HARDWARE REALIZABLE LATTICE-REDUCTION-AIDED DETECTORS FOR LARGE-SCALE MIMO SYSTEMS

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ABSTRACT

Because of their lower complexity and better error performance over K-best detectors, lattice-reduction (LR)-aided K-best detectors have recently proposed for large-scale multi-input multi-output (MIMO) detection. Among existing LR-aided K-best detectors, complex LR-aided K-best detector is more attractive compared to its real counterpart due to its potential lower latency and resources. However, one main difficulty in hardware implementation of complex LR-aided K-best is to efficiently find top K children of each layer in complex domain. In this paper, we propose and implement an LR-aided K-best algorithm that efficiently finds top K children in each layer when K is relatively small. Our implementation results on Xilinx VC707 FPGA board show that, with the aid of LR, the proposed LR-aided K-best implementation can support 3 Gbps transmissions for 16x16 MIMO systems with 1024-QAM with about 2.7 dB loss to the maximum likelihood detector at bit-error rate 10^{-4}.

Index Terms— Lattice reduction, large-scale MIMO, K-best algorithm, field-programmable gate array, very-large-scale integration

1. INTRODUCTION

By transmitting and receiving signals via tens or hundreds of antennas, large-scale multi-input multi-output (MIMO) systems have shown great potential for next generation of wireless communications to obtain high spectral efficiencies. However, because of the non-deterministic polynomial hard of the optimal MIMO detection, a critical challenge of large-scale MIMO systems is to design high performance, high throughput, and low latency detectors. Several detectors such as local neighborhood search and iterative soft interference cancellation detector have been studied for large-scale MIMO systems [1]. However, they still require considerable complexity especially for higher order modulations. In contrast, linear detectors (LDs) and successive interference cancellation (SIC) detectors require polynomial complexity but suffer from significantly degraded error performance.

To improve the error performance of LDs and SIC detectors, lattice reduction (LR)-aided LDs and SIC detectors are proposed [2–4], but their performance gaps to the optimal MIMO detection are still significant when the number of antennas is large [5, 6]. To further bridge the gap, LR-aided K-best detectors are proposed in [7–10]. Among the existing LR-aided K-best detectors, the complex LR-aided K-best detector may be more favorable than its real counterpart for hardware implementation in terms of latency and resource usage. However, one main difficulty of the complex LR-aided K-best detector is to find the best K children of each layer in complex domain. Although several existing methods [10, 11] are proposed to find the top K children in complex domain, the methods are still complicated and inflexible for hardware implementation.

In this paper, we propose a novel complex LR-aided K-best detector for large-scale MIMO systems. We develop a hardware-efficient method to find top K children of each layer in complex domain when K is relatively small. In addition, to verify the efficiency of our design, we implement the complex LR-aided K-best detector on Xilinx FPGA for large-scale MIMO detection. Our implementation results show that the FPGA realization can support up to 3 Gbps MIMO transmissions for 16x16 MIMO systems with 1024-QAM with about 2.7 dB gap to the MLD at bit-error rate (BER) 10^{-4}.

The rest of the paper is organized as follows. Sec. 2 introduces system model and LR-aided detection. Sec. 3 describes complex LR-aided K-best detector and presents our proposed LR-aided K-best search. Sec. 4 discusses the hardware implementation of our proposed LR-aided K-best search algorithm for 16x16 MIMO systems. Sec. 5 concludes the paper.

Notation: Superscript ^{T} denotes the transpose, $^\mathcal{H}$ denotes the Hermite. The real and imaginary parts of a complex number are denoted as $\Re[\cdot]$ and $\Im[\cdot]$. Upper- and lower-case boldface letters indicate matrices and column vectors, respectively. $A_{i,k}$ indicates the (i, k)th entry of matrix $\mathbf{A}$. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix, $\mathbf{0}_{N\times L}$ is the $N \times L$ matrix with all entries zero, and $\mathbf{1}_{N\times L}$ is the $N \times L$ matrix with all entries one. $\mathbb{Z}$ is the integer set, $\mathbf{Z}[j]$ is the Gaussian integer set having the form $\mathbb{Z} + \mathbb{Z}j$, and $j = \sqrt{-1}$. $E\{\cdot\}$ denotes the statistical expectation. $\|\cdot\|$ denotes the 2-norm.

2. SYSTEM MODEL AND LR-AIDED DETECTOR

Consider a transmission model of an MIMO system with N_t transmit antennas and N_r receive antennas as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}, \tag{1}$$

where $\mathbf{s} = [s_1, s_2, \cdots, s_{N_t}]^T, s_i \in \mathcal{S}$ is the complex information symbol vector with $\mathcal{S}$ being a constellation of QAM set, $\mathbf{H}$ is an $N_r \times N_t, (N_r \geq N_t)$ complex channel matrix, whose entries are modeled as independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and unit variance, $\mathbf{y} = [y_1, y_2, \cdots, y_{N_r}]^T$ is the received signal vector, and $\mathbf{w} = [w_1, w_2, \cdots, w_{N_r}]^T$ is the complex additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $N_0\mathbf{I}_{N_r}$.

Given the model in Eq. (1), we consider the following revised problem for LR-aided detectors

$$\hat{\mathbf{s}} = \arg\min_{\tilde{\mathbf{s}} \in \mathcal{U}^{N_t}} \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\|^2 + \frac{N_0}{\sigma_s^2}\|\tilde{\mathbf{s}}\|^2 \tag{2}$$

$$= \arg\min_{\tilde{\mathbf{s}} \in \mathcal{U}^{N_t}} \|\bar{\mathbf{y}} - \bar{\mathbf{H}}\tilde{\mathbf{s}}\|^2 \tag{3}$$

where the finite alphabet set of s_i, $\mathcal{S}$ is relaxed to an unconstrained constellation set $\mathcal{U}$ in the form of $2\mathbb{Z}+1+(2\mathbb{Z}+1)j$ [12], the second term of the right hand side of (2) is the minimum-mean-square-error (MMSE) regularization term that compensates the loss of boundary information and enables diversity-multiplexing tradeoff (DMT) optimality [13] with $E\{\mathbf{s}\mathbf{s}^T\} = \sigma_s^2\mathbf{I}_{N_t}$, and $\bar{\mathbf{H}}$ and $\bar{\mathbf{y}}$ are the MMSE-extended matrices given as

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sqrt{\frac{N_0}{\sigma_s^2}}\mathbf{I}_{N_t} \end{bmatrix}, \qquad \bar{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{N_t \times 1} \end{bmatrix}, \tag{4}$$

with $E\{\mathbf{s}\mathbf{s}^{\mathcal{H}}\} = \sigma_s\mathbf{I}_{N_t}$.

To solve the unconstrained problem in (3) with lower complexity, the LR-aided detection performs LR on the matrix $\bar{\mathbf{H}}$ to obtain a more "orthogonal" matrix $\tilde{\mathbf{H}} = \bar{\mathbf{H}}\mathbf{T}$, where $\mathbf{T}$ is a unimodular matrix, such that all entries of $\mathbf{T}$ are Gaussian integers and the determinant of $\mathbf{T}$ is ± 1 or $\pm j$. Given $\tilde{\mathbf{H}}$ and $\mathbf{T}$, the problem in (3) becomes

$$\hat{\mathbf{s}} = 2\mathbf{T}\arg\min_{\tilde{\mathbf{z}} \in \mathbb{Z}[j]^{N_t}} \|\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\mathbf{z}}\|^2 + (1+j)\mathbf{1}_{N_t \times 1}, \tag{5}$$

where $\tilde{\mathbf{y}}$ is the received signal vector after shifting and scaling as $(\bar{\mathbf{y}} - \bar{\mathbf{H}}(1+j)\mathbf{1}_{N_t\times 1})/2$ and $\tilde{\mathbf{s}} = 2\mathbf{T}\tilde{\mathbf{z}} + (1+j)\mathbf{1}_{N_t \times 1}$. Since $\tilde{\mathbf{H}}$ is more "orthogonal," one could expect low-complexity detectors (e.g., SIC and linear detectors) can achieve better error performance compared to those without LR.

3. LR-AIDED K-BEST DETECTORS

3.1. Complex LR-aided K-best Detector

To further enhance the performance of the LR-aided SIC detector, the complex LR-aided K-best detector [7, 8] is proposed to find a "better" sub-optimal solution to (5).

A general description of the complex LR-aided K-best algorithm is given in Table 1. First, the LR-aided K-best detector performs QR decomposition on $\tilde{\mathbf{H}} = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is an $(N_r+N_t)\times N_t$ orthonormal matrix and $\mathbf{R}$ is an $N_t \times N_t$ upper triangular matrix. Then, the problem in (5) is reformulated as

$$\hat{\mathbf{s}} = 2\mathbf{T}\arg\min_{\tilde{\mathbf{z}} \in \mathbb{Z}[j]^{N_t}} \|\check{\mathbf{y}} - \mathbf{R}\tilde{\mathbf{z}}\|^2 + (1+j)\mathbf{1}_{N_t \times 1}. \tag{6}$$

where $\check{\mathbf{y}} = \mathbf{Q}^{\mathcal{H}}\tilde{\mathbf{y}}$.

Next, given (6), the LR-aided K-best detector performs breadth-first search from the N_tth layer to the 1st layer. For each layer (e.g., the nth layer), only top K partial candidates $\{\mathbf{z}_k^{(n)}\}_{k=1}^K$ among all the children of the K parents $\{\mathbf{z}_k^{(n+1)}\}_{k=1}^K$ are survived, where a partial candidate of the nth layer, $\mathbf{z}_i^{(n)}$, is defined as $\mathbf{z}_i^{(n)} = [z_{i,n}^{(n)}, \cdots, z_{i,N_t}^{(n)}]^T$, the cost is calculated as

$$cost_i^{(n)} = \sum_{\ell=n}^{N_t} |\check{y}_\ell - \sum_{k=\ell}^{N_t} R_{\ell,k} z_{i,k}^{(n)}|^2, \tag{7}$$

and a partial candidate $\mathbf{z}_i^{(n)}$ is a child of $\mathbf{z}_k^{(n+1)}$ if and only if $\mathbf{z}_k^{(n)} = [z_{k,n}^{(n)}, (\mathbf{z}_i^{(n+1)})^T]^T, z_{k,n}^{(n)} \in \mathbb{Z}[j]$.

When the search of the 1st layer is completed, the LR-aided K-best detector outputs $\{\mathbf{z}_k^{(1)}\}_{k=1}^K$ as the K estimates of the symbols in LR domain $\{\hat{\mathbf{z}}_k\}_{k=1}^K$. By transforming the symbols to s-domain, we obtain K estimated symbols that could be served as hard output or soft output for uncoded and coded systems, respectively.

From Table 1, the key task of the complex LR-aided K-best algorithm is to efficiently find the K best partial candidates of each nth layer among all the children of the partial candidates of the previous $(n+1)$st layer.

3.2. Existing Approaches to Find Top K Children

To find the top K children of the $(n+1)$th layer, there are mainly two existing methods:

Input: $\mathbf{R}$, $\check{\mathbf{y}}$, candidate size K	
Output: $\{\hat{\mathbf{z}}_k\}_{k=1}^K$	
(1)	$\mathbf{z}_1^{(N_t+1)} = [], cost_1^{(N_t+1)} = 0, len = 1$
(2)	For $n = N_t : -1 : 1$
(3)	$\left[\{\mathbf{z}_k^{(n)}\}_{k=1}^K, \{cost_k^{(n)}\}_{k=1}^K\right] =$ Find_Kbest_Children($\{\mathbf{z}_k^{(n+1)}\}_{k=1}^{len}, \{cost_k^{(n+1)}\}_{k=1}^{len}$)
(4)	$len = K$
(5)	End for

Table 1. A general description of the complex LR-aided K-best algorithm.

- *Pre-expansion [7]:* The pre-expansion method first finds top K children of each parent, and then chooses the top K partial candidates for the nth layer among all the K^2 children of all K parents.
- *On-demand expansion [8–11,14]:* The on-demand expansion maintains a candidate list with size K that stores the best child of each parent. Then, the on-demand expansion method chooses the best child in the candidate list as one of the best K children of the layer and replaces the best child in the candidate list with its next best sibling of the same parent. After K selections, the best K children of the layer are obtained.

However, these methods exhibit some issues for hardware implementation of complex LR-aided K-best detector.

- The pre-expansion method requires a sorter over K^2 candidates, which may pose considerable hardware cost and latency [7, 15].
- Although the on-demand method reduces the sorter from size K^2 to K, the on-demand expansion requires 2 multipliers in finding next best sibling and evaluating its cost [10, 11], which may result in a critical path.
- Unlike the real K-best algorithm, where the top K children can be easily found in a zig-zag fashion or Schnorr-Euchner (SE) method, generating the top K children of each parent for the complex case is more complicated, and thus existing method [10, 11] may not work well for large-scale MIMO systems.

3.3. Proposed Hybrid Expansion Method

To circumvent the implementation issues of the existing methods, in this paper, we propose a hybrid pre-expansion/on-demand expansion method, which consists of two stages:

1. The pre-expansion stage finds the top K children (ordered) of each parent in complex domain. As we will show in Sec. 4.2, when K is small, by checking some conditions, the top K children (ordered) in complex domain can be uniquely determined.
2. The on-demand expansion stage finds the top K of the layer. When a child in the candidate list is chosen as one of the best K children of the layer, its next best sibling is selected from the top K children list of its parent that is obtained in the pre-expansion stage.

Therefore, the hybrid expansion method only requires a sorter of size K and does not need any multipliers in the on-demand expansion stage. The cost of the hybrid method is the storage of K^2 children, which is same as that of the pre-expansion method.

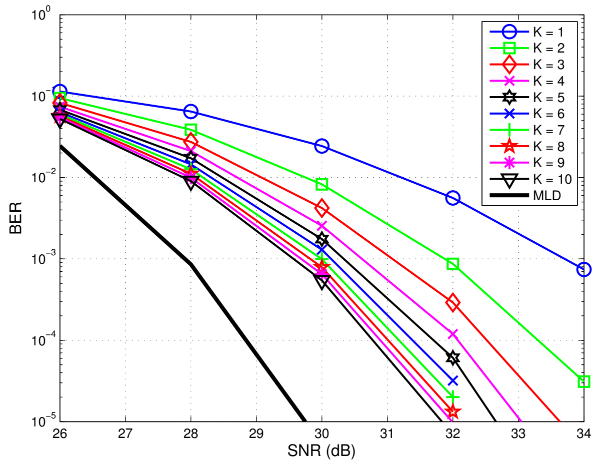

Fig. 1. Performance comparisons of the CLLL-aided K-best detector with MMSE regularization for a 16x16 MIMO system with 1024-QAM and different K's.

4. IMPLEMENTATION OF COMPLEX LR-AIDED K-BEST DETECTOR

In this section, we present the hardware implementation of the proposed complex LR-aided K-best detector for large-scale MIMO systems. We first determine the value of K of the LR-aided K-best. Then, we will present the proposed method to find top K children of a parent in complex domain. Finally, we will describe our VLSI implementation of the proposed complex LR-aided K-best on FPGA for 16x16 MIMO systems.

4.1. Determination of K Value of LR-aided K-best

One key parameter of the LR-aided K-best is the value of K. To decide the value of K with a good performance/complexity tradeoff, we conduct the performance of the complex LR-aided K-best detector with MMSE regularization for a 16x16 MIMO system with 1024-QAM and $K = 1, \cdots, 10$ in Fig. 1. The LR algorithm adopts the CLLL algorithm in [2]. We observe that as K increases, the performance gain decelerates, and the gain of $K = 10$ over $K = 6$ is about 0.5 dB at BER$= 10^{-4}$, while the gain of $K = 6$ over $K = 2$ is about 2 dB. Therefore, we choose $K = 6$ for the implementation.

4.2. Finding Top 6 Children of Each Parent

To simplify the notations, in this subsection, we consider the following model

$$\begin{aligned} c &= |y - rz|^2 \\ &= (\Re[y] - r\Re[z])^2 + (\Im[y] - r\Im[z])^2, \end{aligned} \tag{8}$$

where y is a complex number, $r \neq 0$ is a real number, z is a Gaussian integer that is be determined, and c is the cost. Let us define a sequence of all possible z, i.e., $\{z^{(1)}, z^{(2)}, \ldots\}$ ($z^{(n)} \neq z^{(m)}, \forall n \neq m$) such that their costs $\{c^{(n)} = |y - rz^{(n)}|^2\}_{n=1}^{\infty}$ are in ascend order, i.e., $c^{(1)} < c^{(2)} < \ldots$ for all possible z. We denote $z^{(n)}$ as the nth child.

It is clear that the first child can obtained as

$$z^{(1)} = \lceil y/r \rfloor, \tag{9}$$

$$c^{(1)} = |y - rz^{(1)}|^2. \tag{10}$$

To simplify the equations in deriving the following 2nd to 6th children, let us denote $\Delta_1 = |\Re[y] - r\Re[z^{(1)}]|$, $\Delta_2 = |\Im[y] - r\Im[z^{(1)}]|$, $\delta_1 = \mathrm{sgn}(\Re[y]/r - \Re[z^{(1)}])$, and $\delta_2 = \mathrm{sgn}(\Im[y]/r - \Im[z^{(1)}])$.

Proposition 1. *The 2nd and the 3rd children and their costs can be determined as*

$$z^{(2)} = \begin{cases} z^{(1)} + \delta_1, & \text{if} \quad \Delta_1 > \Delta_2 \\ z^{(1)} + \delta_2 j, & \text{o.w.} \end{cases} \tag{11}$$

$$c^{(2)} = \begin{cases} (|r| - \Delta_1)^2 + \Delta_2^2, & \text{if} \quad \Delta_1 > \Delta_2 \\ \Delta_1^2 + (|r| - \Delta_2)^2, & \text{o.w.} \end{cases} \tag{12}$$

$$z^{(3)} = \begin{cases} z^{(1)} + \delta_2 j, & \text{if} \quad \Delta_1 > \Delta_2 \\ z^{(1)} + \delta_1, & \text{o.w.} \end{cases} \tag{13}$$

$$c^{(3)} = \begin{cases} \Delta_1^2 + (|r| - \Delta_2)^2, & \text{if} \quad \Delta_1 > \Delta_2 \\ (|r| - \Delta_1)^2 + \Delta_2^2, & \text{o.w.} \end{cases} \tag{14}$$

In the following, without loss of generality, we assume $\Delta_1 > \Delta_2$.

Proposition 2. *The 4th child and its cost are given by*

$$z^{(4)} = \begin{cases} z^{(1)} + \delta_1 + \delta_2 j, & \text{if} \quad 2\Delta_1 + 4\Delta_2 > |r| \\ z^{(1)} - \delta_2 j, & \text{o.w.} \end{cases} \tag{15}$$

$$c^{(4)} = \begin{cases} (|r| - \Delta_1)^2 + (|r| - \Delta_2)^2, & \text{if} \quad 2\Delta_1 + 4\Delta_2 > |r| \\ \Delta_1^2 + (|r| + \Delta_2)^2, & \text{o.w.} \end{cases} \tag{16}$$

Proposition 3. *If $2\Delta_1 + 4\Delta_2 > |r|$, the 5th child and its cost are*

$$z^{(5)} = z^{(1)} - \delta_2 j, \tag{17}$$

$$c^{(5)} = \Delta_1^2 + (|r| + \Delta_2)^2, \tag{18}$$

else the 5th child and its cost are

$$z^{(5)} = \begin{cases} z^{(1)} + \delta_1 + \delta_2 j, & \text{if} \quad 4\Delta_1 + 2\Delta_2 > |r| \\ z^{(1)} - \delta_1, & \text{o.w.} \end{cases} \tag{19}$$

$$c^{(5)} = \begin{cases} (|r| - \Delta_1)^2 + (|r| - \Delta_2)^2, & \text{if} \quad 4\Delta_1 + 2\Delta_2 > |r| \\ (|r| + \Delta_1)^2 + \Delta_2^2, & \text{o.w.} \end{cases} \tag{20}$$

Proposition 4. *If $4\Delta_1 + 2\Delta_2 < |r|$, the 6th child and its cost are*

$$z^{(6)} = z^{(1)} + \delta_1 + \delta_2 j, \tag{21}$$

$$c^{(6)} = (|r| - \Delta_1)^2 + (|r| - \Delta_2)^2, \tag{22}$$

else the 6th child and its cost are

$$z^{(6)} = \begin{cases} z^{(1)} + \delta_1 - \delta_2 j, & \text{if} \quad 4\Delta_1 - 2\Delta_2 > |r| \\ z^{(1)} - \delta_1, & \text{o.w.} \end{cases} \tag{23}$$

$$c^{(6)} = \begin{cases} (|r| - \Delta_1)^2 + (|r| + \Delta_2)^2, & \text{if} \quad 4\Delta_1 - 2\Delta_2 > |r| \\ (|r| + \Delta_1)^2 + \Delta_2^2, & \text{o.w.} \end{cases} \tag{24}$$

Summarizing Propositions 1-4, we conclude that the first 6 children can be uniquely determined by the following conditions: $\Delta_1 \lesseqgtr \Delta_2$, $2\Delta_1 + 4\Delta_2 \lesseqgtr |r|$, $4\Delta_1 + 2\Delta_2 \lesseqgtr |r|$, $4\Delta_1 - 2\Delta_2 \lesseqgtr |r|$, and $2\Delta_1 - 4\Delta_2 \lesseqgtr |r|$. Note that, since the multiplications with 2 and 4 can be implemented as shifting operations, all these conditions can be efficiently checked in hardware implementation.

4.3. Overview of the Proposed Implementation

Besides $\check{\mathbf{y}}$ and $\mathbf{R}$, the implementation requires the input of $\{1/R_{i,i}\}_{i=1}^{N_t}$ that is assumed to be pre-computed before the LR-aided K-best. The brief description of the main modules of the implementation is provided as follows.

- Last layer (LL) module generates the best 6 children of the N_tth layer using the method described in Sec. 4.2.
- Pre-expansion (PE) module generates the best 6 children of one parent of a specific layer using the method described in Sec. 4.2.
- On-demand expansion and selection (OES) module chooses the best 6 children of a specific layer given the lists of the best 6 children of all 6 parents using a sorter of size 6.

4.4. Synthesis and Performance Results

The proposed complex LR-aided K-best detector is modeled using Verilog, and the fixed-point (FP) settings for some key parameters are listed in Table 2 (the FP setting is denoted as $[a, b]$, where a is the number of integer bits including one sign bit if applicable, and b is the number of fractional bits).

Variable	$\check{\mathbf{y}}$	$\mathbf{R}$	$1/R_{i,i}$	$\mathbf{z}$	cost
FP setting	[11, 10]	[4, 10]	[4, 10]	[10, 0]	[5, 10]

Table 2. Fixed-point settings of the FPGA implementation.

The Verilog design is synthesized, and placed and routed on Xilinx VC707 evaluation board. The used resources, maximum achievable frequency, latency, and throughput of the proposed LR-aided K-best implementation are summarized in Table 3. For 1024-QAM, the maximum throughput for 16x16 MIMO systems is over 3 Gbps.

The BER performance of the proposed implementation is displayed in Fig. 2 using CLLL. We observe that the proposed implementation can achieve almost the same performance as the floating-point one, and the performance gap to the MLD for 1024-QAM is 2.7 dB at BER $= 10^{-4}$.

Design	16x16 MIMO
Slices	22043
Multipliers (DSP48E1s)	702
Maximum frequency, f_{max}	120.351 MHz
Latency	188 cycles
Max. throughput (64-QAM)	1925 Mbps
Max. throughput (256-QAM)	2567 Mbps
Max. throughput (1024-QAM)	3209 Mbps

Table 3. Implementation results of the proposed LR-aided K-best detector for 16x16 MIMO systems.

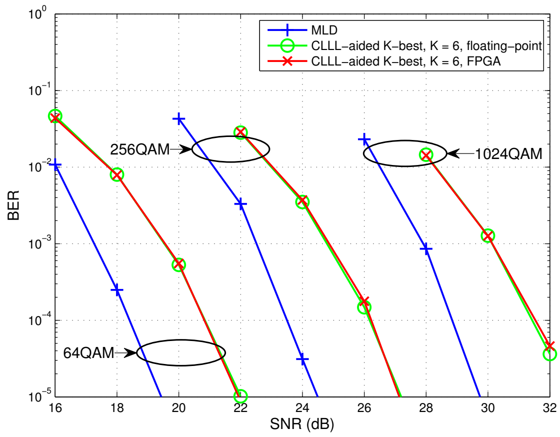

Fig. 2. Performance of the FPGA implementation of the proposed LR-aided K-best detector for 16x16 MIMO systems with different QAMs.

5. CONCLUDING REMARKS

In this paper, we proposed a novel complex LR-aided K-best algorithm that efficiently finds top K children of each layer in complex domain and is suitable for hardware implementation. In addition, we develop a hardware implementation of the proposed complex LR-aided K-best for 16x16 MIMO on Xilinx FPGA. The implementation results show that the proposed LR-aided K-best based on FPGA can achieve 3 Gbps throughput with about 2.7 dB gap at BER$= 10^{-4}$.

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