AUTOMATIC WH-BASED EDGE DETECTOR IN WEIBULL CLUTTER

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ABSTRACT

Assuming a non-stationary Weibull background with no prior knowledge about the presence or not of a clutter edge, we propose and analyze the censoring and detection performances of the automatic censoring Weber-Haykin constant false censoring and alarm rates (ACWH-CFCAR) detector in homogeneous clutter and in the presence of a clutter edge within the reference window. The cfcarness property is assured by use of the Weber-Haykin (WH) adaptive thresholding which bypasses the estimation of the distribution parameters. The censoring algorithm starts up by considering the two most left ranked cells and proceeds forward. The selected homogeneous set is used to estimate the unknown background level. Extensive Monte Carlo simulations show that the performances of the proposed detector are similar to those exhibited by the corresponding fixed-point censoring WH-CFCAR detector.

***Index Terms* —** Weibull Clutter; Clutter Edge; Weber-Haykin Thresholding; Automatic Censoring; Automatic Detection.

1. INTRODUCTION

In radar signal detection systems, automatic target detection is usually performed by comparing return echoes to a threshold. These returned signals are, often, statistically non-stationary, with unknown variance at the receiver input. Thus, adaptive thresholding techniques are known to maintain CFAR by acquiring immunity against all kind of background heterogeneities caused by the presence of a clutter edge and/or interfering targets [1]. To guarantee cfarness in heterogeneous environment, a class of detectors based on order statistics has been developed in the literature [2-9]. In these detectors, the samples within the reference window are first ranked in an ascending order according to their amplitude. Then, those representing unwanted echoes are discarded. The remaining homogeneous set of samples is used to estimate the unknown background level. Nevertheless, these detectors give good results only if the clutter edge position and/or the interferences number are known *a priori*, which is not always available in real applications. To strike this problem, automatic censoring techniques have been widely investigated. In non-Gaussian clutter, the well-known approaches are found in [4-9]. In Lognormal clutter and multiple target situations, Almarshad et al. [4] proposed the Forward Automatic Censored Cell Averaging Detector (F-ACCAD-CFAR). This detector uses ranked transformed normal samples to censor automatically the highest unwanted cells. It starts up the heterogeneity tests from a subset of the ranked reference cells assumed to be homogeneous, and proceeds forward. Both censoring and detection algorithms are based on a biparametric linear threshold for which the normal distribution parameters are estimated using a simple linear approach. In [5], they considered the Forward/Backward Automatic Censoring Order Statistics Detectors (F/B-ACOSD-CFAR) based on the non-parametric Weber-Haykin adaptive threshold introduced by Weber and Haykin [3]. In Weibull clutter and multiple target situations, Chabbi et al. [6, 7] considered the Forward/Backward Order Statistics Automatic Censoring and detection (F/B-OSACD-CFCAR). In [6], they introduced a biparametric linear threshold where the transformed Gumbel distribution parameters are given by the Maximum Likelihood Estimators (MLEs), while in [7]; they used the Weber-Haykin adaptive threshold. Recently, Pourmottaghi et al. [8], proposed an automatic clutter edge detection and localization within the Weibull reference data samples. By eliminating the misleading data, the proposed CFAR scheme improved the performances of the Log-t detector proposed by Goldstein [10]. Meanwhile, Chabbi et al. [9], proposed the Dual Automatic Censoring Best Linear Unbiased (DACBLU-CFCAR) detector in Weibull clutter, when both clutter edge and/or interferences may be present in the reference window. In this detector, both censoring and detection algorithms rely upon the same biparametric linear threshold based on the BLU Estimators (BLUEs) of the Gumbel parameters.

In this paper, we introduce a simple procedure to localize automatically the position of a clutter edge, if any, in a Weibull environment. Explicitly, we analyze CFCAR censoring and target detection of the Automatic Censoring Weber-Haykin (ACWH-CFCAR) detector. The censoring algorithm starts up by considering the two most left ranked cells and proceeds forward. The selected homogeneous set of cells is then used to estimate the unknown background level. The cfcarness property is guaranteed by use of the Weber-Haykin thresholding. To complete the study, we show through extensive Monte Carlo simulations that the performances of the proposed detector are similar to those exhibited by its corresponding fixed-point censoring WH-CFAR detector in both homogeneous and clutter edged background.

2. DETECTION AND CENSORING ALGORITHMS

In the proposed detector, Fig. 1, we consider adaptive thresholding that ensures a constant probability of false censoring (P_{fc}) and a constant probability of false alarm (P_{fa}), i.e., cfcarness should be guaranteed for any values of the Weibull distribution parameters. Let the $X_i, i = 1, 2, \dots, N$ be independent and identically distributed (IID) random variables drawn from a Weibull distribution characterized by a scale parameter α and a shape parameter β. The Weibull probability density function (pdf) of a random variable X is given by [1]

$$f_X(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right],\ x \geq 0,\ \alpha > 0,\ \beta > 0 \quad (1)$$

The Weibull random variables X_i, $i = 1, 2, \dots, N$ are ranked in an ascending order to get the $X_{(i)}, i = 1, 2, \dots, N$; to be first processed by the censoring algorithm and then, in a censored version, by the detection algorithm.

2.1. Detection Algorithm

The detection algorithm of the ACWH-CFCAR detector is based on the following hypothesis test

$$X_0 \underset{H_0}{\overset{H_1}{\gtrless}} T_{\beta_{\widehat{m}}} \quad (2)$$

where the detection adaptive threshold $T_{\beta_{\widehat{m}}}$ is given by

$$T_{\beta_{\widehat{m}}} = \begin{cases} X_{(\widehat{m}+1)}^{\beta_{\widehat{m}}} X_{(N)}^{1-\beta_{\widehat{m}}} & \widehat{m} \in [0, N/2[\\ X_{(1)}^{\beta_{\widehat{m}}} X_{(\widehat{m})}^{1-\beta_{\widehat{m}}} & \widehat{m} \in [N/2, N-1] \end{cases} \quad (3)$$

$\widehat{m} \in [0, N-1]$ is determined by the censoring algorithm; it represents the estimated number of cells located before a clutter transition occurs in the reference window. That is, for $\widehat{m} \in [2, N/2[$, discarding the $\widehat{m}$ lowest cells avoids the capture effect. However, for $\widehat{m} \in [N/2, N-1]$, discarding the $N - \widehat{m}$ highest cells avoids masking effect. Only then after, the detection algorithm estimates the background level

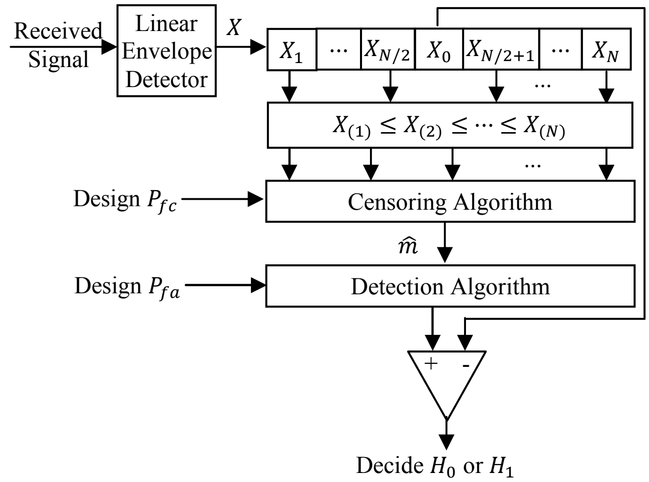


Fig. 1. Block diagram of the ACWH-CFCAR detector.

through the remaining cells. For $\widehat{m} = 0$, the environment is declared homogeneous and, hence the entire reference window is used by the detection algorithm. The detection threshold coefficient $\beta_{\widehat{m}}$ is set according to

$$P_{fa} = Prob\{X_0 > T_{\beta_{\widehat{m}}} \backslash H_0\} = Constant \quad (4)$$

Since analytical expression of the pdf of $T_{\beta_{\widehat{m}}}$ cannot be obtained, therefore, $\beta_{\widehat{m}}$ is determined through Monte Carlo simulations. Furthermore, besides the fact that the pdf of X_0, under H_1, cannot have a closed form, the probability of detection (P_d) also depends on $T_{\beta_{\widehat{m}}}$. Therefore, we had to resort to Monte Carlo simulations to determine P_d, which is given by

$$P_d = Prob\{X_0 > T_{\beta_{\widehat{m}}} \backslash H_1\} \quad (5)$$

where, *Prob* designates probability.

2.2. Censoring Algorithm

The censoring algorithm of the ACWH-CFCAR detector uses the ranked reference cells $X_{(i)}, i = 1, 2, \dots, N$, to get $\widehat{m}$, and then excises the undesirable cells with a constant false censoring rate (CFCR) according to a desired P_{fc}. It is based on the following heterogeneity test

$$X_{(\widehat{m}+1)} \underset{hH}{\overset{nhH}{\gtrless}} T_{\alpha_{\widehat{m}}} \quad (6)$$

The censoring adaptive threshold $T_{\alpha_{\widehat{m}}}$ is given by

$$T_{\alpha_{\widehat{m}}} = X_{(1)}^{\alpha_{\widehat{m}}} X_{(\widehat{m})}^{1-\alpha_{\widehat{m}}} \quad (7)$$

$\alpha_{\widehat{m}}$ is the censoring threshold coefficient set according to a desired P_{fc}. It is written as

$$P_{fc} = Prob\{X_{(\widehat{m}+1)} > T_{\alpha_{\widehat{m}}} \backslash hH\} = Constant \quad (8)$$

P_{fc} \ $\hat{m}$	**0**	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
10^{-1}	**0.204**	0.100	0.085	0.072	0.062	0.054	0.048	0.043	0.038	0.034	0.031	0.029	0.026	0.024	0.022	0.021	0.019	0.017	0.016	0.015	0.014	0.013	0.012
10^{-2}	**0.821**	0.010	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.008	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
10^{-3}	**0.978**	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Table 1. P_{ee} and P_{oe} in a homogeneous environment ($P_{ue} = 0$) for $N = 24$, and $\beta = 3$ with P_{fc} as a parameter.

Since analytical expression of the pdf of $T_{\alpha_{\hat{m}}}$ is not available, $\alpha_{\hat{m}}$ is obtained using Monte Carlo simulations.

The successive tests, starting by $\hat{m} = 2$, are repeated until the environment is declared non-homogeneous, i.e., non- homogeneous hypothesis, *nhH*, or when all the $N - 2$ highest samples are used up and, thus, $\hat{m}$ is set equal to 0 and the clutter edge is declared absent, i.e., homogeneous hypothesis, *hH*. To do this, consider a binary indicator d, which is set to '1' to indicate a *nhH*, and to '0', to designate a *hH*. The censoring algorithm starts by setting $d = 0$. Two scenarios can be taken into consideration. The first one considers a transition located before the CUT ($\hat{m} \in [2, N/2[$). In this case, in order to avoid any capture effect, the lowest $\hat{m}$ cells are discarded. The second considers a transition located after the CUT ($\hat{m} \in [N/2, N-1]$). In this case, in order to avoid any masking effect, the $N - \hat{m}$ highest cells are discarded. Moreover, when $\hat{m} = N/2$, the clutter transition may occur in either the CUT or in the $(N/2 + 1)^{th}$ cell. As the censoring algorithm deals only with the reference window and does not care about any clutter transition within the CUT, here again, the $N - \hat{m}$ highest reference cells are discarded.

We now describe in brief the censoring algorithm.

```
Set m̂ = 2 and d = 0
While m̂ ≤ (N − 1) and d = 0
        Select α_m̂ to satisfy design P_fc and compute T_α_m̂
        Perform the heterogeneity test
        If nhH; Set d = 1
        Else; m̂ = m̂ + 1
        End
End
If m̂ = N; Set m̂ = 0; End
Output m̂
```

Let $m \in [0, N-1]$ represent the unknown number of cells embedded in the clear, and $\hat{m}$ its estimated number. $m = 0$ corresponds to a homogeneous background. Note that if $m = 1$, $\hat{m} = 1$ cannot be obtained. This is because the Weber-Haykin threshold, needs at least two cells to estimate the background level. However, for large reference window, $m = 1$ is assimilated to $m = 0$; i.e., both cases represent homogeneous environment. Let P_{ee} denote the probability of estimating exactly the unknown number of cells located before the clutter transition. Then

$$P_{ee} = Prob\{\hat{m} = m\} = f_{\hat{m}}(m)\ ,\ m \in \Im \qquad (9)$$

where $\hat{m}$ is a discrete random variable having values in $\Im = \{0, 2, \ldots, N-1\}$, a pdf $f_{\hat{m}}(m)$ and a cumulative density function (cdf) of $\hat{m}$, $F_{\hat{m}}(m)$, is given by [1]

$$F_{\hat{m}}(m) = Prob\ \{\hat{m} \le m\} = \sum_{u \le m} f_{\hat{m}}(u),\ m \in \Im \qquad (10)$$

Since we could not find any closed form expression for $f_{\hat{m}}(m)$, we resorted to Monte Carlo simulations to determine P_{ee}.

The estimation of m may also lead to under-estimation ($\hat{m} < m$) or over-estimation ($\hat{m} > m$). That is, when the transition is located before the CUT, under-estimation engenders capture effect, and over-estimation leads to a decrease in the detection probability. On the other hand, when the transition is located after the CUT, under-estimation decreases the detection probability and over-estimation engenders masking effect. Finally, in a homogeneous environment, the over-estimation degrades the detection performance. Thus, both of them are undesirable properties. To quantify these hitches, we usually represent them through their respective probabilities of under-estimation (P_{ue}) and over-estimation (P_{oe}) such that

$$P_{ue} + P_{ee} + P_{oe} = 1 \qquad (11)$$

The probability of under-estimation is given by

$$P_{ue} = Prob\{\hat{m} < m\} \qquad (12)$$

The probability of over-estimation is written, from (9), (10), (11) and (12), as

$$P_{oe} = Prob\{\hat{m} > m\} = 1 - F_{\hat{m}}(m) \qquad (13)$$

Thus, P_{oe} is the complementary cdf of $\hat{m}$. Note that, P_{ue} and P_{oe} are also determined through Monte Carlo simulations.

3. SIMULATION RESULTS

In this Section, we evaluate the performance of the ACWH-CFCAR detector, in Weibull clutter, through a battery of simulation tests. We deal with a single-pulse detection, which corresponds to both Swerling I and Swerling II fluctuating target models. We assume that a clutter edge may be present in the reference window. All censoring and detection curves are obtained after 10,000 Monte Carlo runs. Also, in computing the detection probability, we need to define the Signal-to-Clutter Ratio (SCR). In addition, since the clutter power is assumed to be much greater than the thermal noise power, the presence of a clutter edge refers to a Clutter-to-Clutter Ratio (CCR). A clutter transition may be

P_{fc} \ $\hat{m}$	0	2	3	4	5	**6**	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
10^{-1}	0.091	0.105	0.097	0.096	0.103	**0.125**	0.116	0.087	0.058	0.036	0.023	0.016	0.011	0.008	0.006	0.005	0.004	0.003	0.003	0.003	0.002	0.002	0.001
	0	0.107	0.104	0.113	0.155	**0.520**	0.002	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10^{-2}	0.720	0.011	0.012	0.014	0.018	**0.036**	0.045	0.041	0.031	0.022	0.015	0.010	0.007	0.005	0.004	0.003	0.002	0.002	0.002	0.001	0.001	0.001	0.001
	0.009	0.011	0.012	0.017	0.032	**0.909**	0.010	0.000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10^{-3}	0.957	0.001	0.001	0.002	0.002	**0.005**	0.008	0.007	0.006	0.004	0.003	0.002	0.001	0.001	0	0	0	0	0	0	0	0	0
	0.174	0.001	0.001	0.002	0.004	**0.810**	0.008	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2. P_{ee}, P_{ue} and P_{oe} in the presence of a clutter edge before the CUT ($m = 6$) for $N = 24$, and $\beta = 3$, with P_{fc} as a parameter. First row CCR=10 dB and second row CCR=30 dB.

P_{fc} \ $\hat{m}$	0	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	**18**	19	20	21	22	23
10^{-1}	0.003	0.100	0.086	0.074	0.064	0.058	0.051	0.047	0.043	0.040	0.038	0.037	0.036	0.036	0.038	0.041	0.050	**0.074**	0.050	0.022	0.008	0.003	0.001
	0	0.101	0.086	0.075	0.065	0.058	0.052	0.048	0.044	0.041	0.039	0.038	0.038	0.039	0.042	0.048	0.064	**0.124**	0	0	0	0	0
10^{-2}	0.171	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.012	0.013	0.014	0.017	0.022	0.031	0.057	**0.217**	0.192	0.102	0.042	0.015	0.004
	0	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.011	0.012	0.014	0.016	0.019	0.026	0.041	0.092	**0.687**	0.001	0	0	0	0
10^{-3}	0.542	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.004	0.007	0.017	**0.154**	0.145	0.075	0.028	0.009	0.002
	0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.003	0.005	0.010	0.031	**0.933**	0.002	0	0	0	0

Table 3. P_{ee}, P_{ue} and P_{oe} in the presence of a clutter edge after the CUT ($m = 18$) for $N = 24$, and $\beta = 3$, with P_{fc} as a parameter. First row CCR=10 dB and second row CCR=30 dB

inherent to different shape parameters or different clutter powers. Although not shown in this paper, this would not affect the proposed algorithm. In all forthcoming experiments, the design false alarm probability is set to $P_{fa} = 10^{-4}$. The reference window is assumed to be of size $N = 24$. For comparison purposes, the design false censoring probability is set to $P_{fc} = 10^{-1}$, 10^{-2} and 10^{-3}, and the shape parameter of the Weibull distribution is set to $\sigma = 0.355$ and 0.7. As it would be tedious to exhaustively simulate the behavior of the detector with respect to each parameter taken apart, our attention is focused on $m = 0$, 6 and 18, SCR and CCR with an emphasis on P_{fc}.

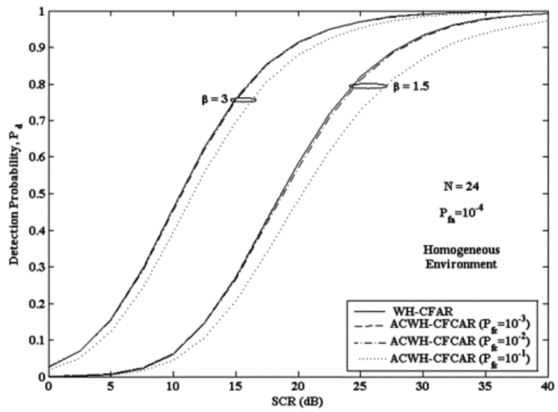
3.1. Evaluation of the Censoring Probabilities

For a homogeneous environment, Table 1 represents P_{ee} and P_{oe}, for $N = 24$, $\beta = 3$, with P_{fc} as a parameter. The largest values are attained for $\hat{m} = 0$; which corresponds to P_{ee}. This is exactly what we would need from a censoring algorithm in a homogenous situation. Also, as P_{fc} increases, the censoring algorithm behaves as if there were a heterogeneity; yielding a decrease in P_{ee}. Finally, although not shown, values of $\beta > 0.01$ have no effect on the censoring algorithm. Tables 2 and 3 show P_{ee}, P_{ue} and P_{oe} in the presence of a clutter edge before ($m = 6$) and after ($m = 18$) the CUT, respectively, at CCR=10 and 30 dB, for $N = 24$, and $\beta = 3$, with P_{fc} as a parameter. In this case, the censoring algorithm still remains robust by outputting, for high CCR and/or low P_{fc}, the largest values of P_{ee}. That is, there should be a trade-off in choosing P_{fc} so that the performances of the ACWH-CFCAR detector become comparable to those of the corresponding WH-CFAR detector. Also, although not shown, as β decreases, P_{ee} decreases in a clutter edge environment.

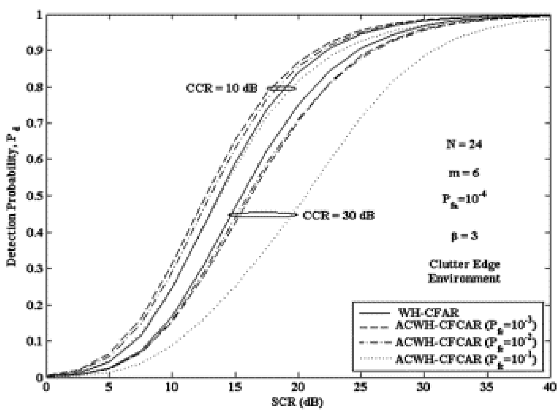
3.2. Evaluation of the Detection Probability

In this section, we compare the detection performances of the proposed ACWH-CFCAR detector and the corresponding fixed-point censoring WH-CFAR detector in both homogeneous and clutter edged environments.

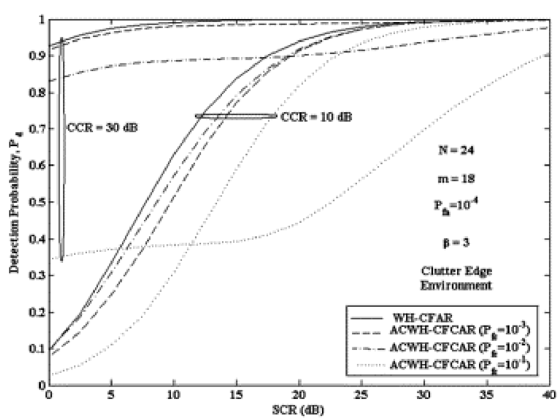
For a homogeneous environment, Fig. 2a shows the detection probability of the ACWH-CFCAR and the WH-CFAR detectors against SCR for $N = 24$, $P_{fa} = 10^{-4}$, with β and P_{fc} as parameters. Note that, for $P_{fc} = 10^{-2}$ and 10^{-3}, independently of β, both detectors exhibit the same detection probability. However, for $P_{fc} = 10^{-1}$, the $\text{CFAR}_{\text{Loss}}$ increases significantly for small values of β and high values of SCR. This is due to the fact that when P_{fc} increases, P_{oe} increases ($P_{oe} + P_{ee} = 1$). Consequently, the proposed detector should be calibrated in a homogeneous environment to a value of P_{fc} that ensures a detection performance comparable to that of the WH-CFAR detector. Fig. 2b and Fig. 2c illustrate the detection probability against SCR of the ACWH-CFCAR and WH-CFAR detectors in the presence of a clutter edge before ($m = 6$) and after ($m = 18$) the CUT, respectively, for $N = 24$, $\beta = 3$, $P_{fa} = 10^{-4}$, with P_{fc} and CCR as parameters. Note that, independently of P_{fc}, when a clutter edge is located before the CUT, an increase in the CCR causes a decrease in the detection performance of both detectors. On the contrary, when the clutter edge is located after the CUT, an increase in the CCR engenders an important increase in the detection performance of both detectors.



(a) Homogeneous environment



(b) Clutter edge at m=6



(c) Clutter edge at m=18

Fig. 2. Detection probabilities against SCR of the ACWH-CFCAR and WH-CFAR detectors.

Note that each ellipse refers to a set of curves having the same CCR. On the other hand, for $P_{fc} = 10^{-2}$, the ACWH-CFCAR and the WH-CFAR detectors give nearly the same detection probability. Finally, as the paper length constraint does not allow us to put more curves, note that lower values of β, i.e., spiky clutter, would yield degradation in the detection probability.

4. CONCLUSION

In this paper, we have analyzed and evaluated through extensive Monte Carlo simulations the censoring and detection performances of the ACWH-CFCAR detector in Weibull clutter when no prior knowledge is made available about the presence or not of a clutter edge in the reference window. We have also compared its detection performance with that of the corresponding fixed-point censoring WH-CFAR detector. While doing this, we have shown that the proposed detector should be first calibrated in a homogeneous environment to a value of P_{fc} which ensures a detection performance comparable to that of the WH-CFAR detector in both homogeneous and clutter edged environments.

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