VISUALIZATION OF THE HILBERT SPECTRUM

Steven Sandoval

Arizona State University
School of Elect., Comp.
and Energy Eng.
Tempe, AZ, U.S.A.
spsandov@asu.edu

Phillip L. De Leon

New Mexico State University
Klipsch School of Elect.
and Comp. Eng.
Las Cruces, NM, U.S.A.
pdeleon@nmsu.edu

DEMO PROPOSAL

In recent work, we presented mathematical theory and algorithms for time-frequency analysis of non-stationary signals [1, 2]. In that work, we generalized the definition of the Hilbert spectrum by using a superposition of complex Amplitude Modulated–Frequency Modulated (AM–FM) components

$$z(t) \equiv \sum_{k=0}^{K-1} \psi_k \left(t; a_k(t), \omega_k(t), \phi_k\right) \tag{1}$$

assuming only the real part of the signal is observed

$$x(t) = \Re\{z(t)\}. \tag{2}$$

where the AM–FM component is defined as

$$\psi_k(t; a_k(t), \omega_k(t), \phi_k) \equiv a_k(t) \exp\left\{ j \left[\int_{-\infty}^{t} \omega_k(\tau) d\tau + \phi_k \right] \right\} \tag{3a}$$

$$= a_k(t) e^{j\theta_k(t)} \tag{3b}$$

$$= s_k(t) + j\sigma_k(t) \tag{3c}$$

and is parameterized by the Instantaneous Amplitude (IA), $a_k(t)$, Instantaneous Frequency (IF), $\omega_k(t)$, and phase reference, ϕ_k.

Although time-frequency analysis has been extensively studied and other AM–FM models have been proposed, the use of a generalized AM–FM model for this analysis, without the harmonic correspondence condition and other limiting conditions, has never been proposed. Using this model leads to *non-unique signal decompositions*. However, by imposing assumptions/constraints on the form of the AM–FM component, a unique parameterization in terms of IA and IF can be obtained. Using our Hilbert Spectral Analysis (HSA) approach, the IA and IF estimates can be far more accurate at revealing underlying signal structure than prior approaches to time-frequency analysis. By using HSA, we gain a degree of freedom in our analysis that may be more useful in describing the underlying physical phenomena.

We have also proposed a novel 3-D visualization of the Hilbert spectrum by plotting $\omega_k(t)$ vs. $s_k(t)$ vs. t as a line in a 3-D space and coloring the line with respect to $|a_k(t)|$ for each component, the simultaneous visualization of multiple parameters for each component is possible. Further, orthographic projections yield common plots: the time-real plane (the real signal waveform), the time-frequency plane (2-D Hilbert spectrum), and the real-frequency plane (analogous to the Fourier magnitude spectrum). The ability to visualize and interpret model parameters is key to the adoption of any analysis method. Often complex AM–FM signals are plotted as a series of real 2-D plots, i.e. $s_k(t)$ vs. t, which for AM–FM components, would provide little insight into the underlying signal model. A more appropriate visualization plots the Hilbert spectrum in its entirety. The proposed method allows one to visualize the assumed underlying signal model.

In this demo, we propose to demonstrate the advantages of the proposed framework and, more specifically, the proposed visualization for acoustic modeling applications using 3-D visualizations and other animations. In particular, we will demonstrate the advantage of utilizing a modeling framework that allows non-unique signal decompositions. We feel that the ASRU demo format is particularly well suited for such a material; the vivid 3-D color illustrations/animations that will be presented allow for a lucid comprehension of the HSA signal models but are not well suited for other presentation formats such as printed articles or posters. Specifically, we intend to provide 3-D HSA plot for: 1) basic narrowband components; 2) wideband components; 3) four different parameterizations of a triangular waveform 4) two parameterizations of a sinusoidal FM waveform 5) two parameterization of the beat phenomenon. In addition, we have several 2-D HSA plots corresponding to real-world signals, and several 3-D Argand diagrams that can be presented.

An example of the visualizations we indent to present, we have included links to two downloadable supplemental enclosures (Enclosure 1 and Enclosure 2). We did not supply the enclosures as email attachments due to the large file size. In order to properly view the embedded media content use a PDF reader with Flash support (e.g, Adobe Reader with Flash plug-in). We note that other PDF readers (e.g., Preview in Mac OS and PDF Viewer in Chrome) will not properly display the embedded media. The media contains low frequency audio, please listen using headphones or high quality speakers or this audio may not be audible. For best results view the files in full screen mode. Enclosure 1 is relatively straighforward and shows 3-D HSA visualizations for several basic narrowband components, including a Simple Harmonic Component (SHC), two AM components, two FM components, an AM–FM component, and a multi-component signal. Enclosure 2 shows an example that highlights the importance of a model framework that allows non-unique signal decompositions. Consider the two following paramerizations of a real signal

$$x(t) = \Re\left\{\exp\left(j\omega_a t\right) + \exp\left(j\omega_b t\right)\right\} \tag{4a}$$
$$= \Re\left\{2\cos\left[\left(\omega_b - \omega_a\right)t/2\right]\exp\left[j\left(\omega_b + \omega_a\right)t/2\right]\right\}. \tag{4b}$$

The left side of Enclosure 2 shows a model consisting of two SHCs as in (4a) and the right side of Enclosure 2 shows a model consisting of one AM component as in (4b). As is well known, when ω_a and ω_b are closely-spaced, the human auditory system perceives a single AM tone rather than two distinct tones, as is shown in Equation (4a) [3]. On the other hand, if ω_a and ω_b are not sufficiently far apart, the human auditory system perceives two distinct tones rather than a single AM tone, as is shown in Equation (4b). Both decompositions are equally valid models for the real signal $x(t)$—the resulting decompositions simply correspond to the different assumptions of the underlying components, i.e. simple harmonic components and AM components, respectively. This results in different instantaneous parameterizations despite both models producing the same $x(t)$ in (2) and ultimately, results in different Hilbert spectral representations based on different assumptions of the underlying components. The difference between the two models is easily discernible using the proposed the 3-D visualization and highlights the importance of visualization for interpreting a signal model in acoustic modeling.

1. REFERENCES

[1] S. Sandoval and P. L. De Leon, "Theory of the Hilbert spectrum," *arXiv*, Apr. 2015, math.cv/1504.07554.

[2] S. Sandoval, P. L. De Leon, and J. M. Liss, "Hilbert spectral analysis of vowels using intrinsic mode functions," *IEEE Automatic Speech Recognition and Understanding Workshop*, Dec. 2015.

[3] D. O'Shaughnessy, *Speech Communications: Human and Machine*, Addison-Wesley, 1987.