FAST RECONSTRUCTION OF NONUNIFORMLY SAMPLED BANDLIMITED SIGNAL USING SLEPIAN FUNCTIONS

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ABSTRACT

In this paper, we present an algorithm for fast reconstruction of bandlimited signal from nonuniform samples using shift-invariant space with Slepian function as a generator. The motivation to use Slepian functions is that they are bandlimited and most of their energy is concentrated in the finite time interval $[-\tau, \tau]$. This allows their truncation in time with controllable error, and results in a reduction of computational complexity of reconstruction process to $\mathcal{O}(NL^2)$, where N is number of samples, and $L \approx \tau$. As decreasing τ increases the truncation error, the algorithm offers a tradeoff between speed and accuracy. The simulation example of signal reconstruction is provided.

Index Terms— nonuniform sampling, signal reconstruction, fast algorithm, Slepian functions, prolate spheroidal wave functions

1. INTRODUCTION

Nonuniform sampling has been used in many engineering systems for decades. Recently, a type of nonuniform sampling, the signal-dependent event-based sampling, receives an increasing attention of researchers in the development of event-driven control [1,2] and signal processing [3–7]. One of key problems related to nonuniform sampling is a recovery of an original signal. This process faces a few general challenges that are discussed in details below.

1) In general, the nonuniform signal reconstruction is modeled by the expansion

$$x(t) = \sum_{n \in \mathbb{Z}} c_n g_n(t) \tag{1}$$

where $\{g_n(t)\}_{n\in\mathbb{Z}}$ called the *reconstruction functions* are properly chosen to allow representation of the signal $x(t)$ that belongs to a certain space $\mathcal{S}$. For perfect reconstruction, the functions $\{g_n(t)\}_{n\in\mathbb{Z}}$ must be a Riesz basis or a frame for the space $\mathcal{S}$ [8]. In the classical model, $\mathcal{S}$ is assumed to be Paley-Wiener space, PW_B, that is a space of B-bandlimited functions with finite energy. However, real physical signals are often time-limited, which contradicts their bandlimitedness because of the uncertainty principle [9]. The symptom of this mismatch is visible when $g_n(t) = \operatorname{sinc}(t-n)$ is used for reconstruction of time-limited signals because slow time-decay of $\operatorname{sinc}(\cdot)$ impedes modelling a rapid signal suppression.

2) Reconstruction of a signal according to (1) can be posed as a linear inverse problem that consists in finding appropriate coefficients c_n to a set of known signal samples $x(t_k)$. In most of the recovery algorithms, a system of linear equations is solved in order to obtain c_n. For a matrix of linear system without any specific structure (as in [7]), this requires $\mathcal{O}(N^3)$ flops where N is a finite number of samples used for truncated recovery. Also alternative methods, such as recovery using time-varying filter [10] requires recomputing of coefficients using least-square method, and solving the linear equations. If the matrix is transformed to the Toeplitz [8] or Vandermonde [5] form, the computational cost is lowered to $\mathcal{O}(N^2)$. Similarly, a direct recovery based on Lagrange formula is of $\mathcal{O}(N^2)$ complexity [11]. In [12], the method allowing to recover the signal according to (1) with a linear complexity $\mathcal{O}(N)$ is reported but it requires the use of functions with finite time support which are generally non-bandlimited.

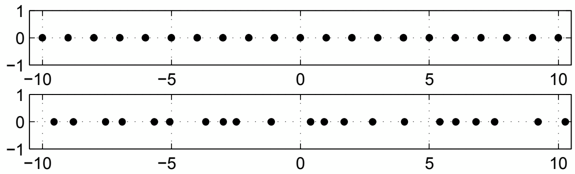

Fig. 1. Example of samples time-location in a) uniform sampling, b) nonuniform sampling - visible gaps and clustering of samples.

3) Nonuniform sampling implies different time intervals between samples. In the gaps between samples, the signal is recovered on the basis of neighboring samples. Due to a limited rate of change of $x(t)$, the values of bunched samples may not differ much, so the recovery depends on such small differences. The more clustered samples, the more vulnerable to noise and unstable recovery algorithm. With such set of samples, the linear system of equations becomes ill-conditioned. Several ways of reducing unstable behavior were presented in [8].

Recently, a signal reconstruction using Slepian functions, known also as prolate spheroidal wave functions, was proposed [7, 13–16]. Thanks to advantageous time-frequency properties (finite bandwidth and high concentration of energy in finite time interval), these functions provide a good choice for $\{g_n(t)\}_{n\in\mathbb{Z}}$ in equation (1) in terms of fast suppression of their values in time. Moreover, the set of Slepian functions $\{\psi_n(t)\}_{n\in\mathbb{Z}}$ forms an orthogonal basis for Paley-

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Wiener space, which results in a relatively well-conditioned linear system, and therefore, a good stability of reconstruction algorithm [7]. Unfortunately, the signal recovery based on the use of Slepian functions is still characterized by high computational requirements for solving an unstructured linear system, $\mathcal{O}(N^3)$, and computing values of Slepian functions. The property of good time localization motivated also the authors of [16] who use the Slepian functions as sampling functions to perform finite number of linear measurements at sub-Nyquist rate and reconstruct continuous-time signal using the discrete-time methods of compressed sensing. This idea, however, cannot be easily applied to the nonuniform sampling, where sampling functions are the $\mathrm{sinc}(\cdot)$.

In this paper, we show an alternative usage of Slepian function allowing a reduction of high computational demands. Instead of using all Slepian functions $\{\psi_n(t)\}_{n\in\mathbb{Z}}$, as in [7], we exploit a shift-invariant space approach that involves only the zeroth Slepian function $\{\psi_0(t-n)\}_{n\in\mathbb{Z}}$. The linear system of such reconstruction can be approximated by a sparse matrix, which significantly reduces computation, yielding $\mathcal{O}(N)$ algorithm. Furthermore, values of only one Slepian function need to be evaluated, in contrast to the method presented in [7, 14], which lowers the overall computational cost of the algorithm. The proposed approach maintains good properties of representing time-limited signals, and allows to control the reconstruction error. We also provide the requirements guaranteeing stability of the recovery algorithm. The paper is organized as follows. Section 2 presents basics of signal reconstruction from nonuniform samples using frame theory which is based on matrix inversion. Reconstruction matrices based on $\mathrm{sinc}(\cdot)$ and Slepian functions are compared. Section 3 introduces an original concept of the reconstruction using shifted Slepian function. The properties of the proposed algorithm are discussed in Section 4. Numerical results of reconstruction using a new algorithm are covered by Section 5.

2. RECONSTRUCTION OF NONUNIFORMLY SAMPLED SIGNALS

Assuming that a finite number of samples N are known, the approximate reconstruction from nonuniform samples can be performed using by truncating the general expression (1). The signal value $x(t_k)$ at the time instant t_k which is known due to sampling operation may be, according to (1), presented as:

$$x(t_k) = \sum_{n\in\mathbb{Z}} c_n g_n(t_k) \tag{2}$$

Let us denote a vector of nonuniform samples as $\mathbf{x} = \{x(t_1), x(t_2), \ldots, x(t_N)\} \in \mathbb{R}^N$, a vector of coefficients by $\mathbf{c} = \{c_1, c_2, \ldots, c_N\} \in \mathbb{R}^N$, and a matrix of sampling functions evaluated at sampling points as $\mathbf{G}_{[n,k]} = g_n(t_k)$, $\mathbf{G} \in \mathbb{R}^{N\times N}$. By successive insertion of $t = t_k$ for $k \in \{1, 2, \ldots, N\}$ into (1), the following system of linear equations is obtained

$$\mathbf{x} = \mathbf{G}\mathbf{c} \tag{3}$$

with a solution

$$\mathbf{c} = \mathbf{G}^{-1}\mathbf{x} \tag{4}$$

Knowing a value of the vector $\mathbf{c}$, it is possible to reconstruct the original signal $x(t)$ using (1). Now, various choices of $g_n(t)$ can be discussed. The classical Whittaker-Shannon interpolation offers a perfect reconstruction of a finite-energy bandlimited signal $x(t)$ from its samples using the famous equation

$$x(t) = \sum_{n\in\mathbb{Z}} x(nT)\,\mathrm{sinc}(2B(t-nT)) \tag{5}$$

where T is the sampling period related to the bandwidth B of the signal $x(t)$ defined as $T = 1/(2B)$, and $\mathrm{sinc}(t) := \sin(\pi t)/\pi t$. In practice, the reconstruction is not perfect because real physical signals are not strictly bandlimited and the number of samples used is not infinite as in (5), which results in aliasing and truncation errors.

To alleviate both errors, the reconstruction using the set of Slepian functions was proposed in [13]. The Slepian functions $\psi_{m,B,\tau}(t)$ are solutions of the following eigenfunction problem

$$\lambda_{m,B,\tau}\psi_{m,B,\tau}(t) = 2B\int_{-\tau}^{+\tau} \psi_{m,B,\tau}(u)\,\mathrm{sinc}(2B(t-u))du \tag{6}$$

where m is an eigenvalue/eigenfunction index. The functions $\psi_{m,B,\tau}(t)$ possess some unique properties [9, 13, 17]:

1) they have bandwidth equal to B and Fourier transform $\psi_{m,B,\tau}(\omega)$ given by

$$\psi_{m,B,\tau}(\omega) = (-1)^m\sqrt{\frac{\tau}{(B\lambda_{m,B,\tau})}}\psi_{m,B,\tau}(\omega)\Pi_B(\omega) \tag{7}$$

where $\Pi_B(\omega)$ is bandlimiting operator.

2) the eigenvalue $\lambda_{n,B,\tau}$ associated with eigenfunction $\psi_{m,B,\tau}(t)$ determines a fraction of total energy inside interval $[-\tau,\tau]$; the eigenvalues order is strictly decreasing: $\lambda_{0,B,\tau} > \lambda_{1,B,\tau} > \lambda_{2,B,\tau}, \ldots$.

3) The energy concentration in the time interval $[-\tau,\tau]$ of the zeroth Slepian function ($m = 0$) is the maximum possible among all functions from PW_B space; although they are not time-limited, the energy outside this interval is very low

4) set $\{\psi_{m,B,\tau}(t)\}_{n\in\mathbb{Z}}$ constitutes an orthogonal basis both for $L_2[-\tau,\tau]$ and for PW_B.

The first four Slepian functions obtained using the algorithm described in [17] are presented in Fig. 2. As a basis for a Paley-Wiener space, Slepian functions can be used as a set of functions $\{g_n(t)\}_{n\in\mathbb{Z}} = \{\psi_{n,B,\tau}(t)\}_{n\in\mathbb{Z}}$ in (1), yielding

$$x(t) = \sum_{n\in\mathbb{Z}} c_n\psi_{n,B,\tau} \tag{8}$$

Let us consider a time-limited signal, spread over time interval of a certain length. When energy concentration interval $[-\tau,\tau]$ is set to be equal to signal length, a good model of time-limited and nearly-bandlimited signal is obtained [13].

From the perspective of nonuniform reconstruction, it is also advantageous to use the matrix $\mathbf{G}$ with elements $\mathbf{G}_{[n,k]} = \psi_{n,B,\tau}(t_k)$ rather than with $\mathbf{G}_{[n,k]} = \mathrm{sinc}(2B(t_k - nT))$ when the samples are located only in the interval $[-\tau,\tau]$ because the set of Slepian functions $\{\psi_{n,B,\tau}(t)\}_{n\in\mathbb{Z}}$ is an orthogonal basis both in $[-\tau,\tau]$ and $[-\infty,\infty]$. At the same time, the set of functions $\{\mathrm{sinc}(2B(t-nT))\}_{n\in\mathbb{Z}}$ maintains orthogonality only in the interval $[-\infty,\infty]$. The advantage of the reconstruction based on Slepian functions comes from the fact that the matrix $\mathbf{G}$ used for nonuniform reconstruction (see the formula (3)) has a lower condition number if the basis is orthogonal. This has a positive impact on the stability of the reconstruction algorithm [7, 8].

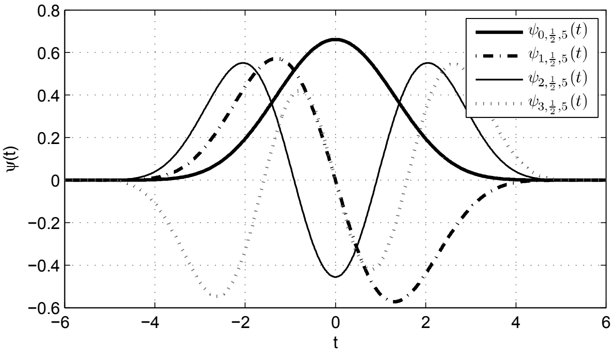

Fig. 2. Slepian functions $\psi_{m,B,\tau}(t), B = \frac{1}{2}, \tau = 5$, $m \in \{0, 1, 2, 3\}$

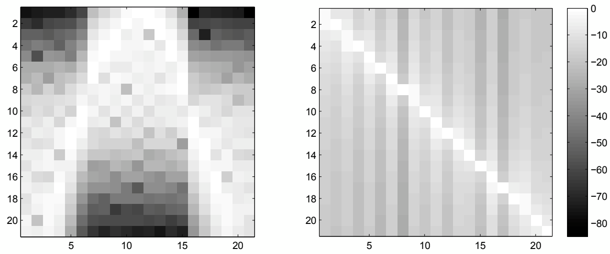

Fig. 3. Magnitudes of matrix $\mathbf{G}$ elements in logarithmic scale ($10\log_{10}|\mathbf{G}|$); reconstruction variants: $\mathbf{G}_{\psi_n}$ with Slepian functions (8), $\tau = 5$, (left); $\mathbf{G}_{sinc}$ with $\mathrm{sinc}(\cdot)$ functions (5) (right)

Fig. 3 illustrates the magnitudes of elements of reconstruction matrices $\mathbf{G}$ built of Slepian functions and $\mathrm{sinc}(\cdot)$ functions respectively for exemplary nonuniform pattern of samples shown in the Fig. 1. The n-th row of the matrix $\mathbf{G}$ contains the vector composed of the values of the reconstruction function $g_n(t)$ at sampling points $\{t_1, t_2, \ldots, t_N\} \in \mathbb{R}^N$. Note, that values of the matrix elements depend on reconstruction function used and sampling instants, being independent of the values of the samples. It is evident, that the vectors from the first rows of matrix $\mathbf{G}_{\psi_n}$ have higher values (and thereby signal energy) concentrated in the middle. In the matrix $\mathbf{G}_{sinc}$, the values of diagonal elements are dominating , although off-diagonal elements do not differ much in the order of magnitude. Such disposition is true for most of the sampling patterns without large gaps. In general, there are no regions in the matrix $\mathbf{G}_{sinc}$ whose elements could be approximated by zero to obtain higher sparsity. Therefore both matrices $\mathbf{G}_{\psi_n}$ and $\mathbf{G}_{sinc}$ can be classified as dense, which results in solving the linear system (3) at the cost of $2N^3/3$ floating point operations [18].

3. SHIFT-INVARIANT SPACE RECONSTRUCTION WITH SLEPIAN FUNCTIONS

From Fig. 3. one can conclude that sparsity of the matrix $\mathbf{G}$ cannot be achieved both for $\mathrm{sinc}(\cdot)$ functions and Slepian functions used as the bases for reconstruction. The only functions with high energy concentration is the collection of the first few Slepian functions, $\psi_{0,B,\tau}(t), \psi_{1,B,\tau}(t), \ldots$. A natural question is if such functions alone can be used as a basis for a Paley-Wiener space, using shift-invariant space approach, i.e. reconstruction (1) with set of functions $\{g_n(t)\} = \{g(t-n)\}$.

As stated in Section 2, Slepian functions are bandlimited, so for each m there exists such linear transformation $h_m(t)$ that

$$\mathrm{sinc}(2Bt) * h_m(t) = \psi_{m,B,\tau}(t) \tag{9}$$

or equivalently, in the frequency domain:

$$\Pi_B(\omega)H_m(\omega) = \Psi_{m,B,\tau}(\omega)$$

Then, from the property (7), $H_m(\omega)$ is given by

$$H_m(\omega) = (-1)^m\sqrt{\frac{\tau}{B\lambda_{m,B,\tau}}}\psi_{m,B,\tau}(\omega) \tag{10}$$

Let us represent certain bandlimited signal $y(t) \in PW_B$ using (5) and convolve it with $h_m(t)$

$$\begin{aligned} h_m(t) * y(t) &= \sum_{n\in\mathbb{Z}} y(nT)\left(h_m(t) * \mathrm{sinc}(2B(t-nT))\right) = \\ &= \sum_{n\in\mathbb{Z}} y(nT)\psi_{m,B,\tau}(t-nT) \end{aligned} \tag{11}$$

The next question is if an arbitrary signal $x(t) \in PW_B$ can be represented as $x(t) = h_m(t) * y(t)$. To respond to this question, note that the following relationships holds in frequency domain:

$$\begin{aligned} X(\omega) &= H_m(\omega)Y(\omega) \\ Y(\omega) &= X(\omega)H_m^{-1}(\omega) \end{aligned} \tag{12}$$

To allow representation by (5), $y(t)$ must also belong to PW_B to keep (10) valid. To maintain finite energy of $y(t)$, the constraint on $H_m(\omega)$ must be imposed

$$\int_{-2\pi B}^{-2\pi B}\left|\frac{1}{H_m(\omega)}\right|^2 d\omega < \infty \tag{13}$$

which is fulfilled as long as $H_m(\omega) \neq 0$ for $\omega \in (-2\pi B, 2\pi B)$. The similar result regarding reconstruction on the basis of a bandlimited functions different from $\mathrm{sinc}(\cdot)$ was presented more formally in [19]. Because according to (10) the zeroes of the $H_m(\omega)$ are related to the zeroes of $\psi_{m,B,\tau}(\omega)$, it can be easily seen from Fig. 2, that only the zeroth Slepian function, $\psi_{0,B,\tau}(t)$, is a valid choice. Resulting reconstruction formula is

$$x(t) = \sum_{n\in\mathbb{Z}} c_n\psi_{0,B,\tau}(t-nT) \tag{14}$$

where $c_n = y(nT)$. Function $\psi_{0,B,\tau}(t)$ has also highest energy concentration among Slepian functions, so the reconstruction matrix $\mathbf{G}_{\psi_0}$ has the highest possible number of elements with numerical values close to zero (Fig. 4).

4. ALGORITHM PROPERTIES

4.1. Computational complexity

The matrix $\mathbf{G}_{\psi_0}$ after rounding small values towards zero can be considered as a band matrix $\widetilde{\mathbf{G}}_{\psi_0}$ with non-zero values present only in the diagonals. Formula (14) can be used for the substitution analogous to (2) to obtain the linear system $\mathbf{x} = \widetilde{\mathbf{G}}_{\psi_0}\mathbf{c}$ with $\mathbf{G}_{\psi_0}$ having $2L+1$ diagonals.

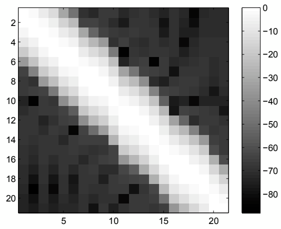

Fig. 4. Magnitudes of matrix $\mathbf{G}_{\psi_0}$ elements in logarithmic scale ($10\log_{10}|\mathbf{G}_{\psi_0}|$); reconstruction with Slepian functions using (14), $\mathbf{G}_{\psi_0}, \tau = 5$

To solve it, the Gaussian elimination algorithm for band matrices can be used [18], yielding computational complexity

$$N(2L^2 + 5L + 1) - 2L^3 - 5L^2 \tag{15}$$

If we denote by L_n the number of $\widetilde{\psi}_{0,B,\tau}(t - nT)$ samples within the support $[-\tau, \tau]$, then $L = \max_{n\in\{1,2,..,N\}} L_n$. If the deviations of sampling instants from their uniform positions are low, $|t_n - nT| \lesssim 0.5T$, then L can be estimated as $\lceil\tau\rceil$, where $\lceil\cdot\rceil$ represents rounding upwards. Decreasing τ is then advantageous from the perspective of lowering computational requirements. On the other hand, the smaller τ, the higher sidelobe level and therefore the higher truncation error $\varepsilon(t,\tau) = |\psi_{0,B,\tau}(t) - \widetilde{\psi}_{0,B,\tau}(t)|$. This tradeoff can be seen in Fig. 5 and Fig. 6.

4.2. Numerical stability

Because of the relation (10), low magnitude of $\Psi_{0,B,\tau}(\omega)$ in the region of high frequencies (near B) causes an amplification of these frequencies in $Y(\omega)$. The energy of the coefficients $\|\mathbf{c}\|^2 = \sum_n y^2(nT)$ can be evaluated (using Parseval identity) from (12), as

$$\|Y(\omega)\|^2 = \|X(\omega)\Psi^{-1}_{m,B,\tau}(\omega)\|^2 \tag{16}$$

Therefore, if the reconstructed signal $x(t)$ contains high frequencies, the coefficients $c_n = y(nT)$ will have large absolute values. The prefered signals for the reconstruction should be oversampled, to avoid the problems with finite precision arithmetic.

The characteristics of the algorithm constrains also a size of the maximum gap between samples to the width of the $\psi_{0,B,\tau}(t) : |t_n - t_{n+1}| < 2\tau$. For larger gaps, the matrix $\widetilde{\mathbf{G}}_{\psi_0}$ becomes rank-deficient because there is no sample to provide information on the magnitude of one of the basis functions.

5. SIMULATIONS

To test the algorithm operation, a simulation was performed. To avoid high frequencies in $x(t)$, an example signal was created using the following formula

$$x(t) = \sum_{n=0}^{49} c_n \psi_{0,\frac{1}{2},\tau}(t - n) \tag{17}$$

The coefficients c_n were created as i.i.d. realizations of a random variable $\mathcal{N}(0, 1)$. The vector of nonuniform sampling instants was produced using

$$\{t_1, t_2, \ldots, t_{49}\} = \{0, 1, \ldots, 49\} + X_1, X_2, \ldots, X_{49} \tag{18}$$

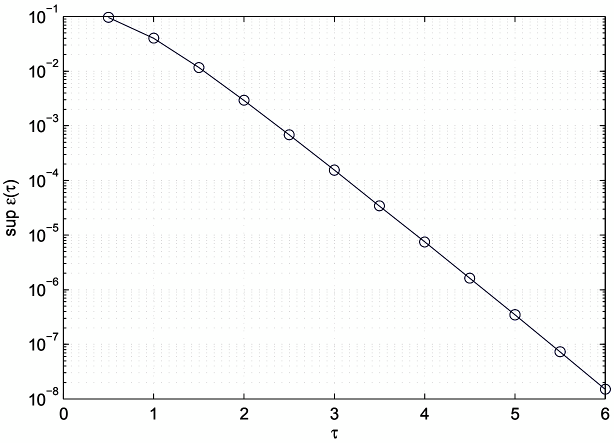

Fig. 5. Maximum sidelobe level, $\sup \varepsilon(t,\tau)$ of $\psi_{0,\frac{1}{2},\tau}(t)$ against τ

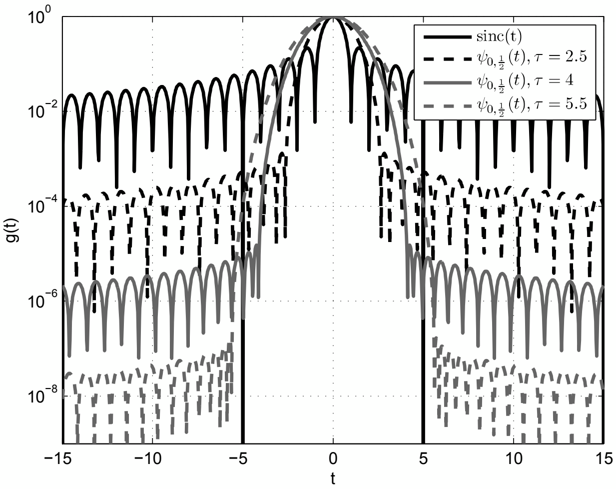

Fig. 6. Sidelobe level of normalized Slepian function, $|\psi_{0,\frac{1}{2},\tau}(t)/\psi_{0,1,\tau}(0)|$ against τ

where random variable was chosen to be independent uniformly distributed $X_n \sim \mathcal{U}(-0.5, 0.5)$ to maintain moderate size of gaps between samples. Finally, the algorithm solving the equation $\mathbf{x} = \widetilde{\mathbf{G}}_{\psi_0}\mathbf{c}$ was run and then (14) was used for reconstruction of $x(t)$. A few sets of parameters L and τ were tested to estimate relation between truncation and reconstruction error $e(t) = x(t) - \widetilde{x}(t)$. This error along with the total mean error are presented respectively in Table 1 and Fig. 7. As follows from Table 1, the reconstruction error $e(t)$ is of the same order of magnitude as the values of truncated sidelobes of $\psi_{0,B,\tau}(t)$ function used for reconstruction.

6. CONCLUSIONS

In this paper, we present a new method for reconstructing a continuous-time bandlimited signal from its nonuniform samples using the shifted Slepian function. In comparison to existing algorithms, the proposed method is characterized by the lowest computational requirements, $\mathcal{O}(NL^2)$. Due to its special numerical properties, it is

τ	L	Mean $\lvert e(t)\rvert$	$\sup \lvert\varepsilon(t,\tau)\rvert$
4	4	$9.3219 \cdot 10^{-6}$	$7.49 \cdot 10^{-6}$
4.5	5	$8.0515 \cdot 10^{-7}$	$1.622 \cdot 10^{-6}$
5	5	$4.700 \cdot 210^{-7}$	$3.479 \cdot 10^{-7}$
5	No truncation	$1.7126 \cdot 10^{-9}$	0

Table 1. Mean reconstruction error and maximum sidelobe level of corresponding $\psi_{0,B,\tau}(t)$

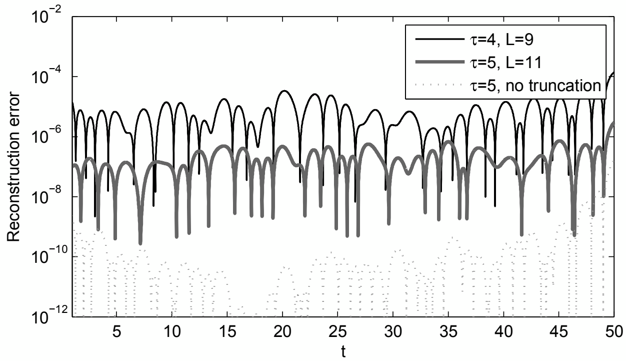

Fig. 7. Reconstruction error $e(t) = |x(t) - \widetilde{x}(t)|$

suitable for the reconstruction of oversampled signals. The simulation results show that the reconstruction error can be controlled by choosing desired value of the parameter τ defining the interval of energy concentration in the reconstruction function $\psi_{0,B,\tau}(t))$, and of the parameter L, which sets the interval of $\psi_{0,B,\tau}(t)$ truncation. The proposed approach yields approximate reconstruction offers of bandlimited function with a set approximated by a set of time-limited kernels $g_n(t) = g(t-n)$, with flexible control of reconstruction error and computation complexity. The relevant problem for further research is to derive an analytical expression for reconstruction error using truncated representation. Another future research task is the problem of improving effectiveness of evaluating Slepian function $\psi_{0,B,\tau}(t)$, possibly by an approximation using polynomials or a splines.

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