A NEW KIND OF ADAPTIVE FREQUENCY SHIFT FILTER

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ABSTRACT

In this paper, two types of adaptive FREquency SHift filters are proposed. One is LMS Adaptive FRESH Filter. The other is Blind Adaptive FRESH Filter. By exploiting the spectral correlation of cyclostationary signals, these adaptive filters can separate the signals which overlap in both frequency and time domain. Theoretical development and simulations of these filters are given in this paper. The results show that for signals which spectrally overlap, the adaptive FRESH filters can perform very well while ordinary adaptive filters fail.

1. INTRODUCTION

In recent years, various filtering techniques to extract signals from interference have been proposed. However these conventional filters cannot separate signals which overlap spectrally. In practical communication systems, signals often overlap in the frequency domain. In order to extract the desired signals from these interference, Gardner and his colleagues have proposed the cyclic Wiener filter[1], which can separate spectrally overlapped signals by using the cyclostationarity of signals.

Consider a cyclic Wiener filter whose input $x(t)$ and output $\hat{y}(t)$ are related by:

$$\hat{y}(t) = \int_{-\infty}^{\infty} h(t,u)x(u)du \qquad (1)$$

where $h(t,u)$ is the impulse response of the filter, which can be expanded as:

$$h(t,u) = \sum_{m=1}^{M} h_m(t-u)exp(j2\pi\alpha_m u) \qquad (2)$$

with α_m being the cyclic frequency of signals and $h_m(t-u)$ being a time-invariant filter. Using Equation(2), we can write the output of the filter as:

$$\hat{y}(t) = \sum_{m=1}^{M} h_m(t) \star [x(t)exp(j2\pi\alpha_m(t))]$$

where $\star$ represents convolution.

It was proven [1] that the Fourier transform $H_m(f)$ of the optimum transfer function $h_m(t)$ must satisfy the following equation:

$$\boldsymbol{S}^{\alpha}_{xx}(f)\boldsymbol{H}(f) = \boldsymbol{S}^{\alpha}_{yx}(f)$$

where: $\boldsymbol{H}(f) = (H_1(f), H_2(f).....H_M(f))^T$, and $\boldsymbol{S}^{\alpha}_{xx}(f)$ is the spectral autocorrelation density matrix of the input such that

$$\boldsymbol{S}^{\alpha}_{xx}(f) = [S^{\alpha_i-\alpha_k}_{xx}(f-(\alpha_i+\alpha_k)/2)]_{M\times M}$$
$$i = 1,2.....M \quad k = 1,2.....M$$

$S^{\alpha_k}_{xx}(f)$ is the Fourier transformation of $R^{\alpha_k}_{xx}(m)$

$$R^{\alpha_k}_{xx}(m) = \langle x(n)x^*(n-m)e^{-j2\pi\alpha_k n}\rangle$$

where $\langle.\rangle$ denotes time average and $*$ denotes conjugate.

Similarly, $\boldsymbol{S}^{\alpha}_{yx}(f)$ is the spectral cross-correlation density function vector between desired signals and input.

$$\boldsymbol{S}^{\alpha}_{yx}(f) = [S^{\alpha_k}_{yx}(f-\alpha_k/2)]_{M\times 1}$$
$$k = 1,2,.....M$$

However, in order to design the optimum filter $\boldsymbol{H}(f)$, $\boldsymbol{S}^{\alpha}_{x}(f)$ and $\boldsymbol{S}^{\alpha}_{yx}(f)$ must be known, which requires the knowledge of $x(t)$ and $y(t)$, $t \in (-\infty,\infty)$. In this paper, we propose two kinds of adaptive FREquency SHift (FRESH) filters. One is LMS Adaptive FRESH Filter (LMSA-FRESH Filter). The other is Blind Adaptive FRESH filter(BA-FRESH Filter). The input of filter is modeled as

$$x(t) = y(t) + i(t) + v(t)$$

where $y(t)$ is the desired signal, $i(t)$ is the interference and $v(t)$ is the stationary noise. We assume that $y(t)$ and $i(t)$ overlap spectrally, but they have different cyclic frequencies.

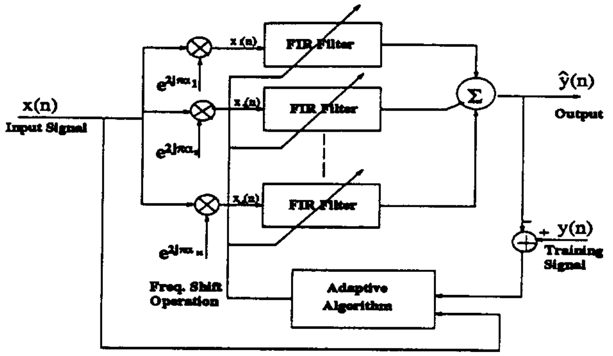

Fig. 2.1 The LMS Adaptive FRESH Filter

2. THE LMS ADAPTIVE FRESH FILTER

The basic idea of the LMS Adaptive FRESH filter is based on following property of cyclic signals.
Property I: The two spectral components of a cyclic signal at $f+\alpha/2$ and at $f-\alpha/2$ are correlated. The magnitude of correlation is measured by spectral correlation coefficients.
When the two spectral components of a signal are correlated, we say that there exist spectrum redundancy between the two components. For a cyclostationary process, the correlation coefficient is defined as:

$$\rho = \frac{S^{\alpha}_{xx}(f)}{(S_x(f+\alpha/2)S_x(f-\alpha/2))^{1/2}}$$

For any particular frequency f at which $\rho = 1$, we have complete spectral redundancy of the two frequency components at $f+\alpha/2$ and at $f-\alpha/2$. When the two spectral components are linearly dependent, we can use one of them to cancel or recover the other. In fact, as a result of this spectral redundancy, certain uncorrupted spectral components in a signal can be used to cancel or recover other corrupted spectral components in that signal. This is the primary mechanism that enables us to separate signals which overlap with each other in spectrum.
If a training signal is known, using the Least Mean Square criterion and the cyclostationarity of signals, we propose the structure of the LMS Adaptive FREquency SHift filter as shown in Fig 2.1. Furthermore, if the time invariant filter of FRESH filter is an FIR filter, the output of the ith filter is given by:

$$\hat{y}_i(n) = \sum_{k=0}^{N-1} h_i(k)x_i(n-k) = \boldsymbol{h}_i^{\dagger}\boldsymbol{x}_i(n) \qquad (3)$$

where

$$\boldsymbol{h}_i = \begin{pmatrix} h_i(0) \\ h_i(1) \\ \vdots \\ h_i(N-1) \end{pmatrix} \qquad \boldsymbol{x}_i(n) = \begin{pmatrix} x_i(n) \\ x_i(n-1) \\ \vdots \\ x_i(n-N+1) \end{pmatrix}$$

and $x_i(n-k) = x(n-k)e^{j2\pi\alpha_i(n-k)}$ $k = 0,1,2,...,(N-1)$ with $x(n)$ being the input of the FRESH filter.
The total output is

$$\hat{y}(n) = \sum_{i=1}^{M} \hat{y}_i(n) = \sum_{i=1}^{M} \boldsymbol{h}_i^{\dagger}\boldsymbol{x}_i(n) = \mathring{\boldsymbol{h}}^{\dagger}\mathring{\boldsymbol{x}}(n) \qquad (4)$$

where

$$\mathring{\boldsymbol{x}}(n) = [\boldsymbol{x}_1^T(n)\boldsymbol{x}_2^T(n)\cdots\boldsymbol{x}_M^T(n)]^T \qquad (5)$$

$$\mathring{\boldsymbol{h}}^{\dagger}(n) = [\boldsymbol{h}_1^{\dagger}(n)\boldsymbol{h}_2^{\dagger}(n)\cdots\boldsymbol{h}_M^{\dagger}(n)] \qquad (6)$$

By minimizing $(y(n)-\hat{y}(n))^2$, we can update $\mathring{\boldsymbol{h}}(n)$ with the LMS algorithm, i.e.

$$\mathring{\boldsymbol{h}}(n+1) = \mathring{\boldsymbol{h}}(n) + 2\alpha e(n)\mathring{\boldsymbol{x}}^*(n)$$

$$= \mathring{\boldsymbol{h}}(n) + \mu e(n)\mathring{\boldsymbol{x}}^*(n)$$

where μ is step size and $e(n) = y(n) - \hat{y}(n)$.
In order to guarantee the convergence of the algorithm, μ should satisfy following condition:

$$0 < \mu < 2/\lambda_{max} \qquad 1 < i < N$$

λ_{max} is the maximum eigenvalue of matrix $\langle\mathring{\boldsymbol{x}}(n)\mathring{\boldsymbol{x}}^{\dagger}(n)\rangle$.

$$\langle\mathring{\boldsymbol{x}}(n)\mathring{\boldsymbol{x}}^{\dagger}(n)\rangle =$$

$$\begin{bmatrix} R^{\alpha_1-\alpha_1}_{xx}(0) & \cdots & R^{\alpha_1-\alpha_M}_{xx}(-N+1) \\ R^{\alpha_2-\alpha_1}_{xx}(1) & \cdots & R^{\alpha_2-\alpha_M}_{xx}(-N+2) \\ \vdots & \vdots & \vdots \\ R^{\alpha_M-\alpha_1}_{xx}(N-1) & \cdots & R^{\alpha_M-\alpha_M}_{xx}(0) \end{bmatrix}$$

3. THE BLIND ADAPTIVE FREQUENCY SHIFT FILTER

When there is no training signal available, we propose an adaptive filter structure which is called the Blind Adaptive FRESH filter as shown in Fig. 3.1. The basic idea of BA-FRESH filter uses the following property of cyclic signals.
Property II: For a cyclic signal $x(n)$, the frequency shifted versions of $x(n)$ are correlated at various cyclic frequencies. The magnitude of correlation is measured

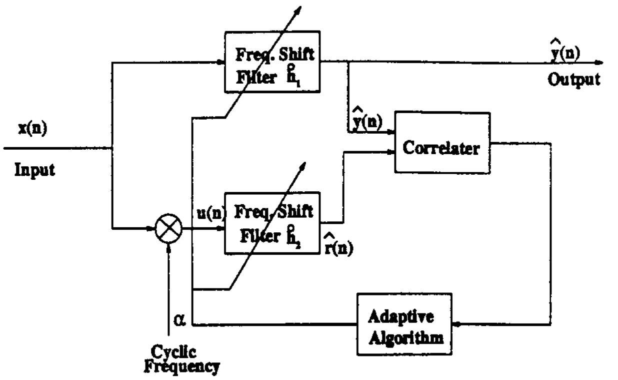

Fig. 3.1 The Blind Adaptive Frequency Shift Filter structure

by cyclic correlation function.

The signal components in the input data $x(n)$ and its frequency-shifted version are $y(n)$ and $y(n)e^{j2\pi\alpha n}$ respectively. At the cyclic frequency α, the desired signal and its frequency shifted versions are correlated but other signal components are not. By maximizing this correlation, we can extract the desired signal from the corrupted input signal.

In Fig.3.1, the FRESH filter is the combination of the frequency shift operation and time invariant filter bank as shown in Fig.2.1. If the filter bank of the FRESH filter in the upper branch is $\mathring{\boldsymbol{h}}_1$ and that of the lower branch is $\mathring{\boldsymbol{h}}_2$, from Eqs.(4) to (6), we have $\hat{y}(n) = \mathring{\boldsymbol{h}}_1^{\dagger} \mathring{\boldsymbol{x}}(n)$ and $\hat{r}(n) = \mathring{\boldsymbol{h}}_2^{\dagger} \mathring{\boldsymbol{u}}(n)$. Since $u(n) = x(n)e^{j2\pi\alpha n}$ with α being the cycle frequency of desired signal, $\hat{y}(n)$ and $\hat{r}(n)$ have strong correlation. For the interference signals, there is no correlation. Hence, we choose the criterion as follow:

$$\max_{\mathring{\boldsymbol{h}}_1, \mathring{\boldsymbol{h}}_2} J(\mathring{\boldsymbol{h}}_1, \mathring{\boldsymbol{h}}_2) = \max_{\mathring{\boldsymbol{h}}_1, \mathring{\boldsymbol{h}}_2} \frac{|\hat{R}_{\hat{y}\hat{r}}|^2}{\hat{R}_{\hat{y}\hat{y}}\hat{R}_{\hat{r}\hat{r}}}, \tag{7}$$

where

$$|\hat{R}_{\hat{y}\hat{r}}|^2 = |\langle \hat{y}(n)\hat{r}(n)\rangle|^2 = |\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xu} \mathring{\boldsymbol{h}}_2|^2$$

$$|\hat{R}_{\hat{y}\hat{y}}|^2 = |\langle \hat{y}(n)\hat{y}(n)\rangle|^2 = |\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xx} \mathring{\boldsymbol{h}}_1|^2$$

$$|\hat{R}_{\hat{r}\hat{r}}|^2 = |\langle \hat{r}(n)\hat{r}(n)\rangle|^2 = |\mathring{\boldsymbol{h}}_2^{\dagger} \hat{\boldsymbol{R}}_{uu} \mathring{\boldsymbol{h}}_2|^2$$

and $\hat{\boldsymbol{R}}_{xu} = \langle \mathring{\boldsymbol{x}}(n) \mathring{\boldsymbol{u}}^{\dagger}(n)\rangle$

Now Eq.(8) is equivalent to maximize the cost function J with respect to the two vectors $\mathring{\boldsymbol{h}}_1$ and $\mathring{\boldsymbol{h}}_2$ i.e.,

$$J(\mathring{\boldsymbol{h}}_1, \mathring{\boldsymbol{h}}_2) = \frac{|\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xu} \mathring{\boldsymbol{h}}_2|^2}{\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xx} \mathring{\boldsymbol{h}}_1 \mathring{\boldsymbol{h}}_2^{\dagger} \hat{\boldsymbol{R}}_{uu} \mathring{\boldsymbol{h}}_2}, \tag{8}$$

Since the desired signal components in $x(n)$ are correlated with that in $u(n)$, we can imagine that $\hat{y}(n)$ and $\hat{r}(n)$ provide the estimates of $y(n)$ and $y(n)e^{j2\pi\alpha n}$ respectively. Therefore, selecting $\mathring{\boldsymbol{h}}_1$ and $\mathring{\boldsymbol{h}}_2$ by maximizing the correlation of $y(n)$ and $y(n)e^{j2\pi\alpha n}$, with normalizing constraint on $\mathring{\boldsymbol{h}}_1$ and $\mathring{\boldsymbol{h}}_2$, it will result in a filter for extracting the signal of interest.

Consider $J(\mathring{\boldsymbol{h}}_1, \mathring{\boldsymbol{h}}_2)$,

$$J(\mathring{\boldsymbol{h}}_1, \mathring{\boldsymbol{h}}_2) = \frac{|\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xu} \mathring{\boldsymbol{h}}_2|^2}{\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xx} \mathring{\boldsymbol{h}}_1 \mathring{\boldsymbol{h}}_2^{\dagger} \hat{\boldsymbol{R}}_{uu} \mathring{\boldsymbol{h}}_2}$$

$$= \frac{|\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xx}^{1/2} \hat{\boldsymbol{R}}_{xx}^{-1/2} \hat{\boldsymbol{R}}_{xu} \mathring{\boldsymbol{h}}_2|^2}{\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xx} \mathring{\boldsymbol{h}}_1 \mathring{\boldsymbol{h}}_2^{\dagger} \hat{\boldsymbol{R}}_{uu} \mathring{\boldsymbol{h}}_2}. \tag{9}$$

Using Cauchey-Scharz inequality, the optimum filters are given by

$$\mathring{\boldsymbol{h}}_1 \propto \hat{\boldsymbol{R}}_{xx}^{-1} \hat{\boldsymbol{R}}_{xu} \mathring{\boldsymbol{h}}_2 \tag{10}$$

$$\mathring{\boldsymbol{h}}_2 \propto \hat{\boldsymbol{R}}_{uu}^{-1} \hat{\boldsymbol{R}}_{xu} \mathring{\boldsymbol{h}}_1 \tag{11}$$

By substituting Eq.(11) into (8), the cost function becomes a generalized Rayleigh quotient in $\mathring{\boldsymbol{h}}_1$, i.e.,

$$J(\mathring{\boldsymbol{h}}_1, \mathring{\boldsymbol{h}}_2) = \frac{\mathring{\boldsymbol{h}}_1^{\dagger} [\hat{\boldsymbol{R}}_{xu} \hat{\boldsymbol{R}}_{uu}^{-1} \hat{\boldsymbol{R}}_{xu}^{\dagger}] \mathring{\boldsymbol{h}}_1}{\mathring{\boldsymbol{h}}_1^{\dagger} \hat{\boldsymbol{R}}_{xx} \mathring{\boldsymbol{h}}_1},$$

Therefore the necessary conditions for maximizing the cost function are given by:

$$\lambda_{max} \mathring{\boldsymbol{h}}_1 = \hat{\boldsymbol{R}}_{xx}^{-1} \hat{\boldsymbol{R}}_{xu}^{\dagger} \hat{\boldsymbol{R}}_{uu}^{-1} \hat{\boldsymbol{R}}_{xu} \mathring{\boldsymbol{h}}_1 .$$

$$\lambda_{max} \mathring{\boldsymbol{h}}_2 = \hat{\boldsymbol{R}}_{uu}^{-1} \hat{\boldsymbol{R}}_{xu}^{\dagger} \hat{\boldsymbol{R}}_{xx}^{-1} \hat{\boldsymbol{R}}_{xu} \mathring{\boldsymbol{h}}_2 .$$

Using the power method[2], $\mathring{\boldsymbol{h}}_1$ and $\mathring{\boldsymbol{h}}_2$ can be solved iteratively by following fomula

$$\mathring{\boldsymbol{h}}_2(n) = g_2(n) \hat{\boldsymbol{R}}_{uu}^{-1}(n) \hat{\boldsymbol{R}}_{xu}^{\dagger}(n) \mathring{\boldsymbol{h}}_1(n-1) \tag{12}$$

$$\mathring{\boldsymbol{h}}_1(n) = g_1(n) \hat{\boldsymbol{R}}_{xx}^{-1}(n) \hat{\boldsymbol{R}}_{xu}(n) \mathring{\boldsymbol{h}}_2(n), \tag{13}$$

where $g_{(.)}(n)$ are the power-normalizing gain constants. The correlation statistics can also be calculated recursively by

$$\hat{\boldsymbol{R}}_{xy}(n) = (1 - 1/n)\hat{\boldsymbol{R}}_{xy}(n-1) + 1/n \boldsymbol{x}(n)\boldsymbol{y}^{\dagger}(n) \tag{14}$$

for arbitrary signals $\boldsymbol{x}(n)$ and $\boldsymbol{y}(n)$.

3.1. Simulation Results

In order to test the two different adaptive algorithms, we carry out some computer simulations. The input signal consist of two BPSK signals with additive white Gaussian noise. One of the two signals is considered to be the desired signal and the other is the interference. Fig.1(a) ($SINR = 0db$) shows that the original spectra overlap each other. Fig.1(b) shows the recovered signal spectrum with LMSA-FRESH Filter. It could be seen that the overlaped interference has been removed after LMSA-FRESH filtering. Figl(c) and Figl(d) show the corresponding eye-diagram for the input signal and the adaptively filtered signal. It can be seen that the eye is open after adaptive filtering. Fig.2 shows the same scenario with BA-FRESH filter. Again, the interfernce has been removed. Similar simulations have been carried out using signals such as QPSK signals and AM signals. The same improvement could be observed.

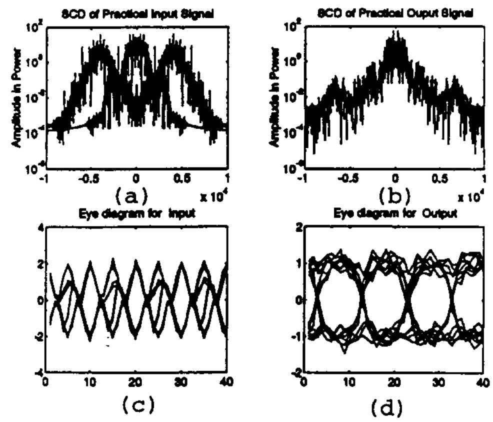

Figure 1: Performance of LMSA-FRESH Filter with BPSK Signals

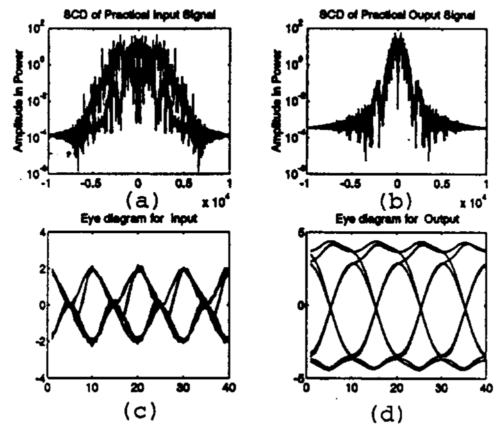

Figure 2: Performance of BA-FRESH Filter with BPSK Signals

4. CONCLUSION

In this paper, using the cyclostationarity of signals, two new kinds of adaptive frequency shift filters are proposed. The cyclic adaptive filtering algorithms and their characteristics are developed and simulated. Among the two new components Frequency Shift Filters, LMSA-FRESH Filter is simpler and it provided acceptable performance for carrier recovery. However, we must provide the training signals. For BA-FRESH Filter, its algorithm is more complex, but the only prior knowledge required is the cyclic frequency of desired signals. Both of these algorithm can adaptively extract desired signals which overlap spectrally with interference. The choice of the adaptive FRESH filtering method depends on the conditions under which it is applied.

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