NONLINEAR DISTORTION REDUCTION FOR ELECTRODYNAMIC LOUDSPEAKER USING NONLINEAR FILTERING

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ABSTRACT

In this paper, we compare the efficiency of compensating nonlinear distortions in electrodynamic loudspeaker system using 2nd- and 3rd-order nonlinear IIR filters. These filters need nonlinear parameters of loudspeaker systems and we used estimated nonlinear parameters for evaluating the efficiency of compensating nonlinear distortions of these filters. Therefore, these evaluation results include the effect of the parameter estimation method. In this paper, we measure the nonlinear parameters using Klippel's measurement equipment and evaluate the compensation amount of both filters. Experimental results demonstrate that the 3rd-order nonlinear IIR filter can realize a reduction by 4dB more than the 2nd-order nonlinear IIR filter on nonlinear distortions at high frequencies.

Index Terms— Loudspeaker system, Nonlinear distortion, Nonlinear IIR filter, Mirror filter

1. INTRODUCTION

Loudspeaker cannot keep the proportional relationship between the velocity of the diaphragm and sound pressure. This is because of the nonlinearity of the voice coil driving system and the mechanical nonlinearity of the edge and damper that support the diaphragm [1]. These distortions lead to the degradation of sound quality and some researchers have attempted to compensate nonlinear distortions by nonlinear digital signal processing, for example, Volterra filter-based compensator [2–7].

One interesting approach to compensating nonlinear distortions is to employ the mirror filter [8, 9]. This filter is derived from the nonlinear differential equation for loudspeaker systems and includes the nonlinearities of the force factor and stiffness of loudspeaker systems. This filter can be realized as a 2nd-order nonlinear IIR filter [10]. Hence, the computational complexity of this filter is lower than that of a Volterra filter-based compensator. However, it cannot compensate nonlinear distortions at high frequencies because this filter does not take into account the nonlinearity of self-inductance, which is a dominant component at high frequencies. In [11], 3rd-order nonlinear IIR filter is proposed to compensate the nonlinear distortions at high frequencies. This filter takes into account the nonlinearity of self-inductance of loudspeaker systems and has 3rd-order IIR filter structure. Hence, this filter can reduce nonlinear distortions at high frequencies while maintaining a lower computational complexity than that of a Volterra filter-based compensator. These filters need nonlinear parameters of loudspeaker systems and we used estimated nonlinear parameters for evaluating the efficiency of compensating nonlinear distortions of these filters. Therefore, these evaluation results include the effect of the parameter estimation method.

In this paper, we measure the nonlinear parameters using Klippel's measurement equipment and evaluate the compensation amount of both filters. Experimental results demonstrate that the 3rd-order nonlinear IIR filter can realize 4dB more reduction of nonlinear distortions at high frequencies than the 2nd-order nonlinear IIR filter.

2. LINEARIZATION OF LOUDSPEAKER SYSTEM USING THIRD-ORDER NONLINEAR IIR FILTER [11]

The 2nd- and 3rd-order nonlinear IIR filters are based on mirror filter. Both the 2nd- and 3rd-order nonlinear IIR filters and mirror filter are governed by the same fundamental equations, which are linear and nonlinear differential equations for electrodynamic loudspeaker system. The difference between the both nonlinear IIR filter and mirror filter is calculation of the filter coefficients. The coefficients of the mirror filter are directly calculated from linear and nonlinear parameters of the loudspeaker. The values of them are in the different ranges and have different units. Hence, the mirror filter cannot be easily implemented in any processor, especially fixed-point processors. On the other hand, the values of the coefficients of both nonlinear IIR filters are in the same number of places and the filters can be easily implemented into any processors (e.g. DSP). In this section, the design of the 3rd-order nonlinear IIR filter is introduced. This is the same concept of the designing of the 2nd-order nonlinear IIR filter.

The concept of the design of 3rd-order nonlinear IIR filter has two steps:

1. Realizing the linear motions (displacement x, velocity v, acceleration a, and jerk j).
2. Obtaining the voltage for realizing the expected linear motions in the real loudspeaker system. This voltage is called the compensation signal.

For realizing the linear motions, the motion equation and Kirchhoff's voltage law (KVL) with linear parameters [12,13] are utilized, which are given by

$$Bl_0 i(t) = m_0 \frac{d^2x(t)}{dt^2} + K_0 x(t) + R_m \frac{dx(t)}{dt}, \quad (1)$$

$$A_0 u(t) = R_e i(t) + Bl_0 \frac{dx(t)}{dt} + L_0 \frac{di(t)}{dt}, \quad (2)$$

where $u(t)$ is the input voltage, $x(t)$ is the displacement of the diaphragm, A_0 is the gain of the analogue part, R_e is the electrical resistance of the voice coil, m_0 is the mechanical mass, R_m is the mechanical resistance, K_0 is the stiffness, Bl_0 is the force factor, and L_0 is the self-inductance. In this case, the displacement of the diaphragm $x(t)$ and its derivatives $v(t) = dx(t)/dt$, $a(t) = d^2x(t)/dt^2$ and $j(t) = d^3x(t)/dt^3$ do not exhibit nonlinearity. From these equations, the linear motions $x(n)$, $v(n)$, $a(n)$, $j(n)$ at a discrete time are given by

$$x(n) = G_0\mathcal{Z}^{-1}\left[H_x(z)\right] * u(n), \tag{3}$$
$$v(n) = G_0\mathcal{Z}^{-1}\left[H_v(z)\right] * u(n), \tag{4}$$
$$a(n) = G_0\mathcal{Z}^{-1}\left[H_a(z)\right] * u(n), \tag{5}$$
$$j(n) = G_0\mathcal{Z}^{-1}\left[H_j(z)\right] * u(n), \tag{6}$$
$$H_x(z) = \frac{h_{x0,P}+h_{x1,P}z^{-1}+h_{x2,P}z^{-2}+h_{x3,P}z^{-3}}{1+B_{1,P}z^{-1}+B_{2,P}z^{-2}+B_{3,P}z^{-3}}, \tag{7}$$
$$H_v(z) = \frac{h_{v0,P}+h_{v1,P}z^{-1}+h_{v2,P}z^{-2}+h_{v3,P}z^{-3}}{1+B_{1,P}z^{-1}+B_{2,P}z^{-2}+B_{3,P}z^{-3}}, \tag{8}$$
$$H_a(z) = \frac{h_{a0,P}+h_{a1,P}z^{-1}+h_{a2,P}z^{-2}+h_{a3,P}z^{-3}}{1+B_{1,P}z^{-1}+B_{2,P}z^{-2}+B_{3,P}z^{-3}}, \tag{9}$$
$$H_j(z) = \frac{h_{j0,P}+h_{j1,P}z^{-1}+h_{j2,P}z^{-2}+h_{j3,P}z^{-3}}{1+B_{1,P}z^{-1}+B_{2,P}z^{-2}+B_{3,P}z^{-3}}, \tag{10}$$

where $*$ is the convolution operator, $\mathcal{Z}^{-1}$ is the inverse Z transform operator,

$$\begin{aligned}
h_{x0,P} &= \frac{h_{x1,P}}{3} = \frac{h_{x2,P}}{3} = h_{x3,P} = \frac{1}{4f_s^2}\Big/\alpha_P,\\
h_{v0,P} &= h_{v1,P} = -h_{v2,P} = -h_{v3,P} = \frac{1}{2f_s}\Big/\alpha_P,\\
h_{a0,P} &= -h_{a1,P} = -h_{a2,P} = h_{a3,P} = 1/\alpha_P,\\
h_{j0,P} &= -\frac{h_{j1,P}}{3} = \frac{h_{j2,P}}{3} = -h_{j3,P} = 2f_s/\alpha_P,\\
B_{1,P} &= \left(-1 + \frac{\omega_0}{2Q_0f_s} + 3\frac{\omega_0^2}{4f_s^2}\right)\Big/\alpha_P\\
&\quad + \frac{2\tau}{T_s}\left(-3 - \frac{\omega_0}{2Q_mf_s} + \frac{\omega_0^2}{4f_s^2}\right)\Big/\alpha_P,\\
B_{2,P} &= \left(-1 - \frac{\omega_0}{2Q_0f_s} + 3\frac{\omega_0^2}{4f_s^2}\right)\Big/\alpha_P\\
&\quad + \frac{2\tau}{T_s}\left(3 - \frac{\omega_0}{2Q_mf_s} - \frac{\omega_0^2}{4f_s^2}\right)\Big/\alpha_P,\\
B_{3,P} &= \left(1 - \frac{\omega_0}{2Q_0f_s} + \frac{\omega_0^2}{4f_s^2}\right)\Big/\alpha_P\\
&\quad + \frac{2\tau}{T_s}\left(-1 + \frac{\omega_0}{2Q_mf_s} - \frac{\omega_0^2}{4f_s^2}\right)\Big/\alpha_P.\\
\alpha_P &= \left(1 + \frac{\omega_0}{2Q_0f_s} + \frac{\omega_0^2}{4f_s^2}\right) + \frac{2\tau}{T_s}\left(1 + \frac{\omega_0}{2Q_mf_s} + \frac{\omega_0^2}{4f_s^2}\right),
\end{aligned}$$

where

$$G_0 = \frac{Bl_0A_0}{R_em_0}, \quad \omega_0 = \sqrt{\frac{K_0}{m_0}}, \quad Q_0 = \frac{\sqrt{m_0K_0}}{R_m + Bl_0^2/R_e},$$
$$Q_m = \frac{\sqrt{m_0K_0}}{R_m}, \quad \tau = \frac{L_0}{R_e},$$

with ω_0 being the lowest resonance frequency and Q_0 the sharpness of the resonance at ω_0, the time constant τ, and $T_s = 1/f_s$ is the sampling period. In the derivation of Eqs. (3) – (6), the bilinear transformation is utilized.

For obtaining the voltage to realize the expected linear motions $x(n)$, $v(n)$, $a(n)$, $j(n)$ in the real loudspeaker system, the nonlinear motion equation and KVL with linear and nonlinear parameters [14] are utilized, which are given by

$$Bl(x)i(t) = m_0\frac{d^2x(t)}{dt^2} + K(x)x(t) + R_m\frac{dx(t)}{dt} - \frac{i^2(t)}{2}\frac{dL(x)}{dx}, \tag{11}$$
$$A_0u(t) = R_ei(t) + Bl(x)\frac{dx(t)}{dt} + \frac{dL(x)i(t)}{dt}. \tag{12}$$

where

$$Bl(x) = Bl_0b(x) = Bl_0(1 + b_1x + b_2x^2), \tag{13}$$
$$K(x) = K_0k(x) = K_0(1 + k_1x + k_2x^2), \tag{14}$$
$$L(x) = L_0l(x) = L_0(1 + l_1x + l_2x^2 + l_3x^3). \tag{15}$$

However, it is impossible to derive the compensation signal from Eqs. (11) and (12) because Eq. (11) includes $i^2(t)$ and does not yield a single solution. Therefore, we consider only self-inductance. First, we focus on the 4th term of Eq. (11), which represents the effect of the nonlinearity of self-inductance. If the self-inductance is linear, i.e, $l_1 = l_2 = l_3 = 0$ in Eq. (15), Eq. (11) is rewritten as

$$Bl(x)i_L(t) = m_0\frac{d^2x(t)}{dt^2} + K(x)x(t) + R_m\frac{dx(t)}{dt}. \tag{16}$$

This $i_L(t)$ represents the current for linearizing the self-inductance. Then, Eqs. (11) and (12) are rewritten using $i_L(t)$ as

$$Bl(x)i(t) = m_0\frac{d^2x(t)}{dt^2} + K(x)x(t) + R_m\frac{dx(t)}{dt} - \frac{i_L^2(t)}{2}\frac{dL(x)}{dx}, \tag{17}$$
$$A_0u(t) = R_ei(t) + Bl(x)\frac{dx(t)}{dt} + \frac{dL(x)i_L(t)}{dt}. \tag{18}$$

From Eqs. (17) and (18), the compensation signal at a discrete time $u_{L,P}(n)$ is given by

$$\begin{aligned}
u_{L,P}(n) = \frac{1}{b\left(x(n)\right)}\Bigg[&\frac{a(n)}{G_0} + \omega_0^2k\left(x(n)\right)\frac{x(n)}{G_0}\\
&+ \left\{1 + \left(1 - \frac{Q_0}{Q_m}\right)\left(b^2(x(n)) - 1\right)\right\}\frac{\omega_0}{Q_0}\frac{v(n)}{G_0}\\
&+ \tau\left\{\Delta\left[l\left(x(n)\right)\right] - \frac{l\left(x(n)\right)}{b\left(x(n)\right)}\Delta\left[b\left(x(n)\right)\right]\right\}\\
&\quad\times\left\{\frac{a(n)}{G_0} + \frac{\omega_0}{Q_m}\frac{v(n)}{G_0} + \omega_0^2k\left(x(n)\right)\frac{x(n)}{G_0}\right\}\\
&+ \tau l\left(x(n)\right)\Bigg\{\frac{j(n)}{G_0} + \frac{\omega_0}{Q_m}\frac{a(n)}{G_0}\\
&\quad + \omega_0^2k\left(x(n)\right)\frac{v(n)}{G_0} + \omega_0^2\Delta\left[k\left(x(n)\right)\right]\frac{x(n)}{G_0}\Bigg\}\Bigg]\\
&+ G(x(n))\left\{\frac{a(n)}{G_0} + \frac{\omega_0}{Q_m}\frac{v(n)}{G_0} + \omega_0^2k\left(x(n)\right)\frac{x(n)}{G_0}\right\}^2,
\end{aligned} \tag{19}$$

$$G(x(n)) = -\frac{A_0\tau}{2Bl_0}\frac{1}{b^3(x(n))}\left\{l_1 + 2l_2x(n) + 3l_3x^2(n)\right\},$$

where $\Delta[\cdot]$ is the difference value given by

$$\Delta\left[f(x(n))\right] = \frac{f(x(n)) - f(x(n-1))}{T_s}.$$

The compensation signal that satisfies the linear displacement expressed by Eq. (3) can be realized with a 3rd-order nonlinear IIR

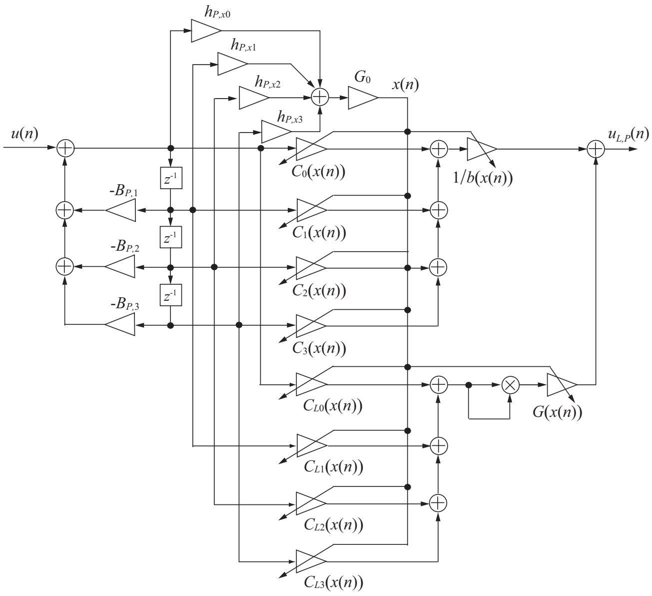


Fig. 1. Block diagram of 3rd-order nonlinear IIR filter [11].

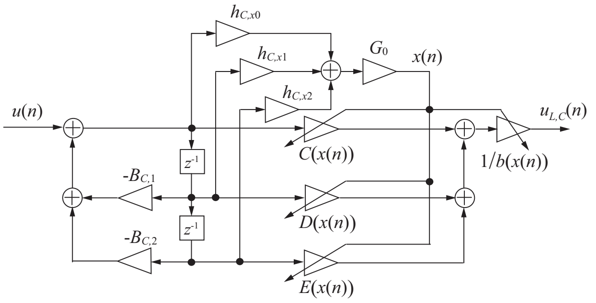


Fig. 2. Block diagram of 2nd-order nonlinear IIR filter [10].

filter as shown in Fig. 1 and can be derived by substituting the linear characteristics expressed by Eqs. (3)–(6) into Eq. (19). The coefficients in Fig. 1 are given by

$$
\begin{aligned}
C_i(x(n)) = {} & h_{ai,P} + \omega_0^2 k(x(n)) h_{xi,P} \\
& + \left\{ 1 + \left(1 - \frac{Q_0}{Q_m} \right) \left(b^2(x(n)) - 1 \right) \right\} \frac{\omega_0}{Q_0} h_{vi,P} \\
& + \tau \left\{ \Delta \left[l(x(n)) \right] - \frac{l(x(n))}{b(x(n))} \Delta \left[b(x(n)) \right] \right\} C_{Li}(x(n)) \\
& + \tau l(x(n)) \left\{ h_{ji,P} + \frac{\omega_0}{Q_m} h_{ai,P} + \omega_0^2 k(x(n)) h_{vi,P} \right. \\
& \left. + \omega_0^2 \Delta \left[k(x(n)) \right] h_{xi,P} \right\}, \\
C_{Li}(x(n)) = {} & h_{ai,P} + \frac{\omega_0}{Q_m} h_{vi,P} + \omega_0^2 k(x(n)) h_{xi,P}, \\
& (i = 0,\ 1,\ 2,\ 3).
\end{aligned}
$$

This filter generates the compensation signal in two steps. First, the linear displacement $x(n)$ is calculated. Next, the coefficients depending on the displacement $x(n)$ are calculated. These coefficients

Table 1. Specifications of experimental loudspeaker system.

Diameter	6.5cm
Rated power	6W
Electrical resistance	4Ω
Enclosure volume	$0.6l$
Enclosure type	Closed-box

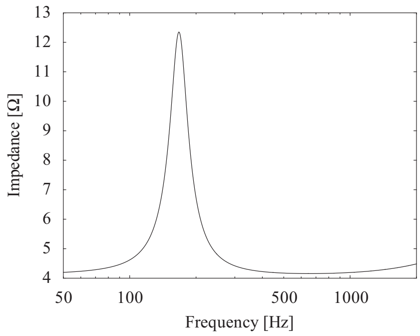


Fig. 3. Impedance characteristic of experimental loudspeaker system.

include the effects of the linear displacement, velocity, acceleration and jerk. If the self-inductance of loudspeaker systems is ignored, the 3rd-order nonlinear IIR filter reduces to the 2nd-order nonlinear IIR filter shown in Fig. 2. That is, the 3rd-order nonlinear IIR filter includes the conventional nonlinear IIR filter.

3. EXPERIMENTAL RESULTS

We conducted experiments on compensating the nonlinear distortions of a loudspeaker system. The specifications of the loudspeaker system and the impedance characteristic are shown in Table 1 and Fig. 3, respectively. It can be seen from Fig. 3 that the impedance starts to rise above 650Hz because of the effect of self-inductance. The 2nd- and 3rd-order nonlinear IIR filters require the linear and nonlinear parameters of the loudspeaker system. First, the linear and nonlinear parameters of the loudspeaker system are measured. Next, IIR filters are designed using the linear and nonlinear parameters. Finally, the compensation signal through the 2nd- and 3rd-order nonlinear IIR filter is generated then sent to the loudspeaker system through an amplifier, and the compensation performance is measured as the sound pressure level at a standard microphone.

3.1. Parameter Measurement

The linear and nonlinear parameters were measured by a measurement instrument made by Klippel GmbH in Germany. The measured linear and nonlinear parameters are given in Table 2 and by Eqs. (20)–(22), respectively.

$$Bl(x) = Bl_0(1 - 108x - 78800x^2), \tag{20}$$

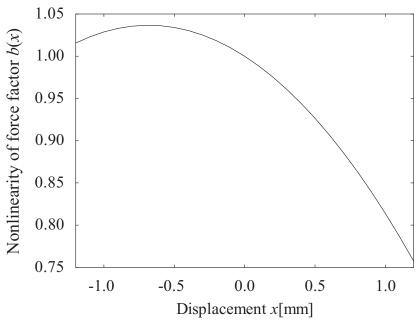
$$K(x) = K_0(1 - 38x + 17100x^2), \tag{21}$$

$$L(x) = L_0(1 - 217x + 5400x^2 + 1.11 \times 10^7 x^3). \tag{22}$$

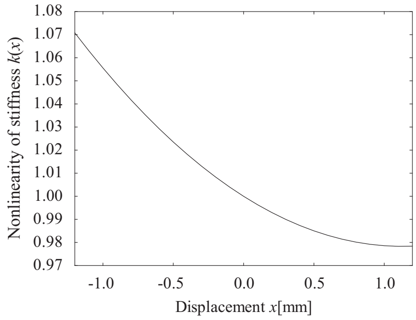
The relationships between the nonlinear parameters and the displacement of the diaphragm obtained from Eqs. (20)–(22) are shown in Fig. 4.

Table 2. Linear parameters of the loudspeaker system.

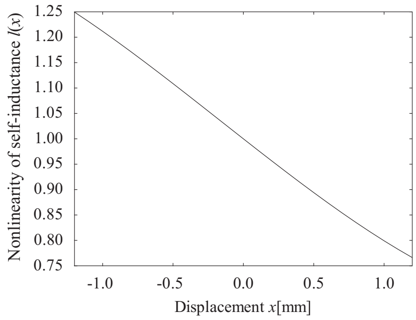
ω_0	1052rad/s
Q_0	1.78
Q_m	5.32
R_e	4.13Ω
R_m	0.62Ns/m
m_0	3.14×10^{-3}kg
K_0	3480N/m
Bl_0	2.26Wb/m
L_0	0.15mH



(a) Force factor



(b) Stiffness



(c) Self-inductance

Fig. 4. Relationships between nonlinear parameters and displacement of diaphragm.

3.2. Compensation Experiments

The 2nd- and 3rd-order nonlinear IIR filters are realized using the measured parameters, and their effectiveness in compensating the nonlinear distortions of the loudspeaker system is compared with each other. The measurement conditions are shown in Table 3.

Table 3. Measurement conditions used for compensating nonlinear distortions.

Input signal	Swept sinusoidal wave
Sampling frequency f_s	32000Hz
Fixed frequency m_1	62Hz
Sweep frequency m_2	30–4000Hz
Average	10
Input voltage	5.0V

Table 4. Comparison between average amounts of compensation of nonlinear distortion of 2nd- and 3rd-order nonlinear IIR filters.

	2nd-order	3rd-order
$2m_2$ characteristic		
30Hz–650Hz	2.6dB	2.2dB
650Hz–4000Hz	-0.6dB	5.8dB
$m_1 + m_2$ characteristic		
30Hz–650Hz	1.5dB	2.4dB
650Hz–4000Hz	1.7dB	4.3dB
$m_2 - m_1$ characteristic		
30Hz–650Hz	3.7dB	4.4dB
650Hz–4000Hz	2.7dB	4.7dB

The sound pressure characteristics of the nonlinear distortions are shown in Fig. 5, and the average amount of compensation of nonlinear distortions are shown in Table 4. As observed in Fig. 5 and Table 4, the 3rd-order nonlinear IIR filter can reduce the nonlinear distortions by about 2 to 5dB more than the 2nd-order nonlinear IIR filter at high frequencies. On the other hand, both filters can reduce the nonlinear distortions by the same amount at low frequencies.

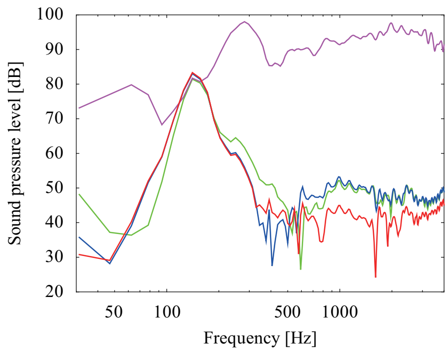
From these results, the 3rd-order nonlinear IIR filter is more effective for compensating the nonlinear distortions of loudspeaker systems than the 2nd-order nonlinear IIR filter.

3.3. Computational Complexity

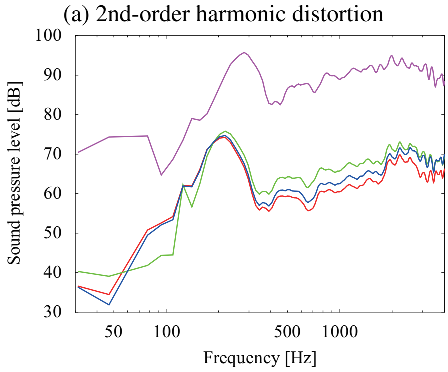
The 2nd- and 3rd-order nonlinear IIR filters have recursive structures. Therefore, their computational complexity is lower than that of Volterra filter-based compensators [2,5]. The 2nd-order nonlinear IIR filter requires 20 multiplications to generate the compensation signal. On the other hand, the number of multiplications required by the 3rd-order nonlinear IIR filter is 110, which is higher than that required by the 2nd-order nonlinear IIR filter. However, the number of multiplications required by the 3rd-order nonlinear IIR filter is much lower than that required by the Volterra filter-based system [5], which is 5829 when the filter length is 128. Hence, it is easy to implement the 3rd-order nonlinear IIR filter in DSP. The compensation results using Volterra filter-based compensator is omitted due to limitations of space.

4. CONCLUSIONS

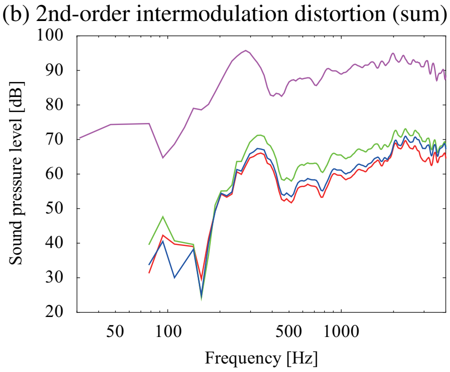
In this paper, we compare an compensation ability of 3rd-order nonlinear IIR filter and 2nd-order nonlinear IIR filter with linear and nonlinear parameters measured by Klippel's measurement equipment. The compensation performance characteristics of the 2nd-order and 3rd-order nonlinear IIR filters were compared through



(a) 2nd-order harmonic distortion



(b) 2nd-order intermodulation distortion (sum)



(c) 2nd-order intermodulation distortion (difference)

Linear characteristic
Before compensation
After compensation using 2nd-order filter
After compensation using 3rd-order filter

Fig. 5. Comparison between abilities of 2nd- and 3rd-order nonlinear IIR filters to compensate nonlinear distortions.

actual experiments. The experimental results indicated that the 3rd-order nonlinear IIR filter can reduce 2nd-order nonlinear distortions by a greater amount than the 2nd-order nonlinear IIR filter. Hence, we conclude that the 3rd-order nonlinear IIR filter is effective for compensating the nonlinear distortions of loudspeaker systems. In the future, we will develop a parameter estimation method to compensate nonlinear distortions effectively.

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