ON THE SEARCH OF MAKING THE MOST OF EVERY DSP OPERATION IN AN IN-VEHICLE ACTIVE NOISE CONTROL SYSTEM.

Pedro Ramos, Raúl Martín, Ana López, Ana Salinas, and Enrique Masgrau.

Communication Technologies Group. Aragon Institute for Engineering Research (I3A).
University of Zaragoza. Spain.

ABSTRACT

In this paper, we present an active noise control (ANC) strategy aimed at speeding up the convergence rate of an in-vehicle ANC system focused on the attenuation of periodic disturbances without significantly increasing its computational complexity. The basic idea is based on the combination of the hierarchical organization of the taps of a filter and their sequential partial update (PU). The inherent reduction in convergence rate due to PU is compensated by the controlled increase of the step size in a frequency-dependent factor called gain in step size. The paper outlines the theoretical basis of the existence of this gain. Computational load of the global control strategy is compared with other adaptive algorithms. Finally, the paper presents experimental results measured in a practical in-vehicle implementation of the control system based on the DSP TMS320C6701.

1. INTRODUCTION

Apart from passive techniques based on the absorption and reflection properties of materials, acoustic noise reduction can be achieved by means of active noise control techniques based on the principle of destructive wave interference, whereby an antinoise is generated with the same amplitude as the undesired disturbance but with an appropriate phase shift in order to cancel the primary noise at a given location by means of secondary sources, generating a zone of silence around an acoustical sensor.

Taking into account that the characteristics of the undesired acoustic disturbance might be time-variant, it is necessary to put in practice adaptive control systems that can carry out the attenuation of noise regardless the fluctuation in power or frequency of the annoying noise [4].

The most popular adaptive algorithm used in DSP-based implementations of ANC systems is the filtered-x LMS algorithm, originally proposed by Morgan [5].

In ANC systems based on adaptive signal processing techniques, a trade-off has been traditionally established among computational load, convergence rate and mean-square error excess. Therefore, the improvement in any of the mentioned parameters is achieved at the expense of degrading the others.

This work tries to take advantage of the benefits of different adaptive algorithms overcoming their drawbacks and limitations by the combination of complementary strategies.

This paper outlines the theoretical gain in step size of the control strategy defined as the ratio between the upper bounds that ensure convergence in the following cases: first, when only a subset of the weights of a hierarchical filter [7] is updated during every iteration and, second, when every tap - regardless the position of the weight in the hierarchy of subfilters - is updated at every cycle. Here, it is important to remark that the degrees of freedom have been constrained by restricting the maximum step size at every level to the same value in order to visualize more easily the factor by which the step size parameter can be increased. The theoretical analysis of the strategy prevents from the use of certain frequencies corresponding to notches which appear in the gain in the step size of the adaptive algorithm. Their width and exact location depend on the length of the slowest subfilter of the hierarchy, the decimation factor and the sampling frequency.

Compared computational cost of different ANC algorithms are given in order to optimize the election of the dimensions of the hierarchical filter and the decimation factor so as to minimize the work load. In other words, the paper proposes a strategy aimed at making the most of every DSP operation involved in the control process.

Computer simulated results confirm the usefulness and feasibility of the proposed idea when this control strategy is used in active attenuation of periodic disturbances consisting of several harmonics - as engine noise can be considered to consist of -. Predicted theoretical gain in step size was successfully compared with the maximum affordable increase in step size obtained by simulation.

The modified filtered-x hierarchical sequential PU LMS algorithm with gain in step size (G_μ - Mod Fx H Seq LMS) has been implemented on a TMS320C6701 DSP-based in-vehicle ANC system. Experimental results achieved inside a van are shown.

2. MODIFIED Fx HIERARCHICAL SEQUENTIAL PU LMS ALGORITHM WITH GAIN IN STEP SIZE

It is well known that the modified filtered-x LMS algorithm [1], [3] makes use of an estimation of the primary noise $\tilde{d}(n)$ to properly swap the order between the secondary path $S(z)$ and the adaptive control filter $W_i^l(z)$ (see Figure 2.1). This secondary path is off-line modeled as $\tilde{S}(z)$. Then, a simultaneous copy of this control filter is used with the reference signal $x(n)$. In this way, the limitations imposed on the step size μ for the standard version of the filtered-x LMS algorithm are overcome in the modified one.

Moreover, the synthesis of an estimate of the undesired noise allows the hierarchical filter proposed by Woo [7] to count on a signal that is necessary to update the coefficients of every subfilter of the hierarchical adaptive controller $W_i^l(z)$, $\forall i \ \forall l$, $W_i^l(z)$ being the i^{th} subfilter on level l, where the level of the hierarchy l varies from 1 to α. Figure 2.2 shows the architecture of a 2-level hierarchical filter. The main idea behind the hierarchical LMS algorithm is to boost the convergence rate of the filter by organizing the taps into a hierarchy consisting of shorter subfilters that, therefore, can converge faster. The number of taps of each subfilter can vary from level to level. Let β_l denote the number of weights of a subfilter for level l. In fact the hierarchy does not even require that all the subfilters at the same level have the same number of taps. For simplicity, this last possibility was not considered in the hierarchical structure shown in Figure 2.2. An input to the subfilter at level l is equal to the output of the corresponding subfilters in the previous level (l – 1). At the last level α, there is only one subfilter. We use y_{ij}^l (or z_{ij}^l) and w_{ij}^l to denote, respectively, the input signal of the slave (or the adaptive) hierarchical filter, and the weight for j^{th} tap of the i^{th} subfilter at the l^{th} level; the error signal of the i^{th} subfilter at the l^{th} level is denoted as e_i^l.

The main drawback of the hierarchical LMS is the high work load inherently associated to its complex structure. In order to reduce the computational costs, sequential PU [2] of the coefficients of the hierarchical filter has been used. The sequential PU LMS algorithm with decimation factor N updates a subset of size L/N, out of a total of L coefficients - $w_l(n)$, $1 \le l \le L$ - per iteration according to

$$w_l(n+1) = \begin{cases} w_l(n) + \mu\, x(n-l+1)\, e(n) & \text{if } (n-l+1) \bmod N == 0 \\ w_l(n) & \text{otherwise} \end{cases} \quad [2.1]$$

where μ is the step size of the algorithm, $x(n)$ is the regressor signal, and e(n) is the error. This sequential PU was applied to every coefficient at every level, from the first tap of the first subfilter to the last tap of the last subfilter.

Convergence rate reduction due to PU is compensated by multiplying the step size by a factor called gain in step size [6], whose characteristics are outlined in Section 3.

The G_μ- Mod Fx H Seq LMS algorithm is given by:

$for\ \ k=1\ \ to\ \ \#\, iterations$

$L = \prod_{l=1}^{\alpha} \beta_l$

$\mathbf{y}^1_{\forall i \forall j}(n) = \mathbf{x}(n)$

$/*\ \ \mathbf{x}(n) = [x(n)\ \ x(n-1)\ \ \ldots\ \ x(n-L+1))]^T\ \ */$

$for\ \ l=1\ \ to\ \ \alpha \qquad /*\ \ Slave\ \ hierarchical\ \ filter\ \ */$

$for\ \ i=1\ \ to\ \ L \Big/ \prod_{r=1}^{l} \beta_r$

$y^{l+1}_{pq}(n) = \mathbf{w}^{l\ T}_{i \forall j}(n)\, \mathbf{y}^l_{i \forall j}\Big|_{p=\left\lceil \frac{i}{\beta_l} \right\rceil,\, q=i-\left\lfloor \frac{i-1}{\beta_l} \right\rfloor \beta_l}$

$end\ \ of\ \ \ for\ \ (i)$

$end\ \ of\ \ \ for\ \ (l) \qquad /*\ \ y(n) = y^{\alpha+1}_{11}(n)\ \ */$

$\tilde{y}(n) = \tilde{\mathbf{s}}^T(n)\, \mathbf{y}^{\alpha+1}_{11}(n)$

$/*\ \ \tilde{\mathbf{s}}(n) = [\tilde{s}_1\ \ \tilde{s}_2,\ \ldots\ \ \tilde{s}_{L_s}]^T\ \ */$

$/*\ \ \mathbf{y}^{\alpha+1}_{11}(n) = [y^{\alpha+1}_{11}(n)\ \ y^{\alpha+1}_{11}(n-1)\ \ \ldots\ \ y^{\alpha+1}_{11}(n-L_s+1))]^T\ \ */$

$e_m(n) = d(n) - y'(n)$

$\tilde{d}(n) = \tilde{y}(n) + e_m(n)$

$x'(n) = \tilde{\mathbf{s}}^T(n)\, \mathbf{x}(n)$

$/*\ \ \mathbf{x}(n) = [x(n)\ \ x(n-1)\ \ \ldots\ \ x(n-L_s+1))]^T\ \ */$

$\mathbf{z}^1_{\forall i \forall j}(n) = \mathbf{x}'(n)$

$/*\ \ \mathbf{x}'(n) = [x'(n)\ \ x'(n-1)\ \ \ldots\ \ x'(n-L+1))]^T\ \ */$

$for\ \ l=1\ \ to\ \ \alpha \qquad /*\ \ Adaptive\ \ hierarchical\ \ filter\ \ */$

$for\ \ i=1\ \ to\ \ L \Big/ \prod_{r=1}^{l} \beta_r$

$z^{l+1}_{pq}(n) = \mathbf{w}^{l\ T}_{i \forall j}(n)\, \mathbf{z}^l_{i \forall j}\Big|_{p=\left\lceil \frac{i}{\beta_l} \right\rceil,\, q=i-\left\lfloor \frac{i-1}{\beta_l} \right\rfloor \beta_l}$

$e^l_i(n) = \tilde{d}(n) - \mathbf{w}^{l\ T}_{i \forall j}(n)\, \mathbf{z}^l_{i \forall j}(n)$

$for\ \ j=1\ \ to\ \ \beta_l$

$if\ \ (k-((i-1)\beta_l + j)+1)\ \bmod\ N == 0$

$w^l_{i\,j}(k+1) = w^l_{i\,j}(k) + G_\mu\, \mu^l\, e^l_i(n)\, z^l_{i\,j}(n))$

$else$

$w^l_{i\,j}(k+1) = w^l_{i\,j}(k)$

$end\ \ of\ \ \ for\ \ (j)$

$end\ \ of\ \ \ for\ \ (i)$

$end\ \ of\ \ \ for\ \ (l) \qquad /*\ \ z(n) = z^{\alpha+1}_{11}(n)\ \ */$

end of for (k)

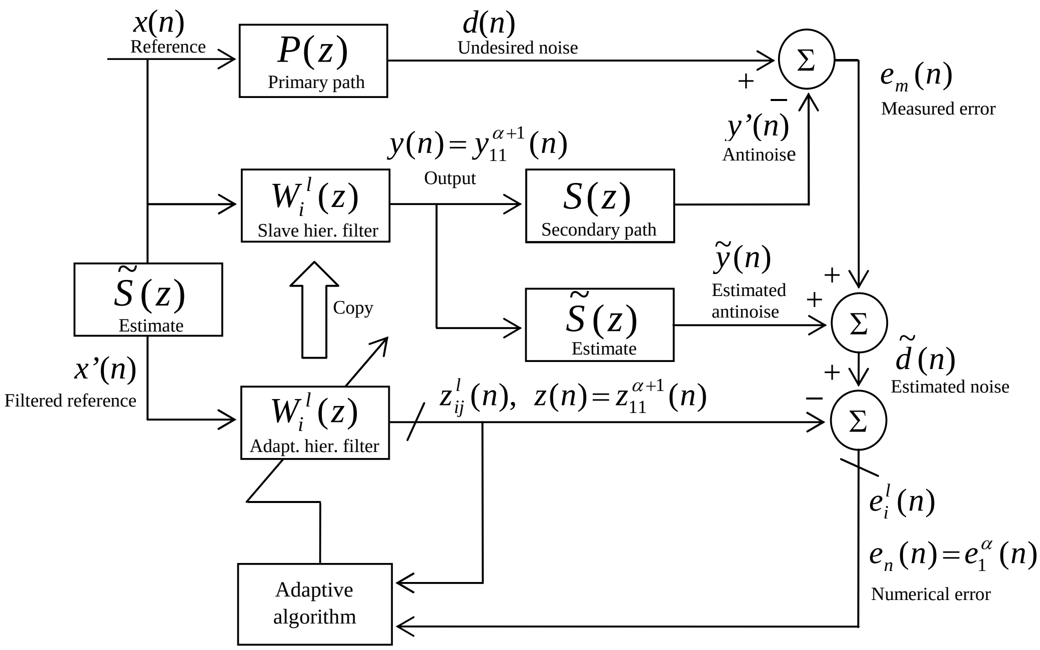

Figure 2.1. Block diagram of the modified filtered-x hierarchical LMS algorithm with sequential partial update.

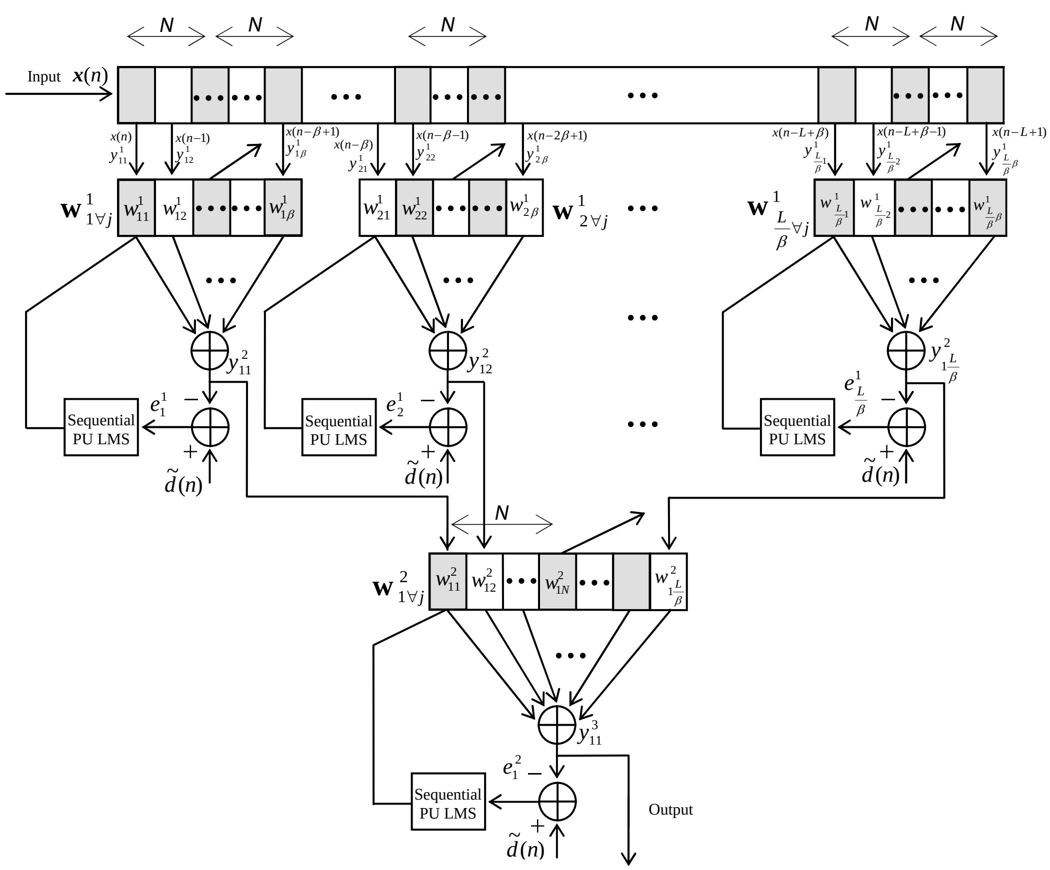

Figure 2.2. Hierarchical filter. Coefficients to be partially updated at first iteration are shadowed.

3. THEORETICAL BASIS

Theoretical derivation of the gain in step size for a simple filter whose coefficients are partially updated according to the sequential algorithm when the regressor signal is periodic can be find in [6]. The referred analysis is based on the determination of the ratio between the upper bounds that ensure convergence in two cases: first, when only a subset of the weights of the filter is updated during every iteration and, second, when the whole filter is updated at every cycle. In order to apply this idea to the hierarchical controller, it is essential to identify the bottle neck during the convergence process. Basically, the largest subfilter of the hierarchy - and therefore, the slowest - determines the maximum convergence rate of the hierarchy. This assumption is more evident if the largest subfilter is located at the first level of the hierarchy provided that the regressor signals of further levels are the output of the immediate previous levels and, consequently, these signals have been smoothed by the filtering.

So as to easily obtain the dependence of the gain in step size on the maximum length of a subfilter β and on the decimation factor N, let a single tone of normalized frequency f_0 be considered as the regressor signal

$$\mathbf{x}'(n) = \cos(2\pi f_0 n + \phi). \qquad [3.1]$$

If the gain in step size G_μ is defined as the ratio between the bounds on the maximum step sizes that ensure convergence of the slowest subfilter in both cases - $N > 1$ and $N = 1$ -, we obtain the factor by which the step size parameter can be multiplied when the adaptive algorithm uses PU. The maximum step size $\mu_{\max}$ is inversely bounded by the largest eigenvalue $\lambda_{\max}$ of the autocorrelation matrix of the regressor signal [4].

$$G_\mu(\beta, f_0, N) = \frac{\mu_{\max}^{N>1}}{\mu_{\max}^{N=1}} = \frac{\dfrac{2}{\lambda_{\max}^{N>1}}}{\dfrac{2}{\lambda_{\max}^{N=1}}} = \frac{\max\left\{\dfrac{1}{4}\left[\beta \pm \dfrac{\sin(\beta 2\pi f_0)}{\sin(2\pi f_0)}\right]\right\}}{\max\left\{\dfrac{1}{4}\left[\left\lceil\dfrac{\beta}{N}\right\rceil \pm \dfrac{\sin\left(\left\lceil\dfrac{\beta}{N}\right\rceil 2\pi N f_0\right)}{\sin(2\pi N f_0)}\right]\right\}}. \qquad [3.2]$$

If β/N were not integer, this ratio must be rounded to the nearest integer towards infinity to determine the length of the "logical" subfilter effectively updated [6].

In the former analysis, it has been assumed that $\beta > N$, that is, that at least 2 weights of the subfilter are updated per iteration. This condition is necessary to apply the idea of the "logical" subfilters presented in [6].

This results can be extrapolated to a regressor signal consisting of several harmonics.

To end this section, a comparison between the theoretical gain in step size with the affordable increase in step size obtained by MATLAB simulation is carried out. The model of this example corresponds to the 1 × 1 × 1 arrangement, that is 1 reference sensor, 1 error microphone and 1 secondary source. The first level of the hierarchical filter consists of 384 coefficients organized in 16 subfilters of 24 taps. In the second and last level of the hierarchy one subfilter of 16 weights can be found. In this example, the reference is a single sinusoidal signal whose frequency varied in 41.6 Hz steps from 41.6 to 4000 Hz. The sampling frequency of the model is 8000 samples/s. Primary and secondary paths - $P(z)$ and $S(z)$ - are pure delays of 50 and 12 samples, respectively. The output of the primary path is mixed with additive white Gaussian noise providing a signal-to-noise ratio of 27 dB. It is assumed that the secondary path has been exactly estimated. In order to provide very accurate results, the increase in step size between every two consecutive simulations looking for the bound is less than 10^{-3} the final value of the step size that ensures convergence. The decimation factor N of this example was set to 3. Figure 3.1 compares the predicted gain in step size with the achieved results. As expected from the analysis of the convergence of the 24-tap length subfilter of the first level, the experimental gain in step size is 3, apart from the notches that appear at 1333.3 Hz and 2666.6 Hz [6].

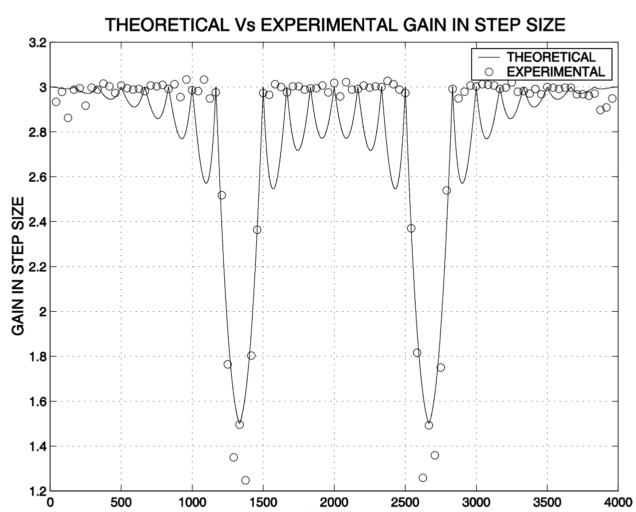

Figure 3.1. Theoretically predicted gain in step size vs. simulated results achieved in a modeled ANC system using the G_μ-Mod FxHLMS + Seq PU algorithm.

4. COMPUTATIONAL COMPLEXITY

This section is devoted to compare the computational complexity of the global strategy proposed with simpler control algorithms. For the sake of simplicity, it has been assumed that the length of every subfilter is β, regardless its level in the hierarchy. L is the total number of taps at the first level of the adaptive hierarchical filter, $L = \beta^\alpha$, with α being the total number of levels. L_s is the length of the off-line estimate of the secondary path. N is the decimating factor used in the sequential partial update of the coefficients.

Table 4.1 shows the number of multiplies required per a single iteration of the modified filtered-x hierarchical LMS algorithm with sequential PU in the different tasks.

Table 4.2 compares the total number of multiplications required per iteration when different control algorithms are used.

TASK	# MULTIPLIES
Computing output of slave filter	$\sum_{l=1}^{\alpha} \frac{L}{\beta^l} \beta$
Filtering of the reference signal with the estimate of the secondary path	L_s
Filtering of the output of the slave filter with the estimate of the secondary path	L_s
Computing output of the adaptive filter	$\sum_{l=1}^{\alpha} \frac{L}{\beta^l} \beta$
Partial update of the coefficients	$\sum_{l=1}^{\alpha} \frac{L}{\beta^l} \left(\frac{\beta}{N} + 1 \right)$

Table 4.1. Computational complexity of the modified filtered-x hierarchical LMS algorithm with sequential partial update in terms of the average number of multiplies per task and iteration.

Algorithm	# TOTAL MUTIPLIES
FxLMS	$2L + L_s + 1$
Mod FxLMS	$3L + 2L_s + 1$
Mod FxHLMS	$\left[\sum_{l=1}^{\alpha} \frac{L}{\beta^l} (3\beta + 1) \right] + 2L_s$
Mod FxHLMS + Seq PU	$\left[\sum_{l=1}^{\alpha} \frac{L}{\beta^l} \left(2\beta + \frac{\beta}{N} + 1 \right) \right] + 2L_s$

Table 4.2. Compared computational complexity of different ANC algorithms in terms of the average number of multiplies per iteration.

Figure 4.1 visually compares the computational load of the algorithms with increasing decimation factors. The choice of the parameters corresponds to a feasible DSP-board based experimental implementation of the strategy inside a van. L, was set to 512 taps, organized in 32 subfilters of β = 16 weights. At the second and last level (α = 2) there is one subfilter of β = 32 taps. The length of the off-line estimate of the secondary path was set to L_s = 64. The decimation factor N varies from 1 to 128.

From Figure 4.1, it can be concluded that it is not necessary to use a very large decimation factor to reduce the computational cost of the modified filtered-x hierarchical LMS algorithm to the order of the of work load of the FxLMS by updating just a subset of the taps of the hierarchical filter per iteration. The computational load for the commonly used values of the decimation factor N = 8 and N = 16 are marked with two circles over the curve.

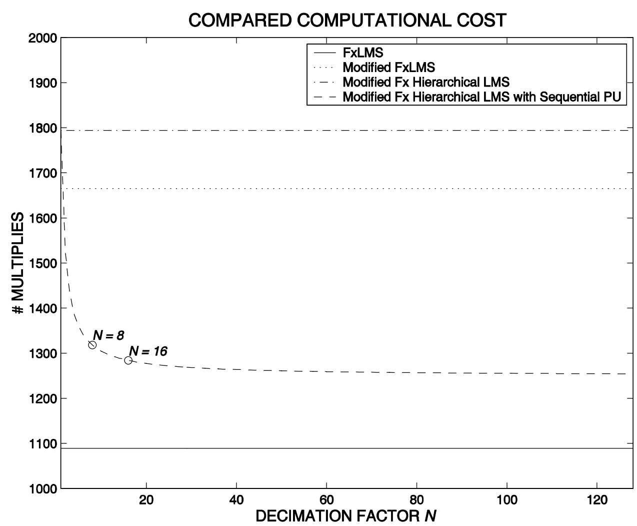

Figure 4.1. Compared computational complexity of different ANC algorithms in terms of the average number of multiplies per iteration. Only the sequential PU algorithm depends on the decimation factor N.

5. EXPERIMENTAL RESULTS

The G_μ - Mod Fx H Seq LMS algorithm was practically put in practice in a two independent channel implementation of an ANC system placed at the front seats of a Nissan Vanette. Two error microphones are located near the head of the driver and the passenger. Low cost microphones and loudspeakers with poor response at low frequencies were used. The distance between a microphone and its respective secondary source in this experiment of local minimization of noise is 6 cm. The main Digital Signal Processor board employed to develop the strategy was the PCI/C6600, based on the DSP TMS320C6701. The selected Input/Output board was the 4 input / 4 output PMCQ20DS board.

The number of taps at first level of the 2-level hierarchy, was set to L = 384 taps, organized in 16 subfilters of β = 24 weights. At the second level there is one subfilter of β = 16 taps. The secondary path was modeled with L_s = 200 taps. The decimation factor N was set to 1.

Performance comparison with a simple FxLMS with 384 (24 x 16) taps was carried out. It is verified that convergence rate of the Mod Fx H LMS algorithm is faster than that of the FxLMS. If necessary, the computational cost of G_μ - Mod Fx H Seq LMS algorithm can be reduced by increasing the decimation factor N.

Figure 5.1 shows the measured learning curves at the position of the driver (Mod Fx HLMS) and front passenger (FxLMS) when the ANC system is dealing with the attenuation of an acoustic disturbance consisting of harmonics at 100, 150, 200, 250, 300, and 350 Hz. This multi tone signal was previously generated and an omnidirectional source Brüel & Kjaer Omnipower 4296 placed inside the van was fed with it. This source acted as the origin of the primary noise.

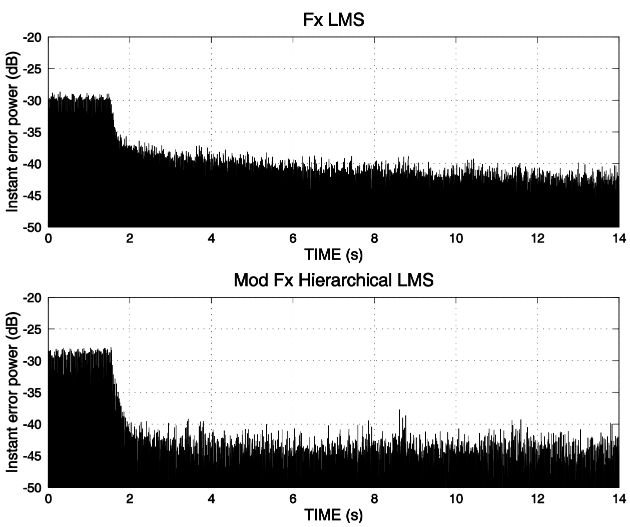

Figure 5.1. Learning curves measured inside the van. ANC system attenuating an acoustic disturbance consisting of harmonics at 100, 150, 200, 250, 300, and 350 Hz. a) FxLMS (front passenger position), b) G_μ - Mod Fx H Seq LMS, N = 1 (driver position).

6. CONCLUSIONS

This work shows the experimental results achieved in the attenuation of periodic disturbances by means of an ANC system based on the combination of different adaptive algorithms.

So as to boost up the convergence rate, the modified filtered-x and the hierarchical LMS algorithms have been used. In so doing, the increase of the computational load turns out to be the main drawback. To reduce the number of operations required per cycle, the sequential PU algorithm has been applied. Finally, the existence of an affordable increase in step size - or gain in step size - allows the global control strategy to compensate the lack of adaptation of most of the coefficients, even when the number of operations per iteration is significantly reduced due to PU. This gain is limited by the slowest subfilter of the hierarchical architectute.

To sum up, this strategy results in a fast algorithm with a computational complexity very close to the conventional FxLMS.

In order to assess the effectiveness of the modified filtered-x hierarchical sequential LMS algorithm with gain in step size in the context of active noise control systems focused on the attenuation of periodic disturbances, the proposed strategy was evaluated and compared in a practical DSP-based implementation inside a van. Simulated and experimental results validate the strategy on condition that frequencies placed at notches in the gain of step size should be avoided.

7. ACKNOWLEDGEMENTS

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