ON THE KERNEL MUSIC ALGORITHM

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ABSTRACT

The kernel MUSIC(Multiple Signal Classification) algorithm was proposed as an improvement over the existing MUSIC algorithm. The proposed algorithm is based on the orthogonality between the image and kernel space of an Hermitian mapping constructed from a signal. The major part of computation is Gaussian elimination of a matrix, required processing time as a function of data number grows slower than that of the existing MUSIC based on eigenstate analysis. And thus this algorithm is advantageous for processing large size data. This algorithm was applied to image reconstruction process of a laser radar and its superior spatial resolution higher than that limited by the wavelength scanning range was demonstrated.

1. INTRODUCTION

When a signal that is a sum of multiple vectors is given, it is important to find the constituent vectors of the signal. Direction–of–arrival estimation in a radar, positions of reflection points detection in a network analyzer are such examples. The MUSIC algorithm was first proposed for estimating arrival direction of incoming waves which were incoherent with each other [1]. And to apply this algorithm to a coherent signal, smoothing techniques of the signal have been studied[2][3]. As the MUSIC algorithm is based on the eigenstate analysis, massive computation is required to calculate eigenvalues and eigenvectors of a large size matrix. And the intuitive meaning of smoothing is not clear although its effectiveness has been studied in detail.

In this paper we propose the kernel MUSIC algorithm that is based on the orthogonality between the image and kernel space of an Hermitian mapping constructed from a signal. Since the major part of computation is Gaussian elimination of a matrix, processing time is reduced compared to the existing MUSIC. And the construction process of the mapping provides the clear meaning of smoothing. We applied this method to our laser radar being developed for the purpose of seeing through a micro–structured object and verified its superior spatial resolution.

2. THE KERNEL MUSIC ALGORITHM

First, a general scheme of signal reconstruction will be explained. When a signal $\boldsymbol{s} = \sum_{i=1}^{I} \boldsymbol{a}_i$ is given, where $\boldsymbol{s}$ and $\boldsymbol{a}_i$'s are in N dimensional complex vector space $\mathbf{C}^N$, the constituent vectors $\boldsymbol{a}_i$'s are estimated by the following steps.

step1 Construct a mapping $\mathbf{H}$ from $\mathbf{C}^N$ to a vector space which is spanned by $\boldsymbol{a}_i$'s.

step2 Examine the characteristics of $\mathbf{H}$ such as dimensions of the kernel space of $\mathbf{H}$.

The method of processing by the kernel MUSIC algorithm is as follows. If an Hermitian mapping $\mathbf{H}$ such that its image space is spanned by $\boldsymbol{a}_i$'s is constructed, then whether or not a test vector $\boldsymbol{u}$ is the constituent of the signal may be decided by estimating the orthogonality between $\boldsymbol{u}$ and the kernel space of $\mathbf{H}$. This decision is based on the orthogonality between the image and kernel space of an Hermitian mapping.

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(Proposition) The image and kernel space of an Hermitian mapping $\mathbf{H}$(Figure 1) are orthogonal.
(Proof) Suppose $\boldsymbol{u} \in$ image($\mathbf{H}$) and $\boldsymbol{v} \in$ kernel($\mathbf{H}$), then there exixts a vector $\boldsymbol{w}$ such that $\mathbf{H}\boldsymbol{w} = \boldsymbol{u}$. So, we denote inner product of $\boldsymbol{u}$ and $\boldsymbol{v}$ by $(\boldsymbol{u},\boldsymbol{v})$, then $(\boldsymbol{u},\boldsymbol{v}) = (\mathbf{H}\boldsymbol{w},\boldsymbol{v}) = (\boldsymbol{w},\mathbf{H}\boldsymbol{v}) = (\boldsymbol{w},\mathbf{0}) = \mathbf{0}$, where conditions that $\mathbf{H}$ is Hermitian and $\boldsymbol{v} \in$ kernel($\mathbf{H}$) were used.

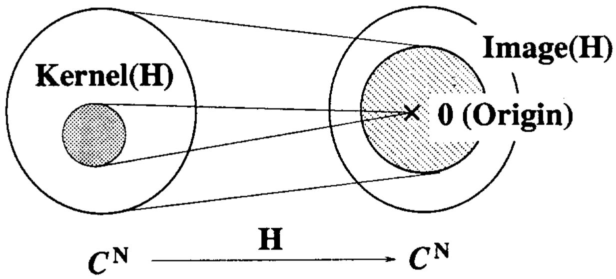

Figure1: Image and kernel space of a mapping $\mathbf{H}$.

The orthonormal basis of kernel($\mathbf{H}$) are easily obtained by utilizing Gaussian elimination of matrix $\mathbf{H}$ and Gram–Schmidt orthogonalization. These procedures are simpler than eigenstate analysis needed in the existing MUSIC. From our computer simulation, it was found that the growth exponents of computation time as a function of data number N were 1.6 and 2.5 in the kernel MUSIC and the existing MUSIC respectively. And thus the kernel MUSIC algorithm is advantageous for processing large size data. Let $\boldsymbol{v}_1, \cdots, \boldsymbol{v}_K$ be the orthonormal basis of kernel($\mathbf{H}$). By evaluating the cost function $f(\boldsymbol{u})$ defined by

$$f(\boldsymbol{u}) = \textstyle\sum_{k=1}^{K} \mid (\boldsymbol{u},\boldsymbol{v}_k) \mid^2,$$

a test vector $\boldsymbol{u}$ that minimizes this cost function $f(\boldsymbol{u})$ may be considerd as the constituent of the signal.

3. AN INTERPRETATION OF SMOOTHING

Finding the constituent vectors $\boldsymbol{a}_i$'s of the signal $\boldsymbol{s}$ is now equivalent to constructing an Hermitian mapping that has the image space spanned by them. Let the non–smoothed mapping $\mathbf{H}$ be $\boldsymbol{s}\boldsymbol{s}^\dagger$. Then the image space of $\mathbf{H}$ has only one dimension, which means that $\mathbf{H}$ projects all vectors to $\boldsymbol{s}$. This $\mathbf{H}$ is decomposed into two terms as follows.

$$\mathbf{H} = \textstyle\sum_{i=1}^{I} \boldsymbol{a}_i\boldsymbol{a}_i^\dagger + \sum_{i\neq j}^{I} \boldsymbol{a}_i\boldsymbol{a}_j^\dagger.$$

If the second term so called interference term were eliminated, then an ideal mapping such that its image space is spanned by $\boldsymbol{a}_i$'s would be obtained. In Ref.[2], to apply the MUSIC algorithm to the coherent signal, the forward smoothed mapping $\mathbf{H}_f$ was studied.

$$\mathbf{H}_f = \textstyle\sum_{j=1}^{J} \boldsymbol{e}_j\boldsymbol{e}_j^\dagger,$$
where $\boldsymbol{e}_j = (\mathrm{s}_j\ , \mathrm{s}_{j+1}\ , \cdots , \mathrm{s}_{j+L-1})^t$,
$J + L - 1 \leq N$ and
s_j is the jth component of $\boldsymbol{s}$.

As the image space of $\mathbf{H}_f$ is spanned by $\boldsymbol{e}_j$'s, its dimension enlarges under the condition that $\boldsymbol{e}_j$'s are linearly independent. Thus, smoothing is naturally interpreted as an extension operation of originally one dimensional image space into multidimensional one that is approximately spanned by the constituent vectors $\boldsymbol{a}_i$'s. This extension is equivalent to the reduction of interference term included in non–smoothed $\mathbf{H}$. Furthermore, from the method of construction of $\mathbf{H}_{fb}$ and $\mathbf{H}_f$, where $\mathbf{H}_{fb}$ is the forward/backward smoothed mapping[3], it is clear that

$$\text{image}(\mathbf{H}_{fb}) \supseteq \text{image}(\mathbf{H}_f).$$

So forward/backward smoothing is expected to give better performance than forward only smoothing.

4. APPLICATION OF THE KERNEL MUSIC ALGORITHM TO A LASER RADAR

A laser radar that can see through a micro–structured object nondestructively has been developed in ATR[4]. Instead of sending a short optical pulse into an object and analyzing echos, the radar illuminates a CW light onto the object and measures the amplitude and phase of the reflected light. This measurement is repeated at a step of wavelength ranging between λ_1 and λ_2. And then the stored data are processed, most simply Fourier transformed to construct the image of the object, and positions of reflection points in the object are displayed. The major advantage of such a scheme is that a high spatial resolution may be realized without complex system needed to generate a short optical pulse. Spatial resolution Δx, which is the most important parameter to evaluate a radar, is given by

$$\Delta x = \frac{\lambda_1 \times \lambda_2}{2|\lambda_2 - \lambda_1|},$$

and it is limited by the wavelength scanning range in principle. If a tunable light source available on the market is used, the obtainable spatial resolution at best is an order of $10\mu m$. With our radar, scanning range is between $1.53\mu m$ and $1.58\mu m$, Δx of $20\mu m$ is expected. To overcome this reoslution limit, we apply the kernel MUSIC algorithm to the image reconstruction processing.

Figure 2 shows the structure of the object used to examine spatial resolution of our radar. A thin film of refractive index 1.64 was deposited on a substrate. There exist two reflection points in the object, at the boundaries between air and film, film and substrate. The back surface of the substrate is roughened to scatter the illuminated light. The thickness of the film measured by our radar is compared to that measured by a mechanical roughness detector.

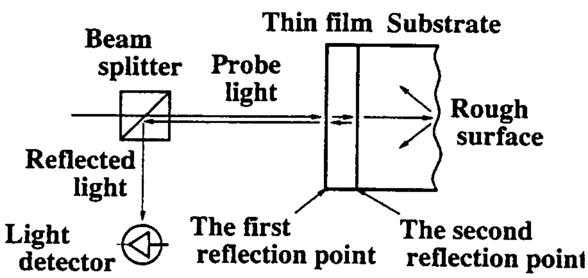

Figure 2: Structure of the object.

Figure 3 shows the comparison of reconstructed images by the kernel MUSIC and Fast Fourier Transform algorithm when the film $10\mu m$ thick was used as the object. The kernel MUSIC algorithm resolved the spacing narrower than the principle resolution limit given above. Moreover, estimated thickness has good agreement with that obtained by the roughness detector.

kernel MUSIC algorithm	roughness detector
$16.5\mu m$	$18.3\mu m$
$9.8\mu m$	$10.9\mu m$
$8.5\mu m$	$6.0\mu m$
unresolvable	$4.8\mu m$

Table1: Estimated thickness of various films.

Table 1 shows estimated thickness of various films in two ways. From these results, we may say that our radar with the kernel MUSIC algorithm has spatial resolution of about $10\mu m$.

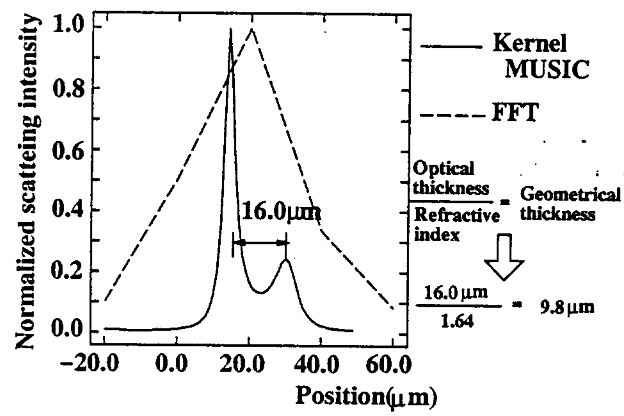

Figure3: Reconstructed images of the film $10\mu m$ thick.

5. CONCLUSION

The kernel MUSIC algorithm was introduced as a signal reconstruction method. This algorithm was based on the orthogonality between the image and kernel space of an Hermitian mapping constructed from a signal. The simplicity of this algorithm reduced processing time of large size data compared to the existing MUSIC algorithm. In addition, the meaning of smoothing operation became clearer. This algorithm was applied to a laser radar to see through a micro–structured object and it was demonstrated that the radar achieved the spatial resolution of about $10\mu m$, which is two times higher than that limited in principle by the wavelength scanning range.

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