CFAR DETECTION OF SPATIALLY DISTRIBUTED TARGETS IN K-DISTRIBUTED CLUTTER WITH UNKNOWN PARAMETERS

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ABSTRACT

The paper deals with Constant False Alarm Rate (CFAR) detection of spatially distributed targets embedded in K-distributed clutter with correlated texture and unknown parameters. The proposed Cell Averaging-based detector automatically selects the suitable pre-computed threshold factor in order to maintain a prescribed Probability of False Alarm *pfa*. The threshold factors should be computed off-line through Monte Carlo simulations for different clutter parameters and correlation degrees.

The online estimation procedure of clutter parameters has been implemented using Maximum Likelihood Moments approach. Performances analysis of the proposed detector assumes unknown shape and scale parameters and Multiple Dominant Scattering centers model (MDS) for spatially distributed targets.

Keywords— Distributed targets, CFAR detection, K-distribution, MDS.

1. INTRODUCTION

In high resolution radar (HRR) detection, the nature of the backscatterers from sea surface is known to depart from the Gaussian form. In fact, sea clutter echoes are better described by compound Gaussian models [1]. In particular, the K-distribution is an adequate statistical model for sea clutter returns [1]. Moreover, the threshold is closely related to the clutter statistics, which are not *a priori* known in realistic cases. To overcome this problem, many techniques of estimating the K-distribution parameters have been proposed [2,3]. In [3], the authors proposed an estimation approach that combines Maximum Likelihood and the Method of Moments (ML/MOM).

On the other hand, real recorded datasets indicate that, when viewed through HRR systems, targets are better described as a reflection from few isolated points, referred to as Multiple Dominant Scattering center model (MDS) [4,5]. In [6], the Generalized Likelihood Ratio Test (GLRT) detection of range-Doppler distributed targets in non Gaussian clutter is examined. The authors showed the impact of the radar resolution on the detection performance. In this work, we focused on the design of a detection scheme for spatially distributed targets, embedded in a partially correlated K-clutter with unknown parameters. The structure of the proposed detector is jointly associated with an estimation procedure, which is an important task in the proposed system. Also, threshold factors, keeping a desired *pfa*, are computed for different clutter parameters using intensive Monte Carlo simulations. Then, the pre-computed factors are stored within a Look-up Table in order to be online selected by the estimated parameters. In fact, the ML/MOM procedure provides an automatic switching to the suitable threshold factor according to the actual environment. In this stage, the estimated shape and scale parameters are compared to the closest integer or half integer parameters among their pre-assigned values in the Look-up Table. Finally, an appropriate statistical hypothesis test has been proposed for distributed targets through two MDS models. Noting that the amount of the backscattered energy and the scatterers locations are specified for each considered model. The performances of the proposed Cell-Averaging detector based on Look-up Tables (CA-LT) have been investigated and compared to those of the Logarithmic Cell-Averaging (CAL) detector [1]. The analysis has been carried out, in terms of the detection probability, under various environment conditions, namely, the texture correlation degree, the target energy profile and the clutter parameters

2. ASSUMPTIONS AND PROBLEM FORMULATION

The K-distributed clutter, which is specified by the shape and the scale parameters (ν,μ), is the product model of a slowly varying component, the texture, modeled by the Gamma distribution, and a rapidly varying component, the speckle, modeled by the exponential distribution. The texture and the speckle components are two independent random variables, and are referred to as τ_i and S_i respectively. Consequently, the square law detected output at range bin 'i' can be written as follows:

$$X_i = \tau_i S_i \tag{1}$$

The probability density function (PDF) of the envelope samples, $Y_i = \sqrt{X_i}$, which are K-distributed, is given by [7]

$$f_Y(Y_i) = \frac{2c^{\nu+1}}{\Gamma(\nu)} Y_i^{\frac{\nu-1}{2}} K_{\nu-1}\left(2c\sqrt{Y_i}\right) \tag{2}$$

with $c = \mu\sqrt{\pi}/2$, where ν and μ correspond respectively to the shape and scale parameters, $\Gamma(.)$ is the Gamma function and $K_{\nu-1}$ represents the modified Bessel function. The speckle is assumed to be uncorrelated from cell to cell, while the texture is described by a first order Markov process. Considering N reference cells and Np primary data, the covariance matrix of the texture, has a Toeplitz structure and can be written as follows

$$M_\tau(i,j) = \rho^{|i-j|} \qquad i,j = 1, \ldots . N + Np \tag{3}$$

we should note here that both the primary and the secondary data (reference cells) share the same covariance matrix M_τ. The correlation structure of the texture is implicit in the multivariate PDF $f_\tau(\tau)$, given by [1]

$$f_\tau(\tau) = f_{\tau_2/\tau_1}\left(\tau_{N+Np}/\tau_{N+Np-1}\right) \ldots . . f_{\tau_2/\tau_1}(\tau_2/\tau_1) . f_{\tau_1}(\tau_1)$$

with conditional probability

$$f_{\tau_2/\tau_1}(\tau_i/\tau_{i-1}) = \frac{\nu}{\mu} \frac{1}{\rho^{(\nu-1)/2}(1-\rho)} \left(\frac{\tau_i}{\tau_{i-1}}\right)^{\frac{\nu-1}{2}} \tag{4}$$

$$\times \exp\left(-\frac{\nu}{\mu} \frac{\tau_i + \rho\tau_i - 1}{1-\rho}\right) K_{\nu-1}\left[\frac{\nu}{\mu} \frac{1}{1-\rho} \sqrt{\rho\tau_i\tau_{i-1}}\right]$$

where ρ corresponds to the one lag-correlation coefficient.
In the MDS model, the target's envelope echoes are independent random variables distributed according to the Chi-square law with mean square value σ^2/Np. Np denotes the number of adjacent range cells that contain the total target energy, also referred to as primary data.
In the MDS representation, the target energy is considered to be spread over Np adjacent cells according to a given energy profile. As a result, the total backscattered energy, namely Δ, becomes the sum of the partial energies reflected by each range location, thus

$$\Delta = \sum_{k=1}^{Np} a_k {X_0}^k \tag{5}$$

where a_k is a multiplicative factor representing the energy proportion of the k^{th} range location within the primary data ${X_0}^k$, k=1, Np. Also, we assume two deterministic MDS models, with Np=3 and Np=5. The amount of the total useful energy backscattered from each cell and the corresponding range locations are given in Table 1.

Model number	Np				
	1	2	3	4	5
1	0.25	0.5	0.25	0	0
2	0.1	0.1	0.6	0.1	0.1

Table 1. Discrete Scatterers Locations and the amount of the total reflected energy

3. LOOK-UP TABLE BASED DETECTOR (CA-LT)

3.1. Working principle of the CA-LT

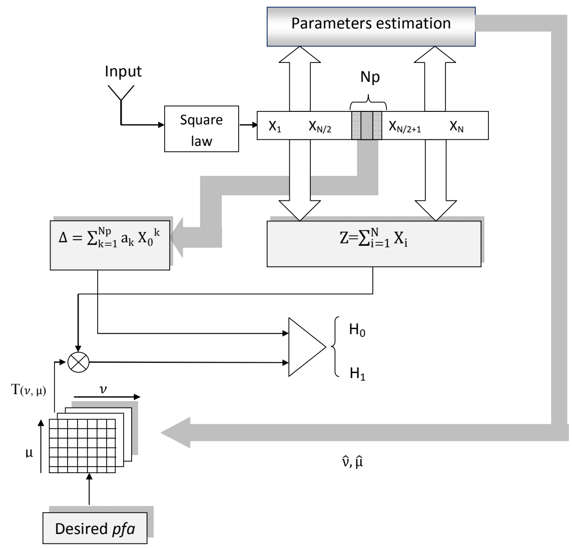

Fig. 1. Structure of the proposed detector (CA-LT).

The proposed detector is based on the cell averaging (CA) concept, by using Look-up-Tables (LT), to decide the presence of a spatially distributed target within the primary data group ${X_0}^k$, k=1, Np.
Since a closed form expression for the *pfa* is not available in K-environments, we resort to Monte Carlo simulations to evaluate the thresholding factors $T(\nu,\mu)$ that achieve a constant *pfa* for a given couple (ν,μ). Noting that pre-computed factors should be stored in Look-up Tables for half integer values of clutter parameters.
As illustrated in Figure 1, the detector CA-LT estimates the local clutter level Z by computing the mean of the N reference cells surrounding the primary data. The target is declared present, among the Np primary cells, if the total backscattered energy, Δ, exceeds the adaptive threshold $T(\nu,\mu)$ multiplied by Z.
According to the actual environment, the suitable threshold factor $T(\nu,\mu)$ is automatically selected via the estimated shape and scale parameters values $\hat{\nu}, \hat{\mu}$, by implementing the ML/MOM technique [3]. In fact, the online estimated values $(\hat{\nu}, \hat{\mu})$ are approximated to the closest half integer values (ν,μ), which have been previously stored in the look-up table, in order to assign the most appropriate value to the threshold factor. Therefore, the binary hypothesis test, including the distribution of the target energy among the Np cells, can be written as

$$\Delta = \sum_{k=1}^{Np} a_k {X_0}^k \underset{<_{H_0}}{\overset{>^{H_1}}{}} T(\nu,\mu) Z \tag{6}$$

Where H_0 and H_1correspond respectively to the null and the alternative hypothesis.
Noting that the hypothesis test (6) reduces to the hypothesis test for unresolved targets, by setting Np to 1 and Δ to X_0, which leads to

$$X_0 \underset{<_{H_0}}{\overset{>^{H_1}}{}} T(\nu,\mu)Z \quad (7)$$

3.2. Clutter parameters estimation by the ML/MOM approach

The ML/MOM approach combines both Maximum Likelihood (ML) technique and the Moments Method (MOM) to estimate the K-distribution parameters. The ML estimation of the shape and the scale parameters ,ν and μ, is derived by maximizing the log likelihood function, which leads to the following [3]

$$\hat{\mu} = \frac{1}{2}\exp\left(\frac{\gamma - \Psi(\hat{\nu}+1)}{2} + \frac{1}{N}\sum_{i=1}^{N}\log(X_i)\right) \quad (8)$$

where $\gamma = 0.5772$ is the Euler constant, $\Psi(.)$ is the digamma function, and Xi are the clutter samples. On the other hand, a relation between the shape and the scale parameter, has been obtained using the k^{th} order moment of the K-distribution [3]

$$\hat{\mu} = \frac{1}{2}\left[\frac{\Gamma(\nu+1)}{\Gamma(0.5k+1)\Gamma(\nu+1+0.5k)}\frac{1}{N}\sum_{i=1}^{N}{X_i}^k\right]^{\frac{1}{k}} \quad (9)$$

Combining (8) and (9), the authors defined the function $g_k(\hat{\nu})$ as [3]

$$g_k(\hat{\nu}) = \frac{1}{N}\sum_{i=1}^{N}\log\left({X_i}^k\right) - \log\left[\frac{1}{N}\sum_{i=1}^{N}{X_i}^k\right] + \frac{k\gamma}{2} + \log\left[\Gamma\left(1+\frac{k}{2}\right)\right] \quad (10)$$

$$g_k(\hat{\nu}) = \log\left[\frac{\Gamma(\hat{\nu}+1)}{\Gamma\left(\hat{\nu}+1+\frac{k}{2}\right)}\right] + \frac{k\Psi(\hat{\nu}+1)}{2} \quad (11)$$

The estimation of the shape parameter $\hat{\nu}$, can be obtained by substituting (10) into (11)

$$\hat{\nu} = g_k^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\log\left({X_i}^k\right) - \log\left[\frac{1}{N}\sum_{i=1}^{N}{X_i}^k\right] + \frac{k\gamma}{2} + \log\left[\Gamma\left(1+\frac{k}{2}\right)\right]\right) \quad (12)$$

Since there is no analytical solution of (12), we propose the following numerical solution. We first tabular the function $g_k(\hat{\nu})$, which is strictly monotonically increasing, according to (11), for different half integer values of the parameter ν. During the detection stage, the function $g_k(\hat{\nu})$ is computed using the experimental clutter data Xi, according to (10). Next, the obtained value of $g_k(\hat{\nu})$ is, in turn, approximated to the nearest pre-assigned value. Finally, the estimated scale parameter can be simply obtained from (8).

4. RESULTS AND DISCUSSION

The performances of the proposed detector (CA-LT) are analyzed in terms of the probability of detection and compared to those of the CAL [1]. We resort to Monte Carlo simulations based on 100/*pfa* independent trials and consider a nominal *pfa* of 10^{-3}.
For the CA-LT structure, we used N=64 reference cells assuming a partially correlated texture and independent speckle component. The performances comparison has been carried out by taking into account various shape and scale parameters for highly correlated texture (ρ=0.9). Since the texture is a correlated Gamma-distributed random variable, it has been generated using 2ν Gaussian exponentially correlated variables with one-lag correlation coefficient ρ.
For the evaluation of the Probability of Detection (Pd) versus the signal to clutter ratio (SCR), we define the SCR as

$$\mathrm{SCR} = 10\log_{10}\frac{2\sigma^2}{\left(\frac{\nu}{\mu}\right)^2 + \frac{\nu}{\mu^2}} \quad (13)$$

where σ is the parameter of a Rayleigh fluctuating target. Noting that the parameter σ is used to generate the Np primary data ${X_0}^k$, k=1……..Np, for different SCR. Assuming a given target energy profile, the primary cells ${X_0}^k$ are scaled by the multiplicative factors a_k and summed according to (5).
The obtained threshold factors $T(\nu,\mu)$, maintaining a *pfa* of 10^{-3}, for different (ν,μ) and for ρ=0.9, are given in Table 2. From Table 2, we show that the CA-LT exhibits low thresholds for high values of ν.However, the threshold factors seem to be not affected by the scale parameter μ.

ν / μ	1	1.5	2	2.5
1	0.248	0.210	0.205	0.169
1.5	0.248	0.211	0.208	0.170
2	0.250	0.211	0.208	0.170

Table 2. Look-up Table versus the clutter parameters

In Figure 2, the *pfa* of the proposed detector is plotted for different values of the scale parameter μ, it highlights how the proposed detector ensures the CFAR property with respect to the scale parameter μ. Figure 3 refers to a comparison of the performances of the CA-LT and CAL detectors assuming 3 primary cells (Np=3) and a highly correlated texture (ρ=0.9). We observe that the Pd of both detectors increases for high values of the shape parameter ν. However, by comparing the curves obtained with the same parameters, we note that the CA-LT outperforms the CAL detector in such environment.

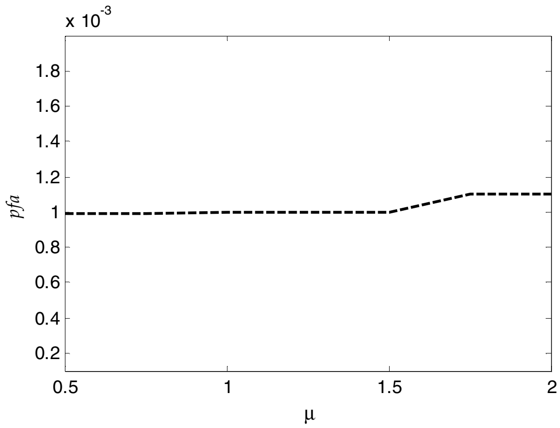

Fig. 2. *pfa* of the CA-LT for different values of μ.

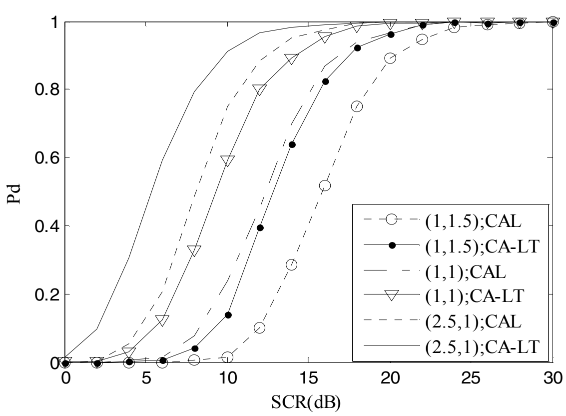

Fig. 3. Pd of CA-LT and CAL detectors for Np=3.

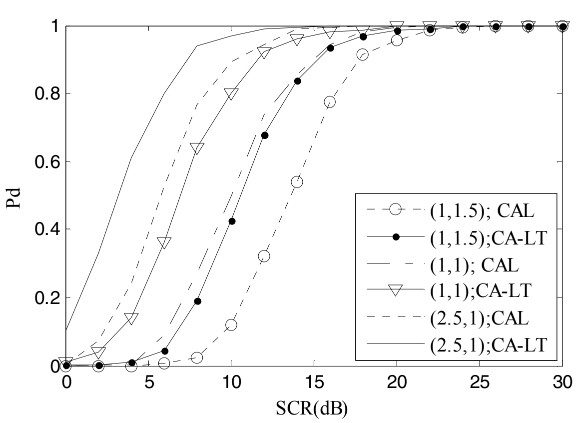

Fig. 4. Pd of CA-LT and CAL detectors for Np=5.

In Figure 4, we present a comparison of the two detectors under investigation using the second MDS model of Table 1. As mentioned in the previous case, the CA-LT exhibits better detection performance than the CAL detector for Np=5 primary cells. Also, we note that the increase of Np results in an improvement of the probability of detection. Indeed, increasing the number of the target scatterers (Np), enhance the radar resolution by the same factor, which produces a significant detection gain.

5. CONCLUSION

In this paper, a CFAR detection algorithm (CA-LT) has been designed for detecting spatially distributed targets, embedded in a K-clutter with unknown statistics. To analyze the detection performances of the CA-LT, we resorted to a two step-approach. First, we computed and stored the threshold factors, maintaining a constant *Pfa*, for different clutter parameters. Then, we implemented the ML/MOM technique, in order to automatically select the suitable thresholding factor to be used in the binary hypothesis test. Also, two MDS models have been investigated for modeling distributed targets, where the expression of the total backscattered target-energy has been taken into account.

Simulation results showed that the CA-LT provides better performance than the CAL detector [1] against K-clutter with unknown parameters. Additionally, the performances of both CA-LT and CAL detectors are enhanced by increasing the number of primary data. Otherwise stated, the detection performance is improved by increasing the radar resolution.

6. REFERENCES

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