ON ESTIMATION ERROR OUTAGE FOR SCALAR GAUSS-MARKOV PROCESSES SENT OVER FADING CHANNELS

Reza Parseh and Kimmo Kansanen

Department of Electronics and Telecommunications
Norwegian University of Science and Technology
O.S. Bragstads plass 2A, 7034 Trondheim, Norway
Email: {reza.parseh,kimmo.kansanen}@iet.ntnu.no

ABSTRACT

Measurements of a complex scalar linear Gauss-Markov process are sent over a fading channel. The fading channel is modeled as independent and identically distributed complex normal random variables with known realization at the decoder. The optimal estimator at the decoder is the Kalman filter with random instantaneous gain and error variance. To evaluate the quality of estimation at the receiver, the probability distribution function of the instantaneous estimation error variance and its outage probability are of interest. For the special case of the Rayleigh fading channels, upper and lower bounds for the outage probability are derived which provide insight and simple means for design purposes.

Index Terms— Estimation Over Fading Channels, Kalman Filter, Outage Probability

1. INTRODUCTION

Low or zero delay transmission of measurements of a dynamic process to a remote controller/observer has a significant role in applications such as network monitoring and control. Generally in real-time signal processing, it is required that the observed signals should be sent over a communication channel without delay. Due to this tight delay requirement, high-performance block-wise coding schemes which incur unacceptable delay should be avoided. For wireless fading channels, it is possible to send the measurements directly over the channel using uncoded transmission and then perform estimation on the channel outputs at the receiver. Analysis of the signal estimation quality is therefore necessary to ensure satisfactory performance.

If the dynamic process follows a linear Gauss-Markov model and the channel realization is independent of the randomness of the process, the optimal estimator is the Kalman filter ([1–3]). Due to the randomness of the fading channel, the Kalman filter is random and does not necessarily converge to a constant value. The instantaneous prediction and estimation error covariance matrices, which are measures of quality within the Kalman filter framework, are then random. It is known that the prediction error covariance matrices develop through a well-known second-order random Riccati equation (RRE). The estimation error covariance matrix also follows the prediction error covariance matrix by simple algebraic operations. Studying the RRE is one way to characterize statistical properties of the prediction error and estimation error covariance. In that line, the exponential stability of the RRE was considered in [4]. The peak covariance stability of the RRE along with boundedness of the error covariance matrix in the usual sense are considered in [5]. Probabilistic convergence of the sequence of random covariance matrices for Kalman filtering over erasure channels was studied in [6]. The stationary distributions for infinitely large random matrices for Riccati and Lyapunov equations was also studied in [7]. Bounds on the mean of the instantaneous covariance matrices in the RRE formulation are also obtained in [8].

In this paper we study the estimation error quality when measurements of a scalar Gauss-Markov signal are sent over a fading channel when channel realizations are assumed i.i.d. and known at the receiver only at the time of the observation. The optimum MMSE filter, i.e. the Kalman filter is then random and the exact value of the instantaneous estimation error variance (IEV) cannot be obtained in advance. In the spirit of outage analysis for fading channels, we incorporate estimation error outage as a criterion for estimation performance assessment. In other words, we study the event where the IEV exceeds a certain threshold. This measure could for instance be a design parameter from a control or real time monitoring system. The notion of end to end average distortion measure for uncoded transmission over fading channels was first introduced in [9] and further studied in [10–12]. Considering settings where delay is of concern, the distortion outage probability for MIMO block fading channels is considered in [13], where a transmitter informed bound for distortion outage probability is studied and it is shown that the source-channel separation achieves the same bound.

In this work and for Kalman estimation over fading channel, we try to find the outage probability and characterize how it is related to average channel quality under certain channel statistics. Due to lack of previous theory for finite dimension vector models, the scalar signal model was chosen as a starting point. We show that in this case, the outage measure takes a simple form for a certain range of outage thresholds, which we believe is insightful for design purposes and further development. We show that using i.i.d channel assumption, the first order pdf of the IEV may be obtained through a recursive integral equation. Using that, we obtain the outage probability for a certain range of thresholds corresponding to higher outage values. We show that for the Rayleigh fading channel model, the outage probability simplifies to a closed-form formula. For such channels, we also provide upper and lower bounds for the outage probability to further simplify evaluation of estimation accuracy.

2. SYSTEM MODEL AND PROBLEM DEFINITION

Consider the following scalar complex Gauss-Markov model.

$$\begin{aligned} &x(n) = \rho x(n-1) + u(n),\, n \geq 1,\, x(0) \sim \mathcal{CN}(0, M(0)) \\ &y(n) = h(n)x(n) + v(n) \end{aligned} \quad (1)$$

with $u(n)$ and $v(n)$ are white circularly symmetric complex Gaussian random variables with variances σ_u^2 and σ_v^2, respectively. Consider $h(n)$ to be a circularly symmetric complex Gaussian random variable. Note that we may call ρ as the correlation coefficient for the process $x(n)$. All the computations are performed in discrete time, and the communication method may be called uncoded analog [10]. The channel is also assumed known at the decoder, while being a realization of the random variable with a known probability density function. The objective at the decoder is optimal estimation of the signal, given the channel outputs.

Given the previous assumptions, and with $h(n)$ independent from $u(n)$ and $v(n)$, the optimal MMSE estimator of $x(n)$ based on the observations $y(n)$ is the well-known Kalman filter with the following prediction and estimation steps adapted from [14].

$$\begin{aligned} &\hat{x}(n|n-1) = \rho \hat{x}(n-1|n-1) \\ &M(n|n-1) = \rho^2 M(n-1|n-1) + \sigma_u^2 \\ &K(n) = M(n|n-1)h^*(n)[\sigma_v^2 + |h(n)|^2 M(n|n-1)]^{-1} \\ &\hat{x}(n|n) = \hat{x}(n|n-1) + K(n)(y(n) - h(n)\hat{x}(n|n-1)) \\ &M(n|n) = (I - K(n)h(n))M(n|n-1) \end{aligned} \quad (2)$$

Given the above equations, we can show that the instantaneous estimation error variance denoted by $M(n)$ (with abuse of notation instead of $M(n|n)$) can be written recursively in terms of its previous values and current value of $h(n)$, i.e. we have that

$$\begin{aligned} M(n) &= (I - K(n)h(n))M(n|n-1) \quad (3) \\ &= \left(I - \frac{M(n|n-1)|h(n)|^2}{\sigma_v^2 + h^2(n)M(n|n-1)} \right) M(n|n-1), \end{aligned}$$

which with necessary manipulations results in

$$M(n) = \frac{\rho^2 M(n-1) + \sigma_u^2}{1 + \gamma(n)\left(\rho^2 M(n-1) + \sigma_u^2\right)}. \quad (4)$$

Note that in (4), $\gamma(n) = |h(n)|^2/\sigma_v^2$ corresponds to the instantaneous channel SNR.

In order to characterize the random estimation outage event, we define estimation error outage probability (EOP) as

$$P_{\text{out}}^n(M_{\text{th}}) = \Pr(M(n) \geq M_{\text{th}}) \quad (5)$$

and in particular the asymptotic EOP which we are interested in, in order to characterize the steady-state distributions, i.e.

$$P_{\text{out}}(M_{\text{th}}) = \lim_{n\to\infty} P_{\text{out}}^n(M_{\text{th}}) = \lim_{n\to\infty} \Pr(M(n) \geq M_{\text{th}}) \quad (6)$$

Clearly $P_{\text{out}}^n(M_{\text{th}}) = 1 - F_{M(n)}(M_{\text{th}})$ and $P_{\text{out}}(M_{\text{th}}) = 1 - F_M(M_{\text{th}})$, where $F_{M(n)}(M)$ ($F_M(M)$) is the cumulative (asymptotic) distribution function of $M(n)$.

3. STATISTICAL PROPERTIES OF INSTANTANEOUS ESTIMATION ERROR VARIANCE

In this section we study the asymptotic probability distribution function of the IEV, i.e. $f_M(M)$. In that way, not only the EOP will readily be obtained with one integration over the pdf, other moments such as mean and variance, if needed, can be obtained or bounded.

3.1. Asymptotic pdf of the instantaneous estimation error variance

We begin by finding $\Pr(M(n) \leqslant M | M(n-1) = m)$.

$$\begin{aligned} &\Pr\left(M(n) \leqslant M | M(n-1) = m\right) \\ &= \Pr\Big(\frac{\rho^2 M(n-1) + \sigma_u^2}{1 + \gamma(n)\left(\rho^2 M(n-1) + \sigma_u^2\right)} \leqslant M | M(n-1) = m\Big) \\ &= \Pr\Big(\frac{1 + \gamma(n)\left(\rho^2 M(n-1) + \sigma_u^2\right)}{\rho^2 M(n-1) + \sigma_u^2} \geqslant \frac{1}{M} | M(n-1) = m\Big) \\ &= 1 - F_\gamma\Big(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\Big) \end{aligned} \quad (7)$$

where the cumulative distribution function of $\gamma(n)$ is given with $F_\gamma(\gamma)$. Clearly by (4), $M(n)$ is only a function of $M(n-1)$ and $\gamma(n)$, i.e. $M(n)$ is a first-order non-linear Markov process, where the transition probability depends on $\gamma(n)$. Then the cumulative distribution function of $M(n)$ conditioned on $M(n-1)$ is given in (7). If $\gamma(n)$ is independent of $M(n-1)$, and in addition $\gamma(n)$'s are independent and identically distributed (i.i.d), we get the following

$$F_{M(n)}(M) = \int_{m \in \mathcal{R}_M} \Pr\big(M(n) \leqslant M | M(n-1) = m\big) f_{M(n-1)}(m) dm. \quad (8)$$

In (8), $\mathcal{R}_M$ is the domain of integration over m (range of M) and may differ depending on different system parameters (exact value in Appendix A). Due to the fact that $\int_{m\in\mathcal{R}_M} f_{M(n-1)}(m)d(m) = 1$, then (8) leads to

$$\begin{aligned} F_{M(n)}(M) &= \\ &\int\limits_{m\in\mathcal{R}_M}\left(1 - F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 M(n-1)+\sigma_u^2}\right)\right) f_{M(n-1)}(m)dm \\ &= 1 - \int\limits_{m\in\mathcal{R}_M} F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 M(n-1)+\sigma_u^2}\right) f_{M(n-1)}(m)dm. \end{aligned} \tag{9}$$

Finally we get $\lim_{n\to\infty} F_{M(n)}(M) = \lim_{n\to\infty} F_{M(n-1)}(M) = F_M(M)$, and $\lim_{n\to\infty} f_{M(n)}(M) = \lim_{n\to\infty} f_{M(n-1)}(M) = f_M(M)$. As a result (9) can be rewritten as

$$F_M(M) = 1 - \int\limits_{m\in\mathcal{R}_M} F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m+\sigma_u^2}\right) f_M(m)dm. \tag{10}$$

In order to get the required pdf, i.e. $f_M(M)$, we provide the following lemma that describes the asymptotic pdf of $M(n)$, i.e. $f_M(M)$ in terms of itself integrated with a kernel that is a function of the instantaneous channel SNR. Solving this equation leads to $f_M(M)$ and with one integration to $P_{\text{out}}(M_{\text{th}})$ for a specific M_{th}, which is the target.

Lemma 1: Asymptotic pdf of $M(n)$, namely $f_M(M)$ can be obtained from the following equation

$$f_M(M) = \begin{cases} \frac{1}{M^2}\int_0^{M_{\max}} f_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m+\sigma_u^2}\right) f_M(m)dm, & M \leqslant \sigma_u^2 \\ \frac{1}{M^2}\int_{\frac{M-\sigma_u^2}{\rho^2}}^{M_{\max}} f_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m+\sigma_u^2}\right) f_M(m)dm, & M > \sigma_u^2 \end{cases} \tag{11}$$

Proof: See Appendix A

Solving (11) then yields $f_M(M)$. We have now described the asymptotic pdf of $M(n)$ using the equations in (11). Hereafter, we focus on the important case of Rayleigh fading channels where $F_\gamma(\gamma) = \lambda e^{-\lambda\gamma}\mathcal{U}(\gamma)$. Note that with this definition, $\lambda = \frac{1}{E(\gamma(n))} = \sigma_v^2/E(|h(n)|^2)$, i.e. average stronger channels yield smaller values for λ and vice versa.

3.2. Bounds on outage probability of the instantaneous estimation error variance under Rayleigh fading channel

We can rewrite (11) given that channel is Rayleigh fading and obtain

$$f_M(M) = \begin{cases} \frac{\lambda}{M^2}\exp(\frac{-\lambda}{M})\int_0^{M_{\max}} Q(\lambda,m)dm, & M \leqslant \sigma_u^2 \\ \frac{\lambda}{M^2}\exp(\frac{-\lambda}{M})\int_{\frac{M-\sigma_u^2}{\rho^2}}^{M_{\max}} Q(\lambda,m)dm, & M > \sigma_u^2. \end{cases} \tag{12}$$

with $Q(\lambda,m) = \exp\left(\frac{\lambda}{\rho^2 m+\sigma_u^2}\right) f_M(m)dm$.

In order to get more insight, (12) can also be written as

$$f_M(M) = \begin{cases} \frac{\kappa\lambda}{M^2}\exp(\frac{-\lambda}{M}), & M \leqslant \sigma_u^2 \\ \frac{\kappa\lambda}{M^2}\exp(\frac{-\lambda}{M}) - \frac{\lambda}{M^2}\exp(\frac{-\lambda}{M})\times \\ \int_0^{\frac{M-\sigma_u^2}{\rho^2}} \exp\left(\frac{\lambda}{\rho^2 m+\sigma_u^2}\right) f_M(m)dm, & M > \sigma_u^2, \end{cases} \tag{13}$$

where

$$\kappa = \int_0^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m+\sigma_u^2}\right) f_M(m)dm. \tag{14}$$

Though in general κ depends on the pdf itself, (13) is insightful in the sense that it shows the exact shape of the pdf for the first part where $M \leqslant \sigma_u^2$. The point $M = \sigma_u^2$ corresponds to the steady-state covariance of the signal $x(n)$ for $\rho = 0$, while $M_{\max}$ corresponds to the upper limit value of the IEV (happens when $\gamma(0) = 0$). It can be shown that for high SNR, the pdf tail vanishes after the break point. Therefore getting bounds on the first part helps with understanding the pdf behavior and at the same time getting approximate values and bounds for P_{out}. In the following and using (13), we find upper and lower bounds for κ and through that, upper and lower bounds for P_{out} for $M \leqslant \sigma_u^2$. Note that in practice, $M \leqslant \sigma_u^2$ corresponds to higher values EOP and thus more important to characterize.

Though the pdf is given by the equation $f_M(M) = \frac{\kappa\lambda}{M^2}\exp(\frac{-\lambda}{M})\,(M \leqslant \sigma_u^2)$, the exact value of κ depends on the whole pdf and cannot be known without solving (13). However, it is possible to obtain the following bounds for κ, namely $\kappa_l < \kappa < \kappa_u$.

Lemma 2: For all $M \leqslant \sigma_u^2$, we have $\kappa_l < \kappa < \kappa_u$, where

$$\kappa_u = \frac{1}{\left(a_\kappa \exp\left(\frac{-\lambda}{\sigma_u^2(1+\rho^2)}\right) + \exp(-\frac{\lambda}{\sigma_u^2})\right)} \tag{15}$$

$$\kappa_l = \frac{1}{\left(a_\kappa \exp(\frac{-\lambda}{\rho^2 M_{\max}+\sigma_u^2}) + \exp(-\frac{\lambda}{\sigma_u^2})\right)} \tag{16}$$

with a_κ defined as

$$a_\kappa = 1 - \int_0^{\sigma_u^2} \exp\left(\frac{\lambda}{\rho^2 m+\sigma_u^2}\right)(\frac{\lambda}{m^2})\exp(\frac{-\lambda}{m})dm \tag{17}$$

Proof: Using the fact that the $\int_0^{M_{\max}} f_M(m)dm = 1$ gives $\int_{\sigma_u^2}^{M_{\max}} f_M(m)dm = 1 - \kappa\exp(\frac{-\lambda}{\sigma_u^2})$. Next, one divides the domain of integral in (14) into $m \leqslant \sigma_u^2$ and $m > \sigma_u^2$ and uses the definition of $f_M(M)$ for the first part. This results in

$$\kappa = \frac{\int_{\sigma_u^2}^{M_{\max}} \exp(\frac{\lambda}{\rho^2 m+\sigma_u^2}) f_M(m)dm}{1 - \int_0^{\sigma_u^2} \exp(\frac{\lambda}{\rho^2 m+\sigma_u^2})\left(\frac{\lambda}{m^2}\right)\exp(\frac{-\lambda}{m})dm} \tag{18}$$

Considering that $\sigma_u^2 \leqslant \rho^2 m + \sigma_u^2 \leqslant \rho^2 M_{\max} + \sigma^2$ and applying it to the two equations involving κ and doing the necessary algebraic manipulations, gives the required result.

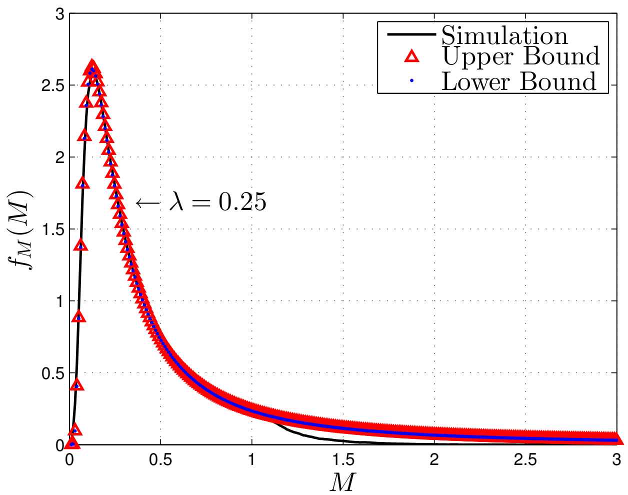

Fig. 1. Asymptotic pdf of $M(n)$ and its estimates using the upper and lower bounds for κ with $\sigma_u^2 = \sigma_v^2 = 1$, $\rho = 0.95$, $\lambda = 0.25$ (Average SNR = 6 dB).

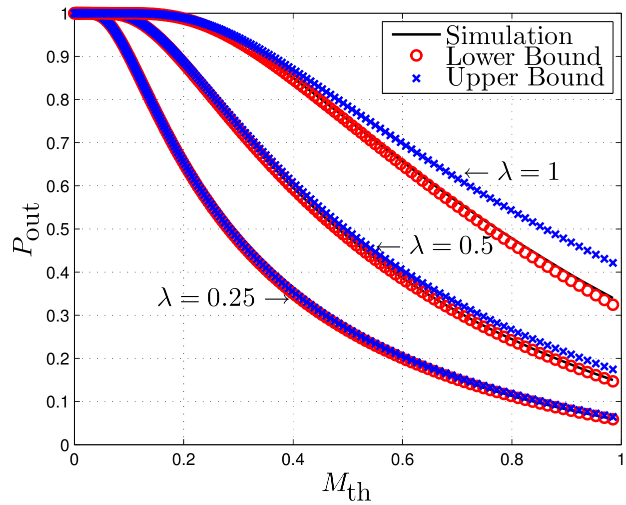

Fig. 2. P_{out} and the upper and lower bounds with $\sigma_u^2 = \sigma_v^2 = 1$, $\rho = 0.95$, $\lambda = 1, 0.5, 0.25$ (Average SNR = 0, 3, 6 dB respectively).

To show how close the bounds for κ are, we have plotted the pdf from simulation and also two approximations using the bounds for κ in Fig. 1 for $\sigma_u^2 = \sigma_v^2 = 1$, $\lambda = 0.25$, and $\rho = 0.95$. Note that the bounds are only guaranteed to hold for $M \leqslant \sigma_u^2$. As a result, one could see that the lower bound does not perform well for $M > \sigma_u^2$ in Fig. 1. It can also be shown that the gap between the bounds and the actual values decreases as SNR goes to infinity, but proof is omitted due to lack of space. With Lemma 2 at hand, we are now ready to present upper and lower bounds for P_{out}. As defined before, P_{out} is given by

$$P_{\text{out}} = \int_{M_{\text{th}}}^{M_{\text{max}}} f_M(M) dM \tag{19}$$

For $M \leqslant \sigma_u^2$, we get

$$P_{\text{out}}(M_{\text{th}}) = \int_{M_{\text{th}}}^{M_{\text{max}}} \frac{\kappa\lambda}{M^2} \text{exp}(\frac{-\lambda}{M}) dM = 1 - \kappa \text{exp}(\frac{-\lambda}{M_{\text{th}}}) \tag{20}$$

As we showed in the previous section, $\kappa_l < \kappa < \kappa_u$. As a result, we get

$$1 - \kappa_u \text{exp}(\frac{-\lambda}{M_{\text{th}}}) < P_{\text{out}}(M_{\text{th}}) < 1 - \kappa_l \text{exp}(\frac{-\lambda}{M_{\text{th}}}) \tag{21}$$

which gives us an upper bound and a lower bound for $P_{\text{out}}(M_{\text{th}})$. Figure 2 depicts the outage probability and the bounds for the case when $\sigma_u^2 = \sigma_v^2 = 1$, $\lambda = 1, 0.5, 0.25$, and $\rho = 0.95$ and for $M \leqslant \sigma_u^2$. As it can be seen in the figure, the bounds are quite good for the aforementioned range and they become tighter as the channel SNR increases (λ decreases).

4. CONCLUSIONS

In this paper, a recursive integral equation approach was presented for finding the pdf of the instantaneous estimation error variance for MMSE estimation of scalar Gauss-Markov signals sent over fading channels. We incorporated the notion of estimation error outage as a means of characterizing the estimation accuracy. It was shown that the pdf can be written as a two-part function over the domain of instantaneous estimation error variance values. The first part of the pdf, which correspond to higher outage probabilities, follows a closed-form non-recursive equation. As a result and for the case of Rayleigh fading channels, the outage probability can be approximated with a closed-form formula for the first part. Upper and lower bounds on the estimation error outage probability were also obtained to simplify characterization of estimation error outage. The presented bounds become visibly tight as the SNR increases.

A. ASYMPTOTIC PDF FOR EOP

In order to get $f_M(M)$, we have

$$\begin{aligned} & f_M(M) \\ & = \frac{\partial}{\partial M}\left[1 - \int\limits_{m \in \mathcal{R}_M} F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm\right] \\ & = -\frac{\partial}{\partial M} \int\limits_{m \in \mathcal{R}_M} F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(y) dm \end{aligned} \tag{22}$$

Note that $P(n) \geqslant 0$, and therefore $M, m \geqslant 0$. Also $\gamma(n) = |h(n)|^2/\sigma_v^2 \geqslant 0$, and therefore we should have $\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2} \geqslant 0$, which results in $m \geqslant \frac{M-\sigma_u^2}{\rho^2}$. Therefore

$$\mathcal{R}_M = [\max\{0, \frac{M-\sigma_u^2}{\rho^2}\}, M_{\max}], \tag{23}$$

or

$$\mathcal{R}_M = \begin{cases} 0 \leqslant m < M_{\max}, & M \leqslant \sigma_u^2 \\ \frac{M-\sigma_u^2}{\rho^2} \leqslant m < M_{\max}, & M > \sigma_u^2, \end{cases} \tag{24}$$

where

$$M_{\max} = \begin{cases} \infty, & |\rho| \geqslant 1 \\ \frac{\sigma_u^2}{1-\rho^2}, & |\rho| < 1. \end{cases} \tag{25}$$

Now if $M \leqslant \sigma_u^2$ (for any ρ), we get

$$\begin{aligned} f_M(M) &= -\frac{\partial}{\partial M}\int_0^{M_{\max}} F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(m)dm \\ &= \frac{1}{M^2}\int_0^{M_{\max}} f_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(m)dm \end{aligned}$$

due to the Leibnitz's differentiation law.

Now we take $M > \sigma_u^2$ and we get

$$\begin{aligned} f_M(M) &= -\frac{\partial}{\partial M}\int_{\frac{M-\sigma_u^2}{\rho^2}}^{M_{\max}} F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(m)dm \\ &= -\int_{\frac{M-\sigma_u^2}{\rho^2}}^{M_{\max}} \frac{\partial}{\partial M} F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(m)dm \\ &+ \frac{\partial}{\partial M}\frac{M-\sigma_u^2}{\rho^2} F_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(m)\Bigg|_{m=\frac{M-\sigma_u^2}{\rho^2}} \\ &= \frac{1}{M^2}\int_{\frac{M-\sigma_u^2}{\rho^2}}^{M_{\max}} f_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(m)dm \\ &+ \frac{1}{\rho^2} F_\gamma(0) f_M(\frac{M-\sigma_u^2}{\rho^2}). \end{aligned} \tag{26}$$

Because $F_\gamma()$ is a CDF, then $F_\gamma(0) = 0$, therefore

$$f_M(M) = \frac{1}{M^2}\int_{\frac{M-\sigma_u^2}{\rho^2}}^{M_{\max}} f_\gamma\left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}\right) f_M(m)dm \tag{27}$$

and the proof is complete.

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