE FILTER USED IN FEEDFORWARD AND FEEDBACK COMPENSATION

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ABSTRACT

This paper proposes robust adaptive algorithms for adjusting coefficients of an adaptive filter which is used for feedforward control together with a feedback compensator. The filtered-x algorithm which is widely employed in adaptive signal processing cannot always assure the stability. In this paper, stability-guaranteed adaptive algorithms are given in time domain and frequency domain on a basis of the strictly positive real property of adaptive systems in the presence of unknown disturbances. Time domain algorithms can be applied to an IIR or FIR adaptive filter, while frequency domain approaches use a structure of adaptive frequency sampling filter. Numerical simulations and experimentally obtained results exhibited significant improvement on convergency and stability of the proposed adaptive algorithms in application to adaptive active noise control.

1. INTRODUCTION

The aim of this paper is to give robustly stabilized adaptive algorithms for adjusting parameters of an IIR or FIR adaptive filter which is incorporated with feedforward and feedback compensators. Such a system structure is utilized in adaptive active noise control attenuating unwanted noise, adaptive feedforward structural control subjected to seismic excitation, and etc. Recently, because of the progress of digital signal processors, a variety of adaptive algorithm can be implemented in practice. Various types of filtered-x LMS algorithms have been typically used in conventional adaptive digital filters [1], however, it sometimes displays serious problems of instability although it does not need a precise error model [2]-[5].

In this paper, we propose stability-guaranteed adaptive algorithms for an IIR or FIR adaptive filter, which can be derived on a basis of the strictly positive real property of the error model treated in adaptive system theory [6], when the error model can be identified a priori. In a case that the error model has large time lag or large relative degree even if it is known a priori, it is very important to assure the robust stability of the adaptive system especially in the presence of unknown disturbances and mismatch in the order of the adaptive filter. Robust adaptive algorithms in frequency domain can also be given by utilizing an adaptive frequency sampling filter structure [7]. Experimental results validate the effectiveness of the proposed stable algorithms.

2. ADAPTIVE SYSTEM CONFIGURATION

Fig.1 shows an example of adaptive feedforward control systems with feedback compensation. An IIR or FIR adaptive filter is utilized in the implementation of the feedforward compensator. This system structure can be employed in an adaptive active noise control system or adaptive feedforward control for response reduction of structure excited by seismic ground motions, etc. In case of active noise control, the purpose of the adaptive filter $G(z^{-1},k)$ is to attenuate the sound pressure of $e(k)$ at the observer point A. The primary source emits an unwanted noise $s(k)$, which is transmitted via the channels $H_1(z^{-1})$ and $H_2(z^{-1})$, and it is detected by the microphones at the location A and B respectively. The signal $r(k)$ detected at B is one of the inputs to the adaptive filter $G(z^{-1},k)$ which generates the control signal $g(k)$ to a loudspeaker C which emits the secondary artificial sound to cancel the unwanted sound at the location A. The feedback compensator $F(z^{-1})$ has an important role of improving the control performance in the presence of the disturbances $v(k)$ which is caused by a multipath effect due to reflections of sounds, and a mismatch of the order of the adaptive filter [8]. The control sound $u(k)(=g(k)+f(k))$ emitted from the loudspeaker C is transmitted to the location A and B via $H_3(z^{-1})$ and $H_4(z^{-1})$ respectively. In adaptive structural control, $s(k)$ is seismic motion detected by ground sensors, $u(k)$ is an actuating control input, $e(k)$ describes response quantities such as displacements, velocities and strains at critical locations.

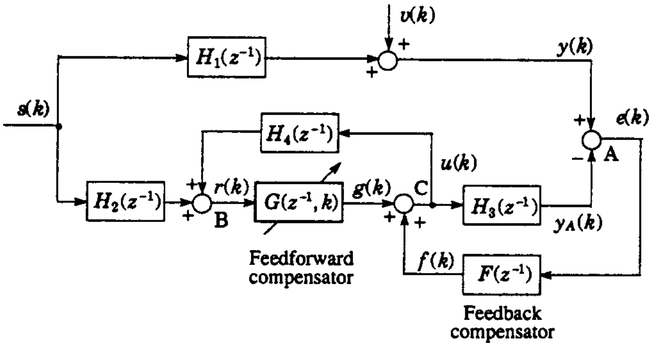

Fig.1 Structure of adaptive feedforward control system

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Now the main purpose of this paper is to give a robustly stable adaptive algorithm for adjusting the parameters of the IIR or FIR adaptive filter in the above system with the feedforward and feedback compensations. On the assumptions that $v(k)$ is unknown but bounded as $|v(k)| < \beta$ and $H_3(z^{-1})$ is known a priori or can be identified, we will clarify that the error model depends on the both $H_3(z^{-1})$ and $F(z^{-1})$ and the proposed robust adaptive algorithm can assure the stability and boundedness of the error.

3. STABILITY-ASSURED ADAPTIVE FILTER

3.1 Description of error model

We shall clarify the conditions for that the sound pressure at A can be attenuated by the compensators $G(z^{-1})$ and $F(z^{-1})$. The transfer functions from $s(k)$ and $v(k)$ to $e(k)$ are given respectively by

$$e(k) = \Big\{\big(H_1(z^{-1}) - [H_1(z^{-1})H_4(z^{-1}) + H_2(z^{-1})H_3(z^{-1})] G(z^{-1})\big)\Big/\big(1 - H_4(z^{-1})G(z^{-1}) + H_3(z^{-1})F(z^{-1})\big)\Big\}s(k) + \frac{1 - H_4(z^{-1})G(z^{-1})}{1 - H_4(z^{-1})G(z^{-1}) + H_3(z^{-1})F(z^{-1})}v(k) \qquad (1)$$

In the absence of the disturbances $v(k)$, the cancellation of $s(k)$ at the point A can be attained perfectly by selecting the feedforward compensator $G(z^{-1})$ as

$$G(z^{-1}) = \frac{H_1(z^{-1})}{H_1(z^{-1})H_4(z^{-1}) + H_2(z^{-1})H_3(z^{-1})} \equiv G^o(z^{-1}) \qquad (2)$$

which does not depend on the feedback compensator $F(z^{-1})$. In the presence of disturbances, if $G(z^{-1})$ is chosen as $G^o(z^{-1})$ in (2), (1) becomes

$$e(k) = \frac{G^o(z^{-1})}{G^o(z^{-1}) + F(z^{-1})}v(k) \qquad (3)$$

It should be noticed that the cancellation error $e(k)$ can be reduced by an appropriate choice of the feedback compensator $F(z^{-1})$.

3.2 Stability analysis of robust adaptive algorithm

When the location of the primary noise source is movable and the multipath effects appear, the transfer functions $H_1(z^{-1})$, $H_2(z^{-1})$ and $H_4(z^{-1})$ are changeable. Therefore we implement the feedforward compensator by the IIR adaptive filter to cope with the above uncertainties, as

$$G(z^{-1},k) = \frac{\hat{b}_1(k)z^{-1} + \cdots + \hat{b}_m(k)z^{-m}}{1 + \hat{a}_1(k) + \cdots + \hat{a}_n(k)z^{-n}} \qquad (4)$$

Let the output of the adaptive filter be denoted by $g(k)$, which is given by

$$g(k) = G(z^{-1},k)r(k) = \hat{\theta}^T(k)\varphi(k) \qquad (5)$$

where $\hat{\theta}(k) \equiv [\hat{a}_1(k),\cdots,\hat{a}_n(k),\hat{b}_1(k),\cdots,\hat{b}_m(k)]^T$, $\varphi(k) \equiv [-g(k-1),\cdots,-g(k-n),r(k-1),\cdots,r(k-m)]^T$. It follows from Fig.1 that

$$e(k) = H_1(z^{-1})s(k) - H_3(z^{-1})(g(k) + f(k)) + v(k)$$
$$f(k) = F(z^{-1})e(k)$$

Then, we have

$$e(k) = W_1(z^{-1})s(k) - W_2(z^{-1})g(k) + w(k) \qquad (6)$$

where $W_1(z^{-1}) = H_1(z^{-1})/(1 + H_3(z^{-1})F(z^{-1}))$, $W_2(z^{-1}) = H_3(z^{-1})/(1 + H_3(z^{-1})F(z^{-1}))$ and $w(k) = W_3(z^{-1})v(k)$ where $W_3(z^{-1}) = 1/(1 + H_3(z^{-1})F(z^{-1}))$.

The problem is formulated as follows: Find the robust adaptive algorithm for adjusting the parameters so as to attenuate $e(k)$, by using the linear combination form $W_2(z^{-1})[\hat{\theta}^T(k)\varphi(k)]$. Thus from (5) and (6) the error model is described by

$$e(k) = W_2(z^{-1})[\bar{\theta}^T\varphi(k)] - W_2(z^{-1})[\hat{\theta}^T(k)\varphi(k)] + w(k) \qquad (7)$$

Now we give a new robust algorithm assuring the stability based on the strictly positive real property of the error model. For the purpose we introduce the auxiliary variable $e_a(k)$ and the extended error $\varepsilon(k)$ as

$$e_a(k) = \hat{\theta}^T(k)W_2(z^{-1})\varphi(k-1) - W_2(z^{-1})\hat{\theta}^T(k-1)\varphi(k-1) \qquad (8a)$$
$$\varepsilon(k) = e(k-1) - e_a(k) \qquad (8b)$$

respectively. Thus we have the new error model form (7) and (8) as

$$\varepsilon(k) = (\bar{\theta} - \hat{\theta}(k))^T\psi(k-1) + w(k-1) \qquad (9)$$

where $\psi(k-1) = W_2(z^{-1})\varphi(k-1)$. From (9) we obtain the gradient type of basic adaptive algorithm as

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \Gamma\psi(k-1)\varepsilon(k) \qquad (10)$$

Because $\varepsilon(k)$ includes $\hat{\theta}(k)$, (10) cannot be directly carried out. Therefore, substituting (10) into (8), and rewriting $\varepsilon(k)$ in (8) gives

$$(1 + \psi^T(k-1)\Gamma\psi(k-1))\varepsilon(k) = e(k-1) + W_3(z^{-1})g(k-1) - \hat{\theta}^T(k-1)\psi(k-1)$$

Thus we can give the RLS type of robust adaptive filter algorithm as

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \alpha(k-1)\Gamma(k-1)\psi(k-1)\varepsilon(k) \qquad (11)$$
$$\varepsilon(k) = \frac{\eta(k)}{1 + \psi^T(k-1)\Gamma(k-1)\psi(k-1)}$$
$$\eta(k) = e(k-1) + W_2(z^{-1})g(k-1) - \hat{\theta}^T(k-1)\psi(k-1)$$
$$\bar{\Gamma}(k) = \Gamma(k-1)\left[I - \frac{\alpha(k-1)\psi(k-1)\psi^T(k-1)\Gamma(k-1)}{1 + \psi^T(k-1)\Gamma(k-1)\psi(k-1)}\right]$$
$$\Gamma(k) = \lambda^{-1}(k)\bar{\Gamma}(k),$$
$$\lambda(k) = \frac{\mathrm{trace}[\bar{\Gamma}(k)]}{\mathrm{trace}[\Gamma(0)]}, \quad \Gamma(0) = \Gamma^T(0) > 0$$
$$\delta(k-1) = \beta'[1 + \psi^T(k-1)\Gamma(k-1)\psi(k-1)]^{1/2}$$
$$\alpha(k-1) = \begin{cases} 0: & \eta^2(k-1) \le \delta^2(k-1) \\ 1: & \eta^2(k-1) > \delta^2(k-1) \end{cases}$$

where, if $\Gamma(k) \equiv \Gamma = \sigma I$, we have a robust LMS algorithm, which is easily implemented.

Theorem: The above algorithm satisfies that

$$\text{(a)}\ \lim_{k\to\infty} \sup (\eta^2(k) - \delta^2(k)) \le 0 \tag{12a}$$

$$\text{(b)}\ \lim_{k\to\infty} \left\| \hat\theta(k) - \hat\theta(k-1) \right\| = 0 \tag{12b}$$

Let the parameter error be $\tilde\theta(k) \equiv \theta - \hat\theta(k)$ and let the Lyapunov function be $V(k) = \tilde\theta^T(k)\Gamma^{-1}(k)\tilde\theta(k)$. Then we can prove the robust stability which assures that the output error $e(k)$ is bounded for any time, by showing that $\Delta V(k) = V(k) - V(k-1) \le 0$ and then the error signal and parameter error are both bounded.

4. STABLE ADAPTIVE ALGORITHM IN FREQUENCY DOMAIN

Adaptive algorithms in the frequency domain are very useful not only in the fields of adaptive digital signal processing [7] but also adaptive control [9]. The frequency domain approach is normally based on FFT, however, which requires batch processing of L time samples of the signal at a time. Therefore use of a frequency sampling filter (FSF) is adequate for real-time processing which is required to adaptive control as we reported in [9]. Let the feedforward controller $G(z^{-1})$ be implemented by an FIR type of filter as

$$\begin{aligned} G(z^{-1}) &= \sum_{k=0}^{L-1} c_k z^{-k} = \sum_{k=0}^{L-1} \frac{1}{L} \sum_{i=0}^{L-1} \breve C_i e^{j\omega_i kT} z^{-k} \\ &= \frac{1-z^{-L}}{L} \sum_{i=0}^{L-1} \frac{\breve C_i}{1 - W_L^i z^{-1}} \end{aligned} \tag{13}$$

where $\{c_i\}$ are the FIR parameters, T is the sampling interval and $W_L \equiv e^{j2\pi/L}$. (13) implies that $G(z^{-1})$ can be constructed by interpolating the L-point frequency response $\breve C_i$ for $i = 0, 1, \cdots, L-1$. If the frequency response is expressed by the M pairs of absolute values and phases on the unit circle as $\{|\breve C_i|, \phi_i; i = 0, 1, \cdots, M; M \equiv (L-1)/2\}$, $G(z^{-1})$ can be determined by the interpolation using the frequency sampling filter as

$$G(z^{-1}) = \sum_{i=0}^{M} \Lambda_i(z^{-1})\Phi_i(z^{-1}) \tag{14}$$

where

$$\Phi_0(z^{-1}) = \frac{1-z^{-L}}{L} \cdot \frac{1}{1-z^{-1}} \tag{15a}$$

$$\Phi_i(z^{-1}) = \frac{1-z^{-L}}{L} \cdot \frac{1}{1 - 2(\cos\omega_i)z^{-1} + z^{-2}} \tag{15b}$$

$$\Lambda_0(z^{-1}) = \lambda_0^A + \lambda_0^B z^{-1}, \quad \lambda_0^B = 0 \tag{16a}$$

$$\Lambda_i(z^{-1}) = \lambda_i^A + \lambda_i^B z^{-1} \qquad \text{for } i = 1, \cdots, M \tag{16b}$$

It is noticed from (14) to (16) that the feedforward controller $G(z^{-1})$ can be constructed by a comb filter $1 - z^{-L}$ followed by a second-order oscillator followed by a two-tapped delay line with two real weights λ_i^A and λ_i^B.

Now, the adaptive feedforward control $u(k)$ is given by

$$\begin{aligned} g(k) &= \sum_{i=0}^{M} (\hat\lambda_i^A(k) + \hat\lambda_i^B(k)z^{-1})\Phi_i(z^{-1})r(k-1) \\ &= \sum_{i=0}^{M} \left(\hat\lambda_i^A(k)\bar r_i(k-1) + \hat\lambda_i^B(k)\bar r_i(k-2)\right) \\ &= \hat\theta^T(k)\varphi(k) \end{aligned} \tag{17}$$

where $\bar r_i(k) = \Phi_i(z^{-1})r(k)$, $\hat\theta^T(k) = (\hat\lambda_1^A(k), \hat\lambda_1^B(k), \cdots, \hat\lambda_1^A(k), \hat\lambda_1^B(k))$, $\varphi(k) = (\bar r_1(k-1), \bar r_1(k-2), \cdots, \bar r_M(k-1), \bar r_M(k-2))^T$.

Thus the error model is represented from (7) as

$$\begin{aligned} e(k) &= W_1(z^{-1})s(k) - W_2(z^{-1})u(k) + n(k) \\ &= W_2(z^{-1})\left((\theta - \hat\theta(k))^T \varphi(k-1)\right) + \overline{w}(k) \end{aligned} \tag{18}$$

where $\overline{w}(k)$ is an uncertain term caused by the unstructured controller error due to the use of an FIR model and disturbances. For assuring stability of the adaptive system, $\bar n(k)$ is assumed to be bounded by β. On the robustness of the adaptive system we will not argue in this paper. Like the time-domain approach, we introduce an auxiliary variable and augmented error to derive a similar RLS algorithm as (11). An LMS algorithm reduces to a more simple form, as given by

$$\hat\theta_i(k) = \hat\theta_i(k-1) + \gamma_i \psi_i(k-1)\varepsilon(k) \tag{19}$$

$$\varepsilon(k) = \frac{e(k-1) + W_2(z^{-1})u(k-1) - \sum_{i=1}^{M} \hat\theta_i^T(k-1)\psi_i(k-1)}{1 + \sum_{i=1}^{M} \gamma_i \psi_i^T(k-1)\psi_i(k-1)}$$

where $i = 0, 1, \cdots, M$, $\gamma_i > 0$, $\hat\theta_i^T(k) = (\hat\lambda_i^A(k), \hat\lambda_i^B(k))$, $\varphi_i(k) = (\bar r_i(k-1), \bar r_i(k-2))^T$, $\psi_i(k-1) = W_2(z^{-1})\varphi_i(k-1)$.

For comparison, an ordinary filtered-x LMS algorithm for updating $\hat\theta_i(k)$ is described by

$$\hat\theta_i(k) = \hat\theta_i(k-1) + \gamma_i \psi_i(k-1)e(k) \tag{20}$$

where $i = 0, 1, \cdots, M$, and it is noticed that (20) can be calculated in parallel at each frequency point, however, its stability cannot be assured, while the computation of $\hat\theta_i(k)$ in the proposed algorithm (19) needs other weights $\{\hat\theta_j(k)\}$ at different frequency points. Further, the extended error $\varepsilon(k)$ is also used in place of the error signal $e(k)$. These modifications enables stabilization of the adaptive algorithm (19).

5. ADAPTIVE ACTIVE NOISE CONTROL

The setup of experiment is given in Fig.2, in which the location of noise source can be changed by switching the two loudspeakers. Fig.3 shows the comparison of convergency of errors between the ordinary filtered-x algorithm (b) and the proposed robust adaptive algorithms (c) and (d). The result for the filtered-x is a limit of stability obtained by choosing the step size γ very carefully. The order of the IIR type of filter was chosen as $n = 31$ and $m = 16$ in

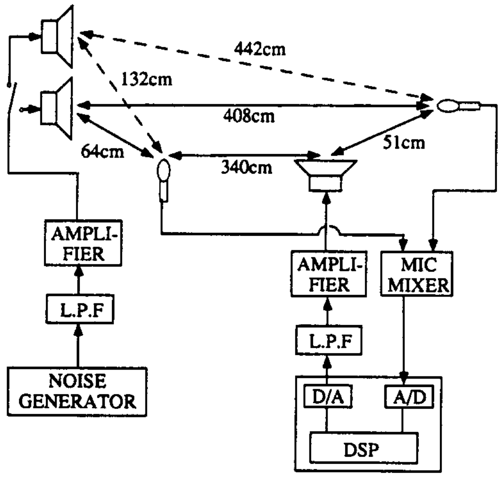

Fig.2 Experimental system for adaptive active noise control.

case (c), which is not sufficient. Therefore the proposed robust adaptive algorithm worked efficiently. Fig.4 gives the power spectrum of the error signal controlled by the proposed adaptive algorithm for the IIR filter, in which noise attenuation could be attained.

6. CONCLUSIONS

The stability-assured adaptive algorithms have been proposed and discussed in time domain and frequency domain for adaptive feedforward control. The adaptive algorithms depends on the controller structure involving the feedback compensator. The validity of the algorithms has been examined by making comparison of the ordinary filtered-x algorithms.

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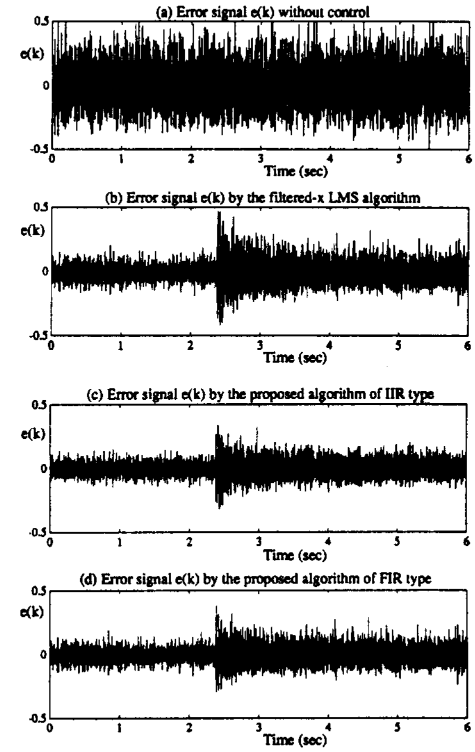

Fig.3 Comparison of controlled results: (a) no control, (b) the filtered-x algorithm for IIR, (c) proposed adaptive algorithm for IIR, and (d) proposed adaptive algorithm for FIR.

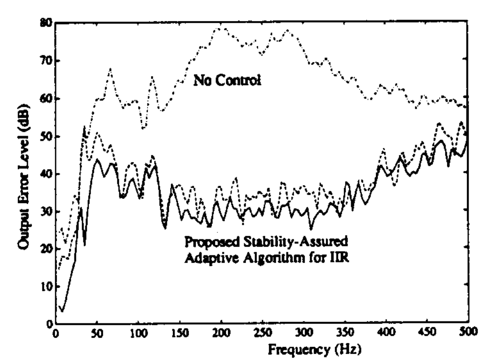

Fig.4 Power spectrum of controlled error signal obtained by the proposed algorithm.